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SECTIONB: HEAT AND THERMO DYNAMIC

CHAPTER1: THERMOMETRY

Heat is the amount of energy which moves from hotter to colder region.

Temperature is a number that expresses the degree of hotness of a body on a given scale. Temperature is measured using a thermometer which has a scale on it. Thermometers use a physical property which is called thermometric property to measure temperature.

Definition

A thermometric property is a physical property which varies linearly and continuously with temperature.

1.1: QUALITIES OF A GOOD THERMOMETRIC PROPERTY

- It should vary linearly and continuously with temperature
- It should vary over a wide range of temperature
- It should correspond to a unique or single value of temperature
- It should respond to small unit temperatures (Sensitive)

TYPES OF THERMOMETERS AND THEIR THERMOMETRIC PROPERTY

Thermometer	Thermometric property
Liquid in glass	Length L of liquid column
Thermocouple	E.M.F " E"
Resistance eg Platinum	Electrical resistance "R" of a wire
Constant Volume gas	Pressure "P" of a fixed mass of a gas
Constant pressure gas	Volume "V" of afixed mass of a gas
Pyrometer	Wavelength λ(quality)

1.1.0: FIXED POINT

This is temperature at which a substance changes states under specific conditions.

1.1.1: ICE POINT

Ice point is temperature at which pure ice can exist in equilibrium with water at standard atmospheric pressure of 760mmHg

1.1.2: STEAM POINT

This is temperature at which pure water can exist in equilibrium with vapour at standard atmospheric pressure (760mmHg). Steam point corresponds to 100℃

1.1.3: TRIPLE POINT OF WATER

This is a unique temperature at which pure ice, pure water and pure water vapour can exist together in equilibrium.

The triple point of water is chosen at fixed point and is defined as 273.16 K.

1.2.0: TEMPERATURE SCALES

This is a scale used to measure the degree of hotness of the body basing on a thermometric property.

1.2.1: TYPES OF TEMPERATURE SCALE

Centigrade or Celsius temperature scale

Kelvin or absolute temperature or abnormal or thermodynamic temperature scale

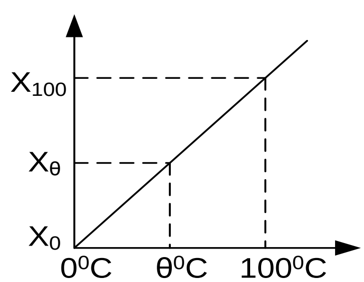
1.2.2: CENTIGRADE/ CELSIUS TEMPERATURE SCALE

Is a temperature scale which uses ice point (0^0C) as it lower fixed point and steam point (100^0C) as its upper fixed point

1.2.3: STEPS IN SETTING UP CELSIUS TEMPERATURE SCALE

- Choose a thermometric properly of substance and let it be X
- Measure the value of the property at ice point, steam point and let values be X_0, X_{100} respectively.
- Measure the value of the properly at unknown temperature θ and let it be X_θ

Plot a graph of property value against temperature.



Slope $=\frac{\Delta y}{\Delta x}$ $\qquad \frac{X_{100}-X_\theta}{100-0}=\frac{X_\theta-X_0}{\theta-0}$ $\qquad \theta=\left(\frac{X_\theta-X_0}{X_{100}-X_0}\right)$ $X100^0C$

Equation above is a defining equation of Celsius scale of temperature

Note: a Celsius scale of temperature can also be defined basing on other thermometric properties

Thermo couple

$\theta=\left(\frac{E_\theta-E_0}{E_{100}-E_0}\right) X100^0C$

Platinum resistance

$\theta=\left(\frac{R_\theta-R_0}{R_{100}-R_0}\right) x100^0c$

Constant volume gas

$\theta=\left(\frac{P_\theta-P_0}{P_{100}-P_0}\right) x100^0c$

Constant pressure gas

$\theta=\left(\frac{V_\theta-V_0}{V_{100}-V_0}\right) x100^0c$

Liquid in glass

$\theta=\left(\frac{L_\theta-L_0}{L_{100}-L_0}\right) x100^0c$

1.2.4: KELVIN TEMPERATURE SCALE

This is a temperature scale which uses triple point of water as a fixed point.

Steps to establish Kelvin scale

- ✓ Select thermometric property X of substance
- ✓ Measure the property at triple point of water, let it be X_t
- ✓ Measure the property at un known temperature θ, let it be X_θ
- ✓ Assuming a linear variation of X with temperature then the un known temperature be determined from

$$\boxed{\theta = \frac{X_\theta}{X_t} x273.16K}$$

1.2.5: DISAGREEMENT OF TEMPERATURE SCALES

Different thermometers give different readings when measuring temperature of the same body except at fixed points where they must agree and this is because different thermometric properties vary differently with temperature.

Example

1) A resistance thermometer has a resistance of 21.42Ω at ice point, 29.10 Ω at steam point and 28.11 Ω at un known temperature θ. calculate θ on scale of this thermometer.

Solution

R_0=21.42 Ω, R_{100}=29.10 Ω

R_θ=28.11 Ω θ=?

θ= $\left(\frac{R_\theta - R_0}{R_{100} - R_0}\right)$ x100^0C

θ= $\left(\frac{28.11-21.42}{29.10-21.42}\right) x100°C$

θ= $\left(\frac{6.69}{7.68}\right)$ x100^{0}c

θ=87.11^0C

2) The resistance of the wire is measured at ice point, steam point and at un known temperature θ and the following values were obtained 2.00Ω, 2.48Ω, 2.60Ω respectively. Determine θ

R_0=2.00Ω

R_{100}=2.48Ω

R_0=2.60 Ω

θ= $\left(\frac{R_\theta - R_0}{R_{100} - R_0}\right)$ x100^0C

θ= $\left(\frac{2.60-2.00}{2.48-2.00}\right)$ x100

θ = $\left[\frac{0.6}{0.48}\right]$ x100^{0}c

θ =125°C

3) The length of mercury column is 2.00cm at ice point, 2.73cm at steam point.

(i) What temperature on the mercury in glass thermometer corresponds to the value of 8.43cm?

(ii) When the above temperature is measured on gas thermometer scale it correspond to a value of 1020^{0}C.Explain the discrepancy

Solution

i) L_θ=2.00

L_θ=8.43

L_{100}=2.73

θ= $\left(\frac{L_\theta - L_0}{L_{100} - L_0}\right)$ x100^0C

θ= $(\frac{8.43-200}{2.73-2.00})$ x100^0C	θ= $(\frac{6.43}{0.73})$ x100	θ= 880.8^0C

(ii) This is because length varies differently from the way pressure of fixed mass of gas varies with temperature.

4) A particular resistance thermometer has resistance of 30Ω at ice point, 41. 58Ω at steam point and 34.58Ω when immersed in a boiling liquid. A constant volume gas thermometer gives readings, 1.333x10^{5}Pa, 1.821x10^{5}Pa and 1.528x10^{5}Pa at the same temperatures. Calculate the temperature at which the liquid is boiling on scale of;

(i) Resistance thermometer (ii) Gas thermometer .

Solution

i) R_0=30 Ω

R_θ=31.58 Ω

R_{100}=41.58 Ω

θ= $(\frac{R_\theta - R_0}{R_{100}-R_0})$ x100℃

θ= $[\frac{34.58-30}{41.58-30}]$ x100℃

θ=39.55℃

ii) P_θ=1.333x10^{5}Pa

P_{100}=1.821X10^{5}Pa

P_θ=1.628x10^{5}Pa

θ= $(\frac{P_\theta - P_0}{P_{100}-P_0})$ x100℃

θ= $(\frac{1.528x10^5-1.333x10^5}{1.821x10^5-1.333x10^5})$ x100℃

θ= $[\frac{19500}{48800}]$ x100℃

θ= 39.959℃

Example on triple point of water or Kelvin scale

5) Pressure recorded by constant volume thermometer at Kelvin temperature T is given by 4.8x10^{4}Nm^{-2}. Calculate T if the pressure at triple point of water is 4.2x10^{4}Nm^{-2}

Solution

T=$\frac{P_\theta}{Pt}$x273.16K P_θ=4.8x10Nm^{-2}	P_t=42x10^{4}Nm^{-2} T=$\frac{4.8x10^4}{4.2x10^4}$x273.16K	T= 312.18K

6) The resistance of platinum wire at triple point of water is 5.16Ω. what will be the resistance at 100℃

Solution

T= $\frac{R_\theta}{R_t}$x273.16K θ= 100^0C T=(273+100)=373K	R_θ=? R_t=5.16 Ω 373=$\frac{R_\theta}{5.16}$X273.16	R_θ=$\frac{1924.6}{273.16}$ R_θ= 7.045Ω

Determining temperature on a scale of one thermometer as read by another
UNEB 2007 Q 5c

1) The resistance, R_θ of a particular resistance thermometer at Celsius temperature θ as measured by a constant volume gas thermometer is. R_θ=50+0.17θ+3x10^{-4} θ^2 Calculate the temperature as measured on a scale of a resistance thermometer which corresponds to a temperature of 60℃ at a gas thermometer.

Solution

θ= $(\frac{R_\theta - R_0}{R_{100} - R_0})$ x100℃

R_θ=50+0.17θ+3x10^{-4} θ^2

When θ= 0℃

R_0=50+0.17x0+3x10^{4}x0^2

When θ= 100℃

R_{100}=50+0.17x100+3x10^{-4}x100^2

R_{100}= 70Ω

When θ= 60^{0}C

R_{60}=50+0.17x60+3x10^{-4}x60^2

R_{60}=61.28Ω

θ= $(\frac{R60-R0}{R100-R0})$ x 100℃

θ= $(\frac{61.28-50}{70-50})$ x100℃

θ= 56.4℃

2) The value of property X of certain substance Xt is given by $X_{t=}X_0$+0.5t+2x10^{-4}t^2 Where t=temperature in ℃ measured in gas thermometer scale. What will be the Celsius temperature at 50℃ on this thermometer scale?

Solution

X_t=X_0+0.5t +2x10^{-4}t^2

X_{100}=X_0+52

X_0=X_0

X_{50} =X_0+25.5

θ= $(\frac{X_{50}-X_0}{X_{100}-X_0})$ x100℃

θ= $(\frac{X0+25.5-X0}{X0+52-X0})x100℃$

θ= $(\frac{25.5}{52})$ x100℃

θ= 49.04℃

3) The resistance of the wire as measured by gas thermometer varies with temperature θ according to the equation. R_θ=R_0 (1+50αθ+200αθ^2).Determine temperature on resistance thermometer that corresponds to 40℃on the gas scale

Solution

R_θ=R_0 (1+50αθ+200αθ^2)

R_{100}= R_0 (1+50xαx100+200α100^2)

R_{100} =[1+α(2005000)]

R_0=R_0

R_{40}= R_0 (1+50αx40+200xα40^2)

R_{40}= R_0 [1+α (322000)]

θ= $[\frac{R40-R0}{R100-R0}]$ x100℃

θ= $(\frac{\mathrm{R0}[1+\alpha(322000)]-\mathrm{R0}}{\mathrm{R0}[1+\alpha(2005000)]\mathrm{R0}})$

θ= $[\frac{322000}{2005000}]$ x100℃

θ= 16.059℃

Exercise 1

1) The resistance of the element in a platinum resistance thermometer is 6.75Ω at triple point of water and 7.166Ω at room temperature. What is the temperature of the room on a scale of resistance thermometer?.state one assumption you have made. **An[290K]**

2) The resistance of platinum wire is 4Ω at ice point and 5.6Ω at steam. Find the temperature at which the resistance is 9.84Ω
3) A particular constant –volume gas thermometer registers a pressure of 1.937x10^4Pa at the triple point of water and 2.618x10^4Pa at the boiling of a liquid. What is the boiling point of the liquid according to this thermometer?**An[369.2K]**

1.3.0: TYPES OF THERMOMETERS

a)-Liquid in glass thermometer;

it uses a thermometric property which is, length of the column of liquid to measure the degree of hotness of the body. Liquids usually used are; mercury, alcohol and water.

Advantages of a Liquid in Glass Thermometer

- It is easy to use
- It is very cheap
- It is very portable
- It has direct readings

Disadvantages of a Liquid in Glass Thermometer

- It has small range of temperature
- It is not very accurate

N.B:

A liquid in glass thermometer is not very accurate because of the following;

1. Parallax errors which contribute about $\pm 0.1°C$
2. Non uniformity of the bore of capillary tube limits accuracy to about 0.1°C
3. The glass contracts and expands and takes long hours to recover its correct size and shape and therefore spoils the calibration

Reasons why mercury is used as thermometric property .

- It doesn't wet the glass
- It is opaque
- It expands uniformly
- It is a good conductor of heat

Reasons why water is not used as thermometric property

- It wets the glass
- It is not opaque
- It is a bad conductor of heat
- It is has non uniform expansitivity.

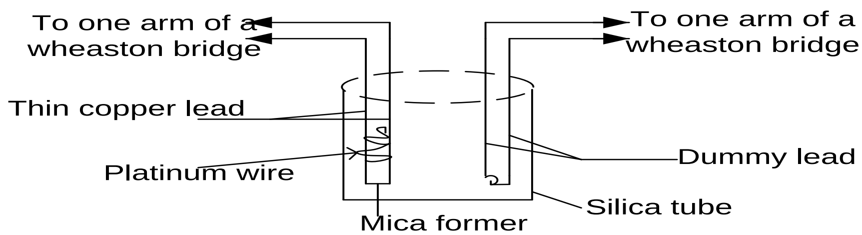
b)-RESISTANCE THERMOMETER [PLATINUM RESISTANCE THERMOMETER]

A resistance thermometer uses resistance(R) of a metal wire as a thermometric property.

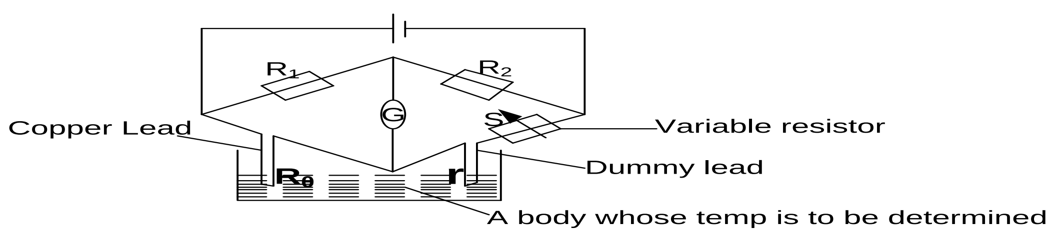
QUALITIES OF A METAL TO BE USED IN A RESISTANCE THERMOMETER

- Material of the wire should have a high temperature co-efficient of resistance (R) so that a small change in temperature causes a measurable change in resistance.
- The variation of resistance with temperature should be linear. Platinum is chosen to be used because it satisfies above 2 conditions.

STRUCTURE OF PLATINUM RESISTANCE THERMOMETER



- Platinum wire is wound on a mica former and put into silica tube. In the same tube dummy leads are also placed in as shown above. They are transferred and connected to Wheatson Bridge as shown below.



- The thermometer is connected in such a way that resistors R_1 and R_2 are equal and using variable a resistor S, a circuit is balanced until galvanometer shows no deflection.
  i.e $I_g=0$

A t balance point $\frac{R_1}{R_2}=\frac{R_\theta+r}{S+r}$

But in practice $R_1=R_2$

$R_\theta + r = S + r$

$R_\theta=S$

- The resistance of platinum wire is measured at ice point (R_0), at steam point (R_{100}) and at un known temperature (R_θ). The temperature of the body can be determined from

$$\theta= \left(\frac{R_\theta-R_0}{R_{100}-R_0}\right) \times 100^0C$$

ADVANTAGES OF PLATINUM RESISTANCE THERMOMETER

- It is used for measuring small unit temperature.

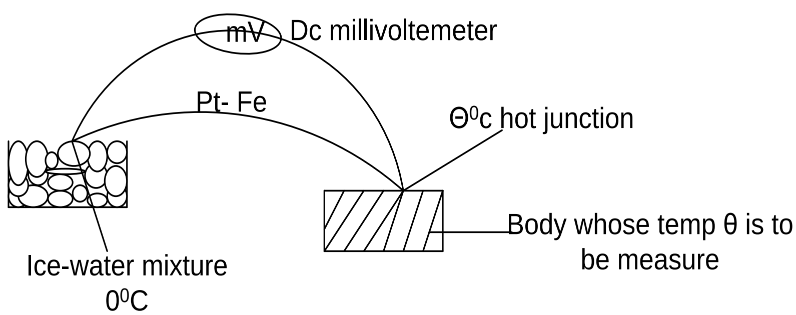
- It is very accurate. It is because the resistance of platinum wire varies linearly with temperature.
- It has a wide range of temperature i.e. from -200℃ to 1200℃
- It is very sensitive to small unit temperatures.

DISADVANTAGES OF PLATINUM RESISTANCE THERMOMETERS

- It cannot measure very rapidly changing temperature. This is because it has low - thermal conductivity and high heat capacity.
- It cannot measure temperature at a point due to size of silica tube.

C) -THERMO COUPLE THERMOMETER

When two wires of different materials are joined together to form two junctions and their junctions maintained at different temperatures, a small E.M.f is created between the junctions. These effects is called thermo electric or **see beck effect** and such an arrangement gives a thermo couple.



MODE OF OPERATION

- One junction is placed on the water-ice mixture and another on the body where temperature is to be measured.
- An $E.m.f$ "E" is created between the two junctions and it is measured by D.C milli-voltemeter.
- The $E.m.f$ is measured at ice point (E_0), steam point (E_{100}) and at unknown temperature θ (E_θ).
- The temperature of the body can then be calculated from

$$\theta = \left(\frac{E_\theta - E_0}{E_{100} - E_0}\right) X100^0C$$

ADVANTAGES OF THERMO COUPLE

- It measure temperature at a point e.g. temperature of crystal since the wires can be made thin.

- It is used to measure rapidly changing temperatures. This is because of its small heat capacity and high thermal conductivity.
- It is portable
- It has a wide range of temperature between -250℃ to 1600℃ and this can be achieved by using different metals.
- It can be used to determine direct readings if connected to galvanometer which has been calibrated to read temperatures directly.

DISADVANTAGES OF THERMO COUPLE

- It cannot measure slowly changing temperatures.
- It is inaccurate because $E.m.f$ doesn't vary linearly with temperature.

N.B an $E.m.f$ can be generated from junction if.

- ✓ The junctions are made from different metals.
- ✓ The junctions are kept at different temperatures.

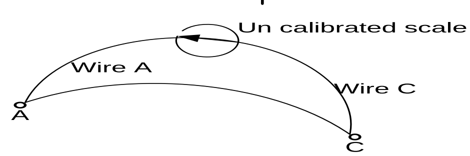
CALIBRATION OF THERMO COUPLE

Question:

Describe briefly how you would calibrate thermo couple and use it to measure the temperature of water.

Solution

Consider un calibrated thermo couple

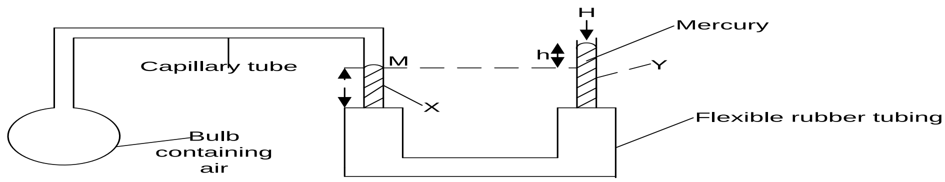


Let junction C and A be placed in water mixture and determine E_0 using m.v.

Divide the interval into two equal parts to complete the calibration.
The temperature of water is measured by placing junction A in it and C in the water mixture.
The temperature of water θ is determined from.

$$\theta = \left(\frac{E_\theta - E_0}{E_{100} - E_0}\right) \times 100^0 C$$

d)-CONSTANT VOLUME GAS THERMOMETER



- The bulb is immersed in the medium e.g. the liquid whose temperature is required.
- As the temperature increases the pressure of gas increases pushing the mercury down in the limb X and up in limb Y.
- By raising Y the mercury level in X is restored to the reference mark M and the volume is kept constant.
- The pressure of gas $P = H + h$
- The pressure is measured at the point P_0, at steam point P_{100}, and at un known temperature P_θ

$$\theta = \left(\frac{P_\theta - P_0}{P_{100} - P_0}\right) \times 100^0C$$

ADVANTAGES OF CONSTANT VOLUME GAS THERMOMETER

- It is very sensitive
- It has wide range of temperature from-270℃ to 1500℃
- It is very accurate since the pressure of fixed mass of gas at constant volume varies linearly with temperature.
- It is used as a standard to calibrate other thermometer e.g. thermo couple thermometer.

DISADVANTAGES OF CONSTANT VOLUME GAS TERMOMETER

- It is bulky i.e. is not portable.
- It has no direct readings, therefore it requires skills to be read it.
- It cannot measure rapidly changing temperatures as the bulb needs time to reach steady states.

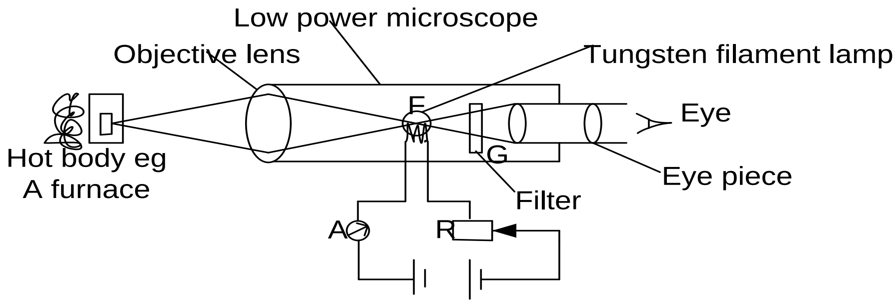
e)-PYROMETERS

They are used to measure very high temperatures e.g. temperature of furnace

They are divided into two;

- Total radiation pyrometer which responds to total radiation i.e. heat and light produced.
- Optical radiation pyrometer which responds to only light produced.

OPTICAL RADIATION PYROMETER



MODE OF OPERATION

- A hot body whose temperature is to be measured is focused by objective lens so that its image lies in the plane of F.
- The light from both the filament and the body pass through filter G to the eye.
- If the body is hotter than filament, the filament appears dark on bright background.
- If the filaments is hotter than the body, the filament appears bright on a dark background.
- Using the reheostat R, the current through filament is adjusted until the filament cannot be distinguished in the background. At that point, the temperature of hot body is then equals that of the filament. And this temperature can then be read from the ammeter (previously calibrated in ℃).

UNEB 2011 Qn 5

(b) (i)Define the term thermometric property and give four examples (02mk)

(ii) State two qualities of a good thermometer property (01marks)

(c) (i)With reference to the a liquid in glass thermometer, describe the steps involved in setting up a Kelvin scale of temperature (03marks)

(ii) State one advantage and disadvantage of the resistance thermometer. (01mk)

(d)A resistance thermometer has a resistance of 21.42Ω at some unknown temperature θ. Calculate θ on the scale of this thermometer. **An[87.11℃]** (03mk)

UNEB 2007 Qn 5

(a) (i) Define a thermometric property and give two examples (02marks)

(ii) When is the temperature **0 K** attained (02marks)

(b) (i) With reference to a constant-volume gas thermometer define temperature on the Celsius scale (02marks)

(ii) State two advantages and two disadvantages of constant-volume gas thermometer.

(02marks)

(c) (i) Define the triple point of water (01mark)

(ii) Describe how you would measure the temperature of a body on thermodynamic scale using a thermo couple. (03marks)

UNEB 2005 Qn 5

(a) (i)What is meant by the term fixed points in thermometry. Give two examples of such points (02marks)

(ii) How is temperature on a Celsius scale defined on a platinum resistance thermometer?
(02marks)

(b) Explain the extent to which thermometer based on different properties but calibrate using the same fixed points are likely to agree when used to measure a temperature

(i) Near one of the fixed points (02marks)

(ii) Midway between the two fixed points (02marks)

(d) What are the advantages of a thermocouple over a constant volume gas thermometer in measuring temperature.

Solution

b)i) They may agree, because for points near the fixed points the values of the thermometric properties vary almost in step for points close to the fixed points.
ii) They may not agree for temperature midway between fixed points the different thermometric properties vary differently with temperature.

UNEB 2004 Q5

(a) What is meant by

(i) Thermometric property (01mark)

(ii) Triple point of water (01mark)

(b) (i)Describe the steps taken to establish a temperature scale
(05marks)

(ii) Explain why the thermometers may give different values for the same unknown the temperature. (02marks)

(c) (i)Describe with the aid of a diagram, how a constant volume gas thermometer may be used to measure temperature (06marks)

(ii) State three corrections that need to be made when using the thermometer in c(i) above.

(iii)State and explain the sources of in accuracies in using mercury-in-glass thermometer.

Solution

Corrections in a constant volume gas thermometer include;

- The temperature of the gas in the dead space because its temperature lies between that of the bulb and the room temperature.
- Thermal expansion of the bulb
- The capillary effect at the mercury surface.

In Accuracies rise Because

The none uniformity of the capillary tube from which the thermometer was made. This causes equal changes in volume of the liquid not producing equal changes in the length of the liquid column.

UNEB 2000 Q7

(a) (i) State the desired properties a material must have to be used as a thermometric liquid substance. (02marks)

(ii) Explain why scales of temperature based on different thermometer properties may not agree (01marks)

(b) (i) Draw a labeled diagram to show the structure of a simple constant volume gas thermometer.

(03marks)

(ii) Describe how a simple-constant volume gas thermometer can be used to establish a Celsius scale of temperature. (05marks)

(iii) State the advantage and disadvantage of mercury in glass thermometer and a constant volume gas (03marks)

(c) The resistance of the element of a platinum resistance thermometer is 4Ω at the point and 5.46Ω at the steam point. What temperature on the platinum resistance scale would correspond to a resistance of a 9.84Ω **An[400^0C]** (03marks)

CHAPTER2: CALORIMETRY

The heat energy of a system is its internal energy and it can be either heat capacity or latent heat.

2.1.0: HEAT CAPACITY AND SPECIFIC HEAT CAPACITY

- **Specific heat capacity** of substance is quantity of heat required to raise the temperature of 1kg mass of substance by 1kelvin.
  Its S.I units are joules per kilogram per Kelvin [$Jkg^{-1}K^{-1}$].

- **Heat capacity** is the amount of heat required to raise the temperature of any mass of the a substance by 1Kelvin.
  Its units are joules per Kelvin [JK^{-1}]
  The heat gained Q or lost by the substance is given by
  Q=mass x S.H.C x temp change

$$\boxed{\boldsymbol{Q = m\, c\, \Delta\, \theta}}$$

Where $\theta = \theta_1 - \theta_2$ C=S.H.C

Heat capacity = mass x S.H.C, which implies

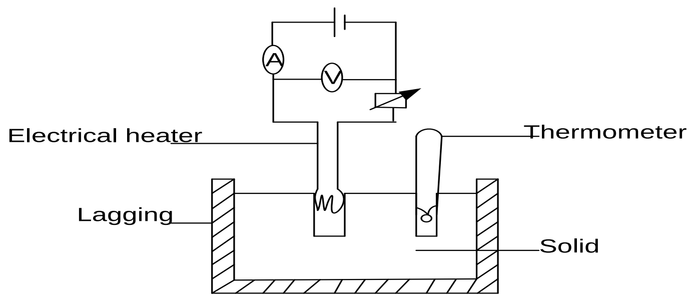
$$\boxed{\text{Q} = \text{Heat capacity x temperature change}}$$

EXERCISE 2

1) Calculate the quantity of heat required to raise the temperature of a metal block with a heat capacity of $23.1J^0C^{-1}$ by 30.0℃. **An [693J]**
2) An electrical heater supplies 500J of heat energy to a copper cylinder of mass 32.4g. Find the increase in temperature of the cylinder (specific heat capacity of copper = $385Jkg^{-1}\,℃^{-1}$) **An[40.1℃]**
3) How much heat must be removed from an object with a heat capacity of $150J℃^{-1}$, in order to reduce its temperature from 80.0℃ to 20.0℃. **An [9×10^3J]**

2.1.2: METHODS OF DETERMINING S.H.C

a) **Determination of S.H.C of a solid by electrical method**



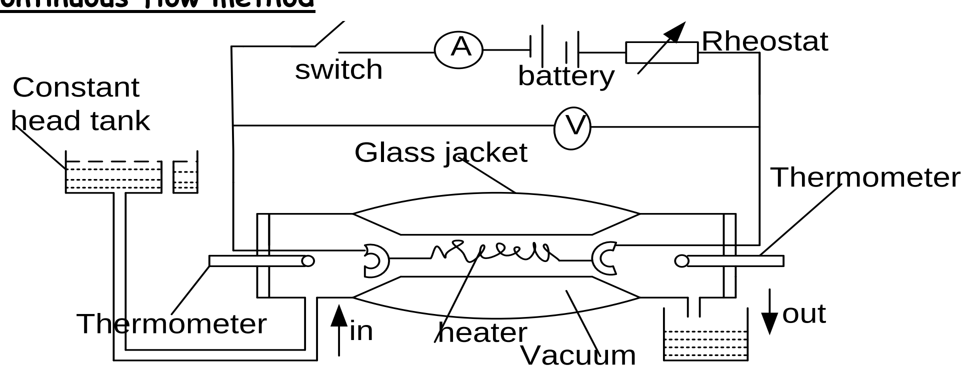
- A material whose S.H.C is to be determined is drilled with two holes, one for thermometer and other for heater. Both the heater and thermometer must be in good thermal contact with material.
- The initial temperature of the material θ_1 is determined using thermometer and recorded before closing the circuit.
- The circuit is now closed and at the same time the stop clock started and heating is done until temperature rises to θ_2
- The time t taken for temperature to rise from θ_1 to θ_2 is recorded and currents I and voltage V for this temperature rise also recorded.
- If m is the mass of the block and C is it S.H.C, then from
- heat supplied by the heater = heat gained by the block.

$$IVt = mC[\theta_2 - \theta_1]$$

$$C = \frac{\mathrm{Ivt}}{\mathrm{m}[\theta_2 - \theta_1]}$$

b)-Determination of S.H.C of a liquid

i)-Using continuous flow method



MODE OF OPERATION

- Liquid is allowed to flow at constant rate from the overhead tank.
- Switch k is closed and system left to run until thermometers indicate steady temperatures.
- The inflow temperature θ_1 and out flow temperature θ_2 are recorded

- The Ammeter reading I_1 and Voltmeter reading V_1 are recorded
- The mass m_1 which flows per second is measured and recorded.
- At steady state $I_1V_1 = m_1c(\theta_2 - \theta_1) + h$ ----- [1] where h is rate of heat loss to surrounding
- The Rheostat is adjusted to vary the current through the circuit until the inflow outflow temperatures are the same as before
- At steady state $I_2V_2 = m_2c(\theta_2 - \theta_1) + h$ ----- [2]

Therefore specific heat capacity of a liquid, c is got from

$$C = \frac{I_2V_2 - I_1V_1}{(m_2 - m_1)(\theta_2 - \theta_1)}$$

MERITS OF CONTINUOUS FLOW METHOD

- The heat capacity of apparatus is not required since at steady states, the apparatus does not absorb any more heat.
- No cooling correction is required since the heat lost to the surrounding is taken care by repeating the experiment.
- The temperature to be measured θ_1 and θ_2 are constant at steady state.
- They can therefore be measured at leisure and accurately using platinum resistance thermometer.
- There are no heat losses by convection since apparatus has vacuum.

DEMERITS OF CONTINUOUS FLOW METHOD

- It can't be used to determine S.H.C of solid
- It requires a large quantity of liquid and therefore it is expensive

Questions

1) In the flow method to determine the S.H.C of the liquid, the following two sets of results were obtained.

	Experiment 1	Experiment 2
P.d across water (V)	5.0	3.0
Current through heater (A)	0.3	0.2
Temperature of liquid at inlet (^{0}C)	25	25
Temperature of liquid at outlet (^{0}C)	41	41
Mass of liquid (kg)	0.15	0.07
Time taken (s)	200	120

a) Calculate the S.H.C of the liquid
b) Heat lost per second

Solution

a) Iv= mc [θ_2-θ_1]+h

Experiment 1

0.3x5 = $\frac{0.15}{200}C[41-25]+h$

1.5 = 0.012 C+ h ------------------ [I]

Experiment 2

0.2x3 = $\frac{0.07}{120}C$ [41-25]+ h

0.6= 9.328x10^{-3}C+h ------- [II]

Subtract equation I from II

0.6-1.5 =9.328x10^{-3}C-0.012C

$\frac{-0.9}{-2.612x10^3}=\frac{-2.672x10^{-3}C}{-2.672x10^{-3}}$

C= 3.3x10^2 Jkg^{-1}K^{-1}

b) From equation1

1.5 =0.012C + h

1.5 =0.012x3.3x10^2+h

h = -2.46J

2) In continuous flow experiment it was found that when applied $p.d$ was 12.0V, current 1.5A, a rate of flow of liquid of 50.0g/minute cause the temperature of inflow liquid to differ by 10℃. When the $p.d$ was increased to 16.0V with current of 1.6A, the rate of flow of 90.0g/minute was required to produce the same temperature difference as before. Find ;
   (i) S.H.C of the liquid
   (ii) Rate of heat loss to the surrounding

Solution

Iv= mc [θ_2-θ_1]+h

1.5x12 = $\frac{50}{1000x60}C$x10+h

18 = $\frac{50}{60000}C$x10+h

18 = 8.33X10^{-3}C+h ---------- [1]

1.6X6 =$\frac{90}{10000}C$x10+h

25.6 = 0.015C+h ------------ [II]

Subtract equation 1 from II

7.6 = 6.67x10^{-3}C

$C = \frac{7.6}{6.67x10^{-3}}$

C= 1.14x10^{3}Jkg^{-1}K^{-1}

ii) 25.6 = 0.015x1.14x10^3+h

h = 25.6-17.09

h=8.50watts

3) Water flow at rate of 0.15kg/minute through a tube and is heated by a heater dissipating 25.2W. The inflow and outflow temperature are 15.2℃ and 17.4℃ respectively. When the rate of flow is increased to 0.232kg/minute and rate of heating to 37.8W. The inflow and outflow temperature are not altered. Find;
   i) S.H.C of water ii) Rate of loss of heat in the tube

solution

Iv= mc [θ_2-θ_1]+h

25.2 = $\frac{0.15}{60}C$[17.4-15.2]+h

25.2 = 2.5x$10^{-3}C$ x 2.2 +h

25.2 = 0.0055C+h ----- [I]

Iv= mc [θ_2-θ_1]+h

37.8 =$\frac{0.232}{60}$xC[2.2]+h

37.8 = 8.506x$10^{-3}C$+h ------- [2]

Subtract 1 from 2

37.8-25.2 = 8.506x$10^{-3}C$-0.0055C

12.6 = 3x$10^{-3}C$

C=4200J$kg^{-1}K^{-1}$

ii)

25.2 = 0.0055C+h

25.2 = 0.0055x4200+h

h= 25.2 -23.1

h=2.1watts

4) In an experiment to measure specific heat capacity of water, stream of water flows at rate of 5gs^{-1}over an electrical heater dissipating 135W and temperature rise of 5K is observed. On increasing the rate of flow to 10$gs^{-1}$the same temperature rises is produced with dissipation of 240W.
   Explain why the power in 2nd case is not twice that needed in 1st case and deduce the value of S.H.C in water.

Solution

Power in the 2nd case is not twice as one is 1st case because some heat is lost to the surrounding [420 J$kg^{-1}K^{-1}$]

Iv= mc [θ_2-θ_1]+h

135 = $\frac{5}{100}$ (5)+h

135=0.25C+h ------------- [1]

I^1v= mc [θ_2-θ_1]+h

240 =$\frac{10}{100}$ (5)+h

240 = 0.5C+h ------- [2]

Subtract 1 from 2

105=0.25C

$C = \frac{105}{0.25}$

C= 420 J$kg^{-1}K^{-1}$

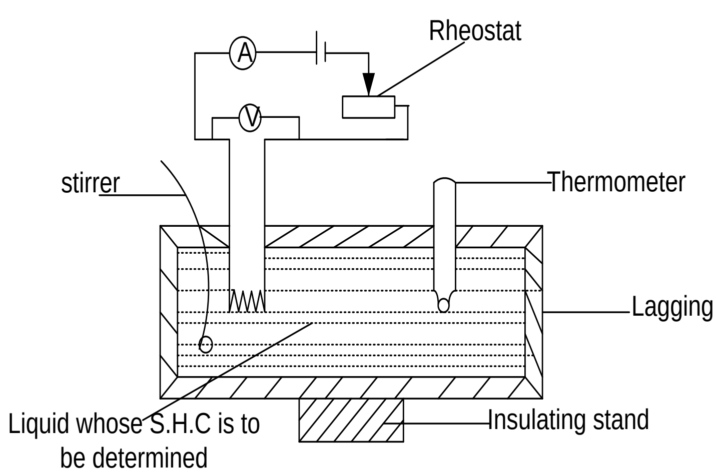
EXERCISE 3

1) In an electrical constant flow experiment to determine the specific heat capacity of a liquid, heat is supplied to the liquid at a rate of 12W. When the rate of flow is $0.060kgmin^{-1}$, the temperature rise along the flow is 2.0K. Use these figures to calculate a value for the specific heat capacity of the liquid. If the true value of the specific heat capacity is $5400Jkg^{-1}K^{-1}$, estimate the percentage of heat lost in the apparatus. **An [6000 $Jkg^{-1}K^{-1}$ 11%]**

2) When water was passed through a continuous flow calorimeter the rise in temperature was from 16 to 20℃, the mass of water flowing was 100g in one minute, the $p.d$ across the heating coil was 20V and the current was 1.5A. Another liquid at 16.0℃ was then passed through the calorimeter and to get the same change in temperature, the $p.d$ was changed to 13V, the current to 1.2A and the rate of flow to 120g in one minute. Calculate the S.H.C of the liquid if the S.H.C of water is 4200 $Jkg^{-1}℃^{-1}$**An[1700 $Jkg^{-1}K^{-1}$]**

3) With a certain liquid, the inflow and outflow temperatures were maintained at 25.20℃ and 26.51℃ respectively for a p.d of 12.0V and current 1.50A, the rate of flow was 90g per minute, with 16.0V and 2.00A, the rate of flow was 310g per minute. Find the S.H.C. of the liquid and also the power lost to the surrounding. **An [2910 $Jkg^{-1}K^{-1}$, 12.3W]**

ii)Electrical method



- When $d.c$ is switched on for time t, the temperature of the liquid and calorimeter changes from θ_1 to θ_2.
- The resistant is then adjusted to get a suitable value of I and V when the mixture is uniform after stirring. Assuming that there is not heat gained by the thermometer, then there is no heat lost to the surrounding.

- The electric energy supplied by heater=heat gain by calorimeter and liquid.

$$Ivt = M_L C_L(\theta_2-\theta_1) + M_C C_C(\theta_2-\theta_1)$$

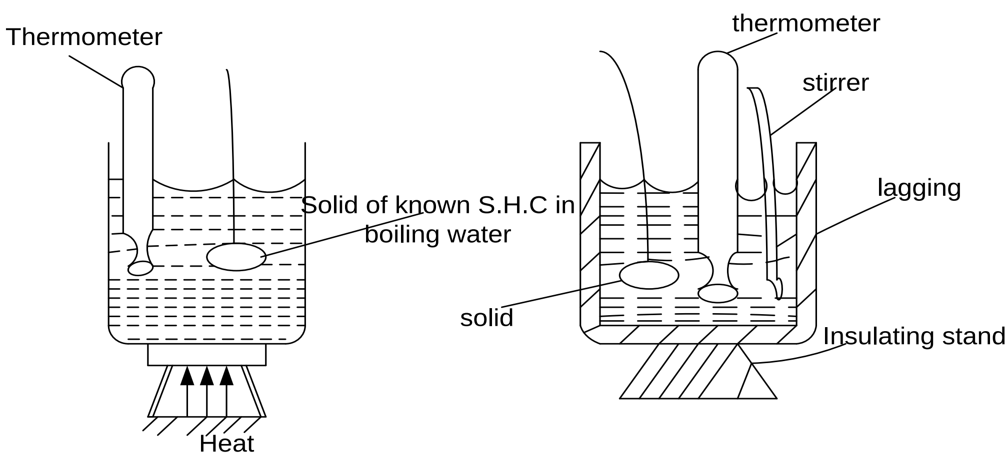
$$C_L = \frac{Ivt - Mc\,Cc\,(\theta 2-\theta 1)}{ML(\theta 2-\theta 1)}$$

M_C = mass of calorimeter $\quad$ C_c = S.H.C of calorimeter

M_L = mass of liquid $\quad$ C_L = S.H.C of liquid

iii)USING METHOD OF MIXTURES

This S.H.C of liquid can be determined using method of mixture as follows



- The solid of mass M_s and S.H.C C_s in boiling water at temperature θ_1 is transferred to liquid of mass M_L whose S.H.C [C_L] is to be determined in calorimeter of mass M_c and S.H.C C_c both at temperature θ_2.
- The mixture is stirred uniformly until final steady temperature θ_3 is obtained
- Assuming there is no heat gained by the stirrer and thermometer and no heat is lost to the surrounding.
- Heat lost by solid= heat gained by calorimeter +heat gained by liquid

$$M_S C_S(\theta_1-\theta_3) = M_L C_L(\theta_2-\theta_3) + M_c C_c(\theta_2-\theta_3)$$

$$C_L = \frac{M_S C_S(\theta 1-\theta 3) - McCc(\theta 2-\theta 3)}{M_L(\theta 2-\theta 3)}$$

PRECAUTIONS TAKEN IN DETERMINING S.H.C BY METHOD OF MIXTURES

- The solid should be transferred as soon as possible to liquid in calorimeter.
- The liquid in calorimeter should be well stirred to ensure uniformly of temperature.
- The calorimeter should be supported on an insulated stand and should also be lagged to reduce heat loss by conduction.
- The calorimeter should be well polished to minimize heat loss by radiation.

DISADVANTAGES OF METHODS OF MIXTURE

- Some heat is lost to the surrounding

- Some heat is absorbed by stirrer and thermometer.
- Some heat losses by conduction and convection

N.B: heat losses that cannot be eliminated can be catered for by a cooling correction

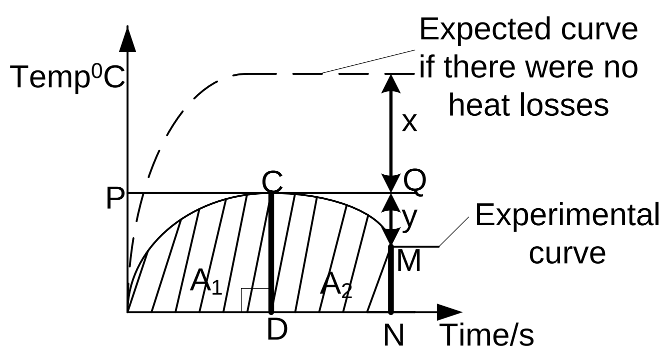
2.1.3: COOLING CORRECTION

Is the number of degree Celsius that should be added to the observed maximum temperature to cater for heat losses during rise or fall.

OR

Is the extra temperature that is added to the observed maximum temperature to compensate for the heat lose to the surrounding.

2.1.4: GRAPHICAL DETERMINATION OF COOLING CORRECTION OF A POOR CONDUCTOR E.G. RUBBER



- A bad conductor eg glass, rubber is heated in boiling water. It is then transferred quickly to calorimeter containing water at room temperature.
- Temperature is recorded at different time interval.
- The body is then allowed to cool by about 1^0c below the maximum rate.
- A graph of temperature against time is plotted.
- A parallel line P to the time axis is drawn which is a tangent at the maximum value of experimental curve.
- The experimental curve is also drawn with an assumption that there were no heat losses.
- Values of x and y are then determined and perpendicular lines CD and MN sub dividing the areas into A_1 and A_2 are also drawn.
- The cooling collection X, then determined from.

$$\frac{A_1}{A_2} = \frac{X}{Y}$$

$$\boxed{X = \frac{A_1 Y}{A_2}}$$

Examples

1) A metal of mass 0.2kg at 100℃ is dropped into 0.08kg of water at 13℃ contained in calorimeter of mass 0.12kg and S.H.C 400J$kg^{-1}K^{-1}$. The final temperature reached is 35℃. Determine the S.H.C of the solid.

Solution

Ms=0.2kg	θ_2=15℃	Cw =4200J$kg^{-1}K^{-1}$
θ_1=100℃	Mc=0.12	θ_3 =35℃
Mw=0.08kg	Cc=400J$kg^{-1}K^{-1}$	

Heat lost by the solid=heat gained by calorimeter + heat gained by water

MsCs (θ_1-θ_2) = McCc(θ_3-θ_2)+Mw Cw (θ_3-θ_2)

0.2x Cs (100-35)= 0.12x 400 (35-15)+0.08x4200(35-15)

13Cs =960 + 6120

Cs=590.769J $kg^{-1}K^{-1}$

2) Hot water of mass 0.4kg at 100℃ is poured into calorimeter of mass 0.3kg and S.H.C 400J$kg^{-1}K^{-1}$ and contains 0.2kg of a liquid at 10℃. The final temperature of mixture is 40℃ determines the S.H.C of a liquid.

Solution

Mw=0.4kg	Mc=0.3kg	M_L=0.2kg θ_2=10^{0}C
θ_1=100℃	Cc=400J$kg^{-1}K^{-1}$	θ_3= 40℃

Heat lost by the hot water =heat gained by the colorimeter +heat gain by liquid

MwCs (θ_3-θ_2)= McCc(θ_3-θ_2 + M_LC_L (θ_3-θ_2)

0.4x 4200$(100-40)$=0.3x400$(40-10)$ +0.2$xC_L(40-10)$

$100800 = 3600 + 6C_L$

C_L=16200J$kg^{-1}K^{-1}$

3) A 15W heating coil is immersed in 0.2kg of water and switched on for 560 seconds during which time; the temperature rises by 10℃. When water was replaced by some volume of another liquid of mass 0.15kg, the power required for same time is 8.3W. Calculate the S.H.C of the liquid.

Solution

Ivt= $M_LC_L\Delta\theta$	C_L =$[\frac{8.3x560}{0.15x10}]$
8.3x 560=0.15xC_Lx10	C_L=3.1x10^3 $J$$kg^{-1}K^{-1}$

Assumption, same temperature rise occurs.

4) When a block of metal of mass 0.11kg and S.H.C $400Jkg^{-1}K^{-1}$ is heated to 100°C and quickly transferred to a calorimeter containing a liquid at 10°C, the resulting temperature is 13°C. On repeating the experiment with 0.4kg of the liquid in the same container at same temperature of 10°C, the resulting temperature is 14.5°C. Calculate;

   a) S.H.C of the liquid b) Thermal capacity of the container.

Solution

$M_S = 0.11Kg$
$Cs=400Jkg^{-1}K^{-1}$
$\theta_1 = 100°C$ $\theta_2=10°C$ $\theta_3=18°C$
$M_L=0.2kg$

$M_L=0.4kg$
$\theta_2=10°C$
$\theta_3=14.5°C$

Heat lost by solid = heat gained by liquid + heat gained by container

$MsCs\ (\theta_1-\theta_3) = M_LC_L\ (\theta_3-\theta_2) + McCc\ (\theta_3-\theta_2)$

$0.11 \times 400(100-18) = 0.2 \times C_L\ (18-10) + H(18-10)$

$3608 = 1.6C_L+8H$ ……………………………………(1)

$MsCs\ (\theta_1-\theta_3) = M_LC_L\ (\theta_3-\theta_2) + McCc\ (\theta_3-\theta 2)$

$0.11 \times 400(100-14.5) = 0.4 \times C_L\ (145-10) + H\ (14.5-10)$

$3762 = 1.8\ C_L + 4.5H$ ……………………………………………(2)

Solving equation1 and equation2 simultaneously

$C_L = 1935Jkg^{-1}K^{-1}$ $H= 66JK^{-1}$[thermal capacity of the container]

EXERCISE 4

1) A piece of copper of mass 100g is heated to 100°C and is then transferred to a well lagged copper can of mass 50g containing 200g of water at 10°C. Neglecting heat loss, calculate the final steady temperature of water after it has been well stirred. Take S.H.C of copper and water to be $400Jkg^{-1}K^{-1}$ and $4200Jkg^{-1}K^{-1}$ respectively. **An[14**°C]
2) A heating coil is placed in thermal flask containing 0.6kg of water for 600s. The temperature of water rises by 25°C during this time. Water is replaced by 0.4kg of another liquid. And the same temperature rise occurs in 180s. Calculate the S.H.C of the liquid given that S.H.C of water is $4200Jkg^{-1}K^{-1}$. State any assumption. **An [$1890Jkg^{-1}K^{-1}$]**

3) Copper calorimeter of mass 120g contains 100g of paraffin at 15°C. If 45g of aluminum at 100°C is transferred to the liquid and the final temperature is 27°C. Calculate the S.H.C of paraffin [S.H.C of aluminum and copper are 1000 $Jkg^{-1}K^{-1}$ and 400 $Jkg^{-1}K^{-1}$ respectively]. **Ans. $2.4 \times 10^3 Jkg^{-1}K^{-1}$**

4) A steady current of 12 A and $p.d$ of 240V is passed, through a block of mass 1500g for 1 ½ minutes. If the temperature of the block rises from 25℃ to 80℃, calculate

(i) The specific heat capacity of the block

(ii) The heat capacity of 4kg mass of the block. **An [3141.82Jkg^{-1}K^{-1}, 12567.28JK^{-1}]**

5) A liquid of mass 250g is heated to 80℃ and then quickly transferred to a calorimeter of heat capacity 380JK^{-1} containing 400g of water at 30℃. If the maximum temperature recorded is 55℃ and specific heat capacity of water is 4200Jkg^{-1}K^{-1}. Calculate the S.H.C of the liquid. **An [8,240Jkg^{-1}K^{-1}]**

6) 500g of water is put in a calorimeter of heat capacity 0.38Jk^{-1} and heated to 60℃. It takes 2minute for the water to cool from 60℃ to 55℃. When the water is replaced with 600g of a certain liquid, it takes 1 ½ minute for the liquid to cool from 60℃ to 55℃. Calculate the S.H.C of the liquid. **An [2624.8kgJ^{-1}K^{-1}]**

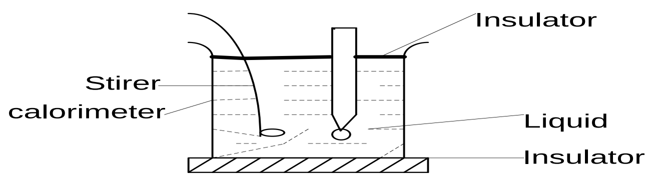
7) When a metal cylinder of mass 2.0x10^{-2}kg and specific heat capacity 500Jkg^{-1}k^{-1} is heated by an electrical heater working at a constant power, the initial rate of rise of temperature is 3.0Kmin^{-1}. .After a time the heater is switched off and the initial rate of fall of temperature is 0.3Kmin^{-1}. What is the rate at which the cylinder gains heat energy immediately before the heater is switched off? **An[0.45W]**

8) A copper block has a conical hole bored in it into which a conical copper plug just fits. The mass of the block is 376g and that of the plug is 18g. The block and plug are initially at room temperature 10℃ and almost completely surrounded by a layer of insulating material. The plug is removed from the block, cooled to a temperature of -196℃ and then quickly inserted into the block again. The temperature of the block falls to 3℃ and then slowly rises. Calculate the value of the mean specific heat capacity of copper (in the range -196℃ to 3℃) obtained by ignoring heat flow into the block from the surrounding. (S.H.C of copper to the temperature range 3℃ to 10℃ is 380Jkg^{-1}K^{-1}). **An [279kg^{-1}K^{-1}]**

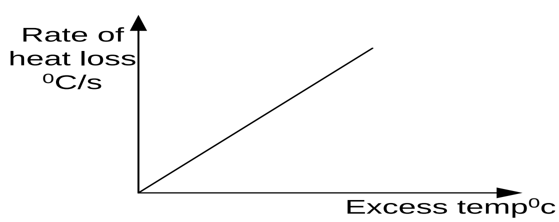
2.1.5: NEWTON'S LAWS OF COOLING

It states that under conditions of forced convection, the rate of heat loss is directly proportioned to excess temperature over the surrounding

EXPERIMENT TO VERIFY NEWTON'S LAW OF COOLING



- Hot liquid is poured in a calorimeter and then transferred to an open window.
- A thermometer is suspended in air and the room temperature θ_R is recorded.
- Temperature θ of the liquid is recorded at different time interval for about 20 minutes.
- A cooling curve of temperature θ against time t is plotted as shown below.
- Different rates of temperature fall i.e. G_1, G_2, G_3 at different temperatures $\theta_1, \theta_2, \theta_3$ are determined.
- For each temperature the excess temperature, $\theta - \theta_R$ is calculated
- A graph of rate of heat loss [$G=\frac{d\theta}{dt}$] against excess temperature is plotted and straight line is obtained which verifies Newton's law of cooling.



2.1.6: HEAT LOSS AND TEMPERATURE CHANGE

A part from excess temperature ($\theta - \theta_R$), the rate of heat loss also depends on;

- Surface area of the body
- The nature of the temperature of the body i.e. Dull surface lose heat faster than shining

A body having a uniform surface area and uniform temperatures, heat loss per second is given by $\frac{d\theta}{dt}$.

Since Q=ML$\Delta\theta$

$\frac{dQ}{dt}$.= $-MC\frac{d\theta}{dt}$

$\frac{d\theta}{dt}$.= $-\frac{d\theta}{dt}/MC$

But $M = \rho x vol$

$\frac{d\theta}{dt}$.= $\frac{dQ}{dt}/\rho x vol x C$

$\frac{d\theta}{dt}$.= $\frac{dQ}{\rho x vol x C dt}$

$\frac{d\theta}{dt}$= $\frac{-1}{\rho x vol x C}$ $\frac{dQ}{dt}$

So the rate of cooling is inversely proportional to the volume "V" of the body. Hence a body with a small volume cools faster than one with a large volume for a given material.

Question: Explain why a small body cools faster than larger bodies of the same material.

2.2.0: LATENT HEAT

This is the amount of heat required for the substance to change state at constant temperature.

$$\text{Solid} \underset{\text{Heat lost}}{\overset{\text{Heat absorbed}}{\rightleftharpoons}} \text{Liquid} \quad \underset{\text{Heat absorbed}}{\overset{\text{Heat lost}}{\rightleftharpoons}} \text{Gas}$$

When melting a solid, latent heat of fusion is absorbed to break the intermolecular forces of attraction between solid molecules and increase their energy to allow molecules to move a part.

When evaporating a liquid, latent heat of vaporization is absorbed to break the intermolecular forces of attraction and increase their energy by higher amount since their molecules become widely spaced when they are in vapour state.

LATENT HEAT OF FUSION

This is heat required to change any mass of substance from solid to a liquid at constant temperature.

SPECIFIC LATENT HEAT OF FUSION

Is the quantity of heat required to change **1kg** mass of a solid to a liquid at **constant temperature**.

It is measured in Jkg^{-1}

LATENT HEAT OF VAPOURIZATION

Is the quantity of heat required to change any mass of substance from liquid to gas at a constant temperature.

SPECIFIC LATENT HEAT OF VAPOURIZATION

Is the quantity of heat required to change **1kg** mass of liquid to gas at **a constant temperature.** It is measured in Jkg^{-1}

2.2.1: WHY LATENT HEAT OF VAPOURIZATION IS HIGHER THAN LATENT HEAT OF FUSION

- In fusion, heat is required to <u>weaken</u> the intermolecular bonds accompanied with increase in volume hence negligible work done against atmospheric pressure.

- While in vaporization, heat is required to <u>break</u> intermolecular attractions and form a gas followed by <u>a large</u> increase in volume and more work is done against atmospheric pressure in expanding the gas.

Example

A calorimeter with heat capacity of $80J°C^{-1}$contains 50g of water at 40°C what mass of ice at 0°C needs to be added in order to reduce the temperature to 10°C. Assume no heat is lost to the surround (S.H.C of water = 4200J$kg^{-1}°C^{-1}$, S.L.H of the of ice = $3.4x10^5Jkg^{-1}$).

Solution

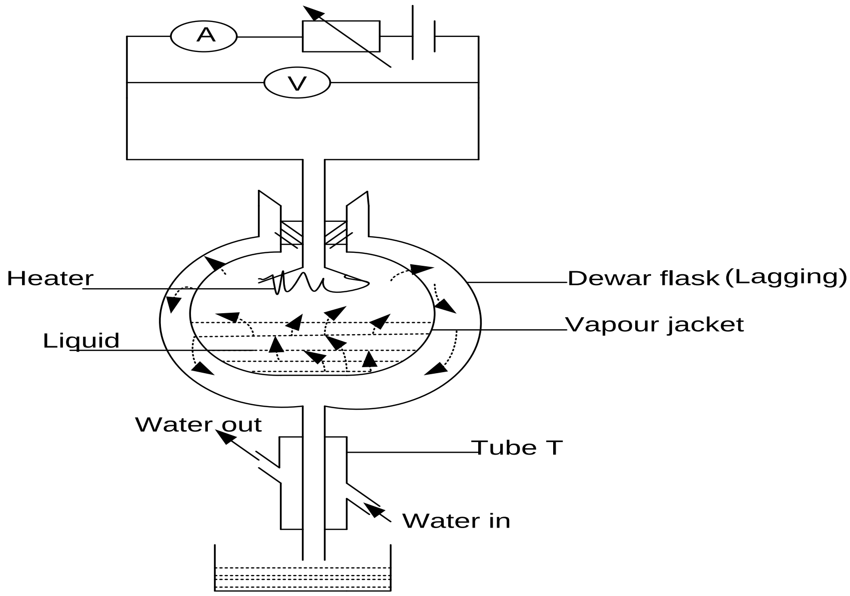
$$\begin{pmatrix} Heat\ lost\ by \\ calorimeter \\ from \\ 40°C\ to\ 10°C \end{pmatrix} + \begin{pmatrix} Heat\ lost\ by \\ water \\ from \\ 40°C\ to\ 10°C \end{pmatrix} = \begin{pmatrix} Heat\ gained \\ by\ ice \\ at\ 0°C \end{pmatrix} + \begin{pmatrix} Heat\ gained \\ by\ melting \\ ice \\ from\ 0°C\ to\ 10°C \end{pmatrix}$$

$$M_c C_c (40-10) + M_w C_w (40-10) = M_1 L + C + M_1 C_I x (10-0)$$

$$80 \times 30 + \frac{50}{1000} \times 4200 \times 30 = M_1 (3.4 \times 10^5 + 4200 \times 10)$$

M_1 = 0.023kg   Mass of ice required = 23g

2.2.2: DETERMINATION OF THE S.L.H OF VAPOURIZATION (L_V) OF LIQUID BY a)ELECTRIC METHOD [DEWAR FLASK METHOD]



- Switch k is closed and liquid is heated until it starts boiling
- A stop clock is started and mass m_1 of liquid collected in a time in a time t.
- The Ammeter reading, I_1 and Voltmeter reading V_1 are recorded.
- At steady state, $I_1\ V_1\ t\ = m_1 \times l_V + h$…………………………………… (1) where $h = ht$ heat lost to surrounding
- The Rheostat is adjusted and a new Ammeter reading I_2 and Voltmeter reading V_2 are recorded
- New mass m_2 of the liquid collected in the same time t is obtained

$$I_2\ V_2\ t\ = m_2 \times l_V + h \quad \text{..(2)}$$

The specific latent heat of vapourization is obtained from

$$\boxed{L_V = \frac{(I_2V_{2-}\ I_1V_1)t}{(M_{2-}\ M_1)}}$$

EXAMPLES

1) When electrical energy is supplied at a rate of 12W to a boiling liquid 1.0x10^{-2} Kg of the liquid evaporates in 30 min .On reducing the electrical power to 7W, 5.0x10^{-3} Kg of the liquid evaporates in the same time. Calculate;

a) S.L.H of vapouration
b) Power loss to the surrounding

Solution

P_1 = 12W, t_1=30min ,M_1 = 1.0x10^{-2}kg, P_2 = 7W ,t_2 =30min , M_2 = 5.0x10^{-3}kg

$I_2\ V_2 = M_1L_v + h$

12 = $\frac{1.0x10-3}{30x60}$. Lv + h ……… (1)

$I_2\ V_2 = M_1L_v + h$

7 = $\frac{5.0x10-2}{30x60}$ Lv +h ……… (2)

Subtract equation (2) from (1)

$[12-7] = [\frac{1.0x10-2}{30x60}]$ Lv – $[\frac{5.0x10-3}{30x60}]$ Lv

$5 = \frac{1.0x10-2}{1800}$ Lv - $\frac{5.0x10-3}{1800}$ Lv

5= $5.555x10^{-6}$ Lv- $2.7777x10^{-6}$Lv

Lv = $1.8x10^{+6}$ Jkg^{-1}

b)

-Using equation 1

$12 = \frac{1.0x10-2}{30x60}x1.8x10^{+6}$

12 = $5.555x10^{-6}$ x $1.8x\ 10^{+6}$ + h

h= 2W

2) An experiment to determine S.L.H of vapourization of alcohol using dewar flask gave the following results.

EXPERIMENT I	EXPERIMENT II
V_1 =7.4V	V_2 = 10.0V
I_1 = 2.6A	I_2 = 6.6A
M_1= 5.8 x 10^{-3}kg	M_2 = $11.3x10^{-3}$ kg
t_1 = 300 seconds	t_2 = 300seconds

a) Find S.L.H of vapourization of alcohol

b) Heat lost to surrounding per unit time.

Solution

a) $I_1V_1 = M_1Lv + h$

$2.6x\ 7.4 = \frac{5.8\ x10-3}{300}$ Lv + h… ………..(1)

$I_2\ V_2 = M_2Lv + h$

$6.6x10 = \frac{11.3x10-3}{300}$ Lv + h ……… …(2)

Subtract (I) from (II)

[66-19.24] = [$3.766X10^{-3}$ Lv – $1.933x10^{-5}$]

46.76 = $1.833x\ 10^{-5}$ Lv

Lv = $2.55x10^{+6}$ J/kg

b)-Using equation (I)

19.24 = $1.933x10^{-5}$ x 2.55 x 10^{-6}+ h

h= 0.39W

3) When electrical power is supplied at rate of 12W, mass of liquid of $8.6x10^{-3}$ kg evapourates in 30minutes. On reducing the power to 7W, $5x10^{-3}$kg of the liquid evapouration in same time. Calculate;

(i) S.L.H of evapouration of liquid. **An $5x10^{-6}$ Jkg^{-1}**

(ii) Power lost to the surrounding. **An 0.056Js^{-1}**

4) In an experiment to determine S.L.H.V of a liquid using Dewar flask in the following results were obtained.

Voltage V(V)	Current I(A)	Mass collected in 300s/g
7.4	2.6	5.8
10.0	3.6	11.3

Calculate the power of the heater to evaporate 3.0g of water in 2 minutes.

Solution

$IVt = MLv$

P= $\frac{MLv}{t}$

But $I_1V_1 = M_1Lv + h$……….[1]

Also $I_2V2 = M_1Lv + h$ ……..[2]

Equation2 – equation1

$Lv = \frac{I_2V_2 - I_1V_1}{M_2 - M_1}$

$Lv = \frac{10x3.6 - 7.4x2.6}{\frac{11.3}{\left(\frac{300}{10^{-3}}\right)} - \frac{5.8}{\left(\frac{300}{10^{-3}}\right)}}$

$Lv = 9.14\text{x } 10^5\, Jkg^{-1}$

Put into equation (2)

$I_2V2 = M_1Lv + h$ ……..[2]

$10\text{x } 3.6 = \frac{11.3}{\frac{300}{100}}\text{x}9.14\text{x}10^5 + h$

h= 1.57w

$I_3\ V_3 = M_3\ Lv + h$

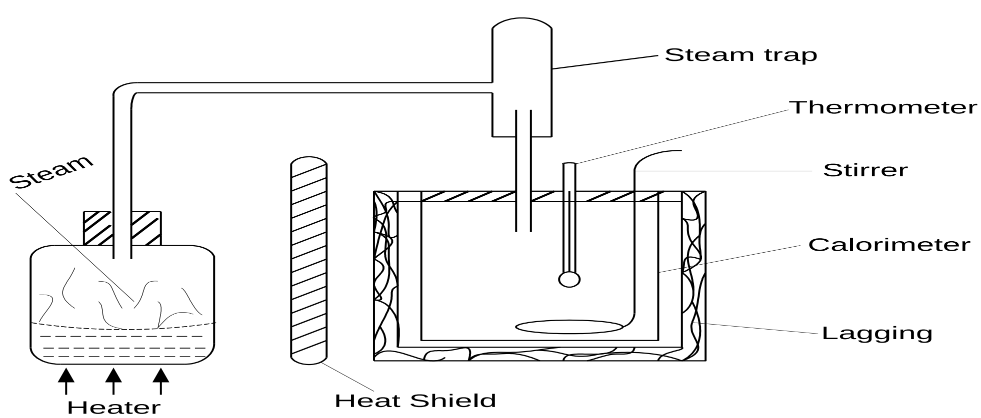
$P_3 = \left(\frac{3x10-3}{2x60}\text{x}9.14\text{x}10^5\right) + 1.57$

$P_3 = 24.42W$

MERITS OF DEWAR FLASK

- Heat capacity of apparatus is not required
- Heat loss to the surrounding is taken care of by repeating apparatus

b)DETERMINATION OF S.L.H.V BY METHOD OF MIXTURE



- ❖ The liquid whose **S.L.H.V** is to be determined, is heated to boiling point.
- ❖ Steam passes through tube T which is then trapped by steam trape and led down.
- ❖ Steam is condensed by cold water contained in a calorimeter.
- ❖ The mass M of condensed steam is found by weighing
- ❖ If θ_1 and θ_2 is the initial and final temperature respectively then the **S.L.H.V** of can be obtained from

$$\begin{pmatrix}\text{Heat given by}\\ \text{steam condensing}\end{pmatrix} + \begin{pmatrix}\text{heat given by}\\ \text{condensed water}\\ \text{from } 100°\text{C to } \theta°\text{C}\end{pmatrix} = \begin{pmatrix}\text{heat taken}\\ \text{by calorimeter}\end{pmatrix} + \begin{pmatrix}heat\ taken\\ by\ water\end{pmatrix}$$

$MsLv + McwCw(100-\theta_2) = McCc[\theta_2-\theta_1] + MwCw[\theta_2-\theta_1]$

Ms = Mass of steam

Mcw = mass of condensed water

Mc = mass of calorimeter

Ms = mass of water in calorimeter

EXAMPLE

1) An electrical heater rated 500W is immersed in liquid of mass 2.0kg contained in large thermal flask of heat capacity $840Jkg^{-1}$ at 28℃. Electrical power is supplied to heater for 10minutes. If S.H.C of liquid is $2.5x10^3$ $Jkg^{-1}K^{-1}$. Its **S.L.H.V** is $8.54x10^3$ $Jkg^{-1}k$ and its boiling point is 78℃. Estimate the amount of liquid which boils off.

Solution

Heat supplied by heater = heat gained by flask + heat gained by liquid + heat used for evaporating the liquid.

$Ivt = M_f C_f (\theta_2-\theta_1) + M_L C_L (\theta_2-\theta_1) + MsLv$

$500x10x60 = 840 (78-28) + 2x2.5x10^3 [78-28] + M_s (8.54x10^3)$

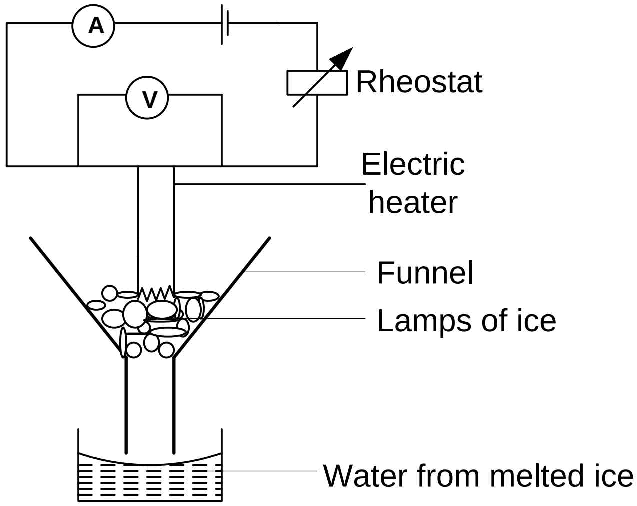
$300000 = 42000 + 250000 + 8.54x10^3 Ms$

$Ms = \frac{300000-292000}{8.54x103}$

Ms = 0.936kg

2) Ice at 0℃ is added to 200g of water initially at 70℃ in a vacuum flask. When 50g of ice is added and has all melted, the temperature of the flask and content is 400℃. When further 80g of ice has been added and has been melted, the temperature of the whole becomes 10℃. Calculate the S.L.H of fusion of the neglecting any heat loss of surrounding. **Ans $3.78x10^5$ Jkg^{-1}**

c) DETERMINATION OF S.L.H.F OF ICE BY ELECTRICAL METHOD



- The rheostat is adjusted until suitable values of I and V are obtained
- The heat supplied by the heater is used to melt the ice and water, and water from melted ice is collected and weighed per unit time.

$$\begin{pmatrix}\text{Heat supplied by}\\ \text{heater per second}\end{pmatrix} + \begin{pmatrix}\text{heat absorbed from}\\ \text{surrouding per second}\end{pmatrix} = \text{latent heat absorbed by ice}$$

$$IV + h = ML_F \text{ .... } (1)$$

- The experiment is repeated with values of I_1V_1 and M_1 is also determined by

$$I_1V_1 + h = M_1L_F \text{.............. } (2)$$

Equation (1) – equation (2)

$$\boxed{L_f = \frac{I\,V - I_1V_1}{M - M_1}}$$

Exercise 5

1) Calculate the heat required to melt 200g of ice at 0℃ . (S.L.H of ice= $3.4\times10^5 Jkg^{-1}$)
   An 6.8×10^4 J

2) Calculate the heat required to turn 500g of Ice at 0℃ into water at 100℃. (S.L.H of ice= $3.4\times10^5 Jkg^{-1}$ S.H.C of water = $4200 Jkg^{-1}$**)** **An [$3.8\times10^5 J$]**

3) Calculate the heat given out when 600g of steam at 100℃ condenses to water at 20℃ [S.L.H of steam = 2.26×10^6 Jkg^{-1}, S.H.C of water = 4200 Jkg^{-1}]. **An [1.56×10^6J]**

4) 1kg of vegetables, having a specific heat capacity 2200 Jkg^{-1} at a temperature 373K are plugged into a mixture of ice and water at 273K. How much is melted. [S.L.H of fusion of the = $3.3x10^5$ Jkg^{-1}] **An [0.67kg]**

5) 3kg of molten lead (melting point 600K) is allowed to cool down until it has solidified. It is found that the temperature of the lead falls from 605K to 600K in 10s, remains constant at 600K for 300s, and then fall to 595K in a further 8. 4s. Assuming that the rate of loss of energy remains constant and that the specific heat capacity of solid lead is $140Jkg^{-1}K^{-1}$. Calculate.

(a) Rate of loss of energy from the lead.

(b) The specific latent heat of fusion of lead.

(c) The specific heat capacity of liquid lead

An [250W, $2.5x10^4Jkg^{-1}$, $167Jkg^{-1}K^{-1}$]

6) 0.02kg of ice and 0.10kg water at 0°C are in a container. Steam at 100°C is passed in until all the ice is just melted. How much water is now in the container?
S.L.H of steam = $2.3x10^6Jkg^{-1}$, S.L.H of ice = $3.4x10^5Jkg^{-1}$,
S.H.C of water = 4.2 $x10^3Jkg^{-1}K^{-1}$ **An [0.1225kg]**

7) When apiece of ice of mass $6x10^{-4}$kg at a temperature of 272K is dropped into liquid nitrogen boiling at 77K in a vacuum flask 8x $10^{-4}m^3$ of nitrogen, measured at 294K and 0.75m mercury pressure are produced. Calculate the mean specific heat capacity of ice between 272K and 77K. Assume that the S.L.H of vaporization of nitrogen is $2.13x10^5Jkg^{-1}$ and that the density of nitrogen at S.T.P is $1.25kgm^{-3}$.**An $1.67x10^3Jkg^{-1}K^{-1}$**

8) Wet clothing at a temperature of 0°C is hung out to dry when the air temperature is 0°C and there is a dry wind blowing. After some time, it is found that some of the water has evaporated and the remainder has frozen. What is the source of the energy required to evaporate the water. Estimate the proportion of the water originally in the clothing which remains as ice, state any assumptions you make.
(S.L.H of fusion of ice at 273K = 333k Jkg^{-1}
S.L.H of vaporization of water at 273K = 2500k Jkg^{-1}) **An [88%]**

9) In a factory heating system water enters the radiators at 60°C and leaves at 38°C. The system is replaced is by one in which steam at 100°C is condensed in the radiators, the condensed steam leaving at 82°C. What mass of steam will supply the same heat as 1.00kg of hot water in the first instance.

(The S.L.H of vapourisation of water is $2.26 \times 10^{6} Jkg^{-1}$ at 100℃. The S.H.Cof water is $4.2 \times 10^{3} Jkg^{-1}℃^{-1}$) **An [0.0396kg]**

10) A beaker containing ether at a temperature of 13℃ is placed in a large vessel in which the pressure can be reduced so that the ether boils, this results in cooling of the remaining ether. What proportion of the ether has evaporated when the temperature of the remainder has been reduced to 0℃ (assume no interchange of heat between the ether and its surrounding)
(Mean S.H.C of ether over the temperature range 0-13^{0}C = $2.4 \times 10^{3} Jkg^{-1}℃^{-1}$)
(Mean S.L.H of vapourisation of ether in the temperature range 0-13^{0}C = $3.9 \times 10^{5} Jkg^{-1}℃^{-1}$) **An[7.4%]**

11) In an express coffee machines, steam at 100^{0}C is passed into milk to heat it. Calculate
(i) The energy required to heat 150g of milk from room temperature (20^{0}C) to 80^{0}C.
(ii) The mass of steam condensed. **An [3.6×10^{6}J, 15.8g]**

UNEB 2013 Q5

(a) Define
(i) Specific heat capacity (01mark)
(ii) Specific latent heat of vaporization of a liquid (01mark)
(b) With the aid of a labeled diagram, describe an electrical method of determining the specific heat capacity of a solid (07marks)
(c) An electrical heater rated 48W, 12V, is placed in a well insulated metal of mass 1.0kg at a temperature of 18℃. When the power is switched on for 5minutes, the temperature of the metal rises to 34℃. Find the specific heat of the metal.
An (900 $Jkg^{-1}K^{-1}$,)
(04marks)
(d) (i) State **Newton's law of cooling**
(01marks)
(ii) Use Newton's law of cooling to show that

$$\frac{d\theta}{dt} = -k(\theta - \theta_R)$$

Where $\frac{d\theta}{dt}$ is the rate of fall of temperature and θ_R is the temperature of the surrounding (03marks)
(e) Explain why evaporation causes cooling (03marks)

UNEB 2012 Q5

(a) (i) Define the terms specific heat capacity and specific latent heat of fusion (2mk) (ii)Explain the changes that take place in the molecular structure of substances during fusion and vaporization. (04marks)

c) With the aid of a labeled diagram describe an experiment to determine the S.H.C of a liquid using the continuous flow method (08marks)

d) Steam at 100°C is passed into a copper calorimeter of mass 150g containing 340g of water at 15°C. This is done until the temperature of the calorimeter and its content is 71°C. If the mass of the calorimeter and its contents is found to be 525g. Calculate the specific latent heat of vaporization of water. (06marks)

Solution

Mass of calorimeter Mc = 150g

Mass of water Mw = 340g

Mass of steam Ms = 525 – (150+340) = 35g

$$\begin{pmatrix}\text{Heat supplied}\\ \text{by steam}\\ \text{at } 100^\circ\text{C}\end{pmatrix}+\begin{pmatrix}\textit{Condensing steam}\\ \textit{from}\\ 100^\circ\text{C } \textit{to } 71^\circ\text{C}\end{pmatrix}=\begin{pmatrix}\text{heat gained by}\\ \text{calorimeter}\\ \text{from}\\ 15^\circ\text{C to} 71^\circ\text{C}\end{pmatrix}+\begin{pmatrix}\textit{heat gained by}\\ \textit{water from}\\ 15^\circ\text{C } \textit{to } 71^\circ\text{C}\end{pmatrix}$$

Ms Lv + MsCs (100-71) = McCc (71-15) + Mw Cw (71-15)

$\frac{35}{1000}Lv + \frac{35}{1000} \times 4200 \times 29 = \frac{150}{1000} \times 400 \times 56 + \frac{340}{1000} \times 4200 \times 56$

$Lv = 2.26 \times 10^6$ Jkg^{-1}

UNEB 2011 QN. 6

a) Define S.H.C of a substance and states its units (02marks)

b) (i) Describe how S.H.C of a liquid can be obtained by the continuous flow method (07marks)

(ii) State one disadvantage of this method (01mark)

c) An electric kettle rated 1000W, 240V is used on 220Vmains to boil 0.52kg of water. If the heat capacity of the kettle is 400Jkg^{-1} and the initial temperature of the water is 20°Chow long will the water take to boil. **An[246s]** (04marks)

UNEB 2009 QN 5

(b) (i) Define specific heat capacity of a substance (01mark)

(ii)Hot water at 85°C and cold water at 10°C are run into a bath at a rate of 3.0×10^{-2}m^{3}min^{-1} and V respectively. At the point of filling the bath the temperature of the mixture of water was 40^{0}C. Calculate the time taken to fill the bath if its capacity is 1.5m^3 (05marks)

(c) The specific latent heat of fusion of a substance is significantly different from its specific latent heat of vaporization at the same pressure. Explain how the difference arises

(04marks)

(d) Explain in terms of S.H.C why water is used in a car radiator than any other liquid. (02marks)

Solution

Let M_h = be mass of hot water

Mc = be mass of cold water

Heat supplied by hot water = heat gained by cold water

From 85℃ to 40℃ from 10℃ to 40℃

M_h C (85-40) = Mc (40-10)

$M_h = \frac{30}{45}$ Mc ………………………………………… (1)

Let t be the time taken to fill

But M_h = ρ x volume

$M_h = \rho x\ (3\times10^{-2})\ t$ ……………………………….,(2)

Also Mc = $\rho\ xV\ t$ ………………………………….(3)

Put equation (2) and equation (3) to equation (1)

$\rho x\ (3\times10^{-2})\ t = \frac{30}{45}\rho vt$

$V = 4.5\times10^{-2}\ m^3\ min^{-1}$

If the total volume = $1.5m^3$

If the volume of cold and hot water at filling temperature are V_1 and V_2 respectively.

$V_1+V_2 = 1.5m^3$

$3\times10^{-2}\ t + 4.5\times10^{-2}\ t = 1.5$

t=20minutes

c)Water has a very high S.H.C which means that a small volume of water can absorb a lot of heat without causing significant increases in the temperature. However other liquids have low S.H.C so a large amount of these liquids will be needed to absorb the heat that causes the same temperature increases as in water consequently this wound require a larger radiator which is un economical.

UNEB 2008 Q 5

(a) Define the following terms

(i) S.H.C of vaporization of a liquid (01mark)

(ii) Coefficient of thermal conductivity (01mark)

(b) With the aid of a well labeled diagram describe an experiment to measure the S.L.H of vaporization of water by an electrical method (07marks)

(c) An appliance rated 240V, 200W evaporates 20g of water in the 5minutes. Find the heat loss if S.L.H of vaporization is $2.26x10^6Jkg^{-1}$ (03marks) **An[14800J]**

(d) Explain why at a given external pressure a liquid boils at a constant temperature (4marks)

(e) With the aid of a suitable sketch graph explain the temperature distribution a long a lagged and un lagged metal rods, heated at one end (04marks)

UNEB 2007 Q 6

(a) (i) Define latent heat (01mark)

(ii)Explain the significance of latent heat in regulation of body temperature (3marks)

(b) (i)Using kinetic theory, explain boiling of a liquid. (03marks)

(ii)Describe how you would determine the S.L.H of vaporization of water using the method of mixtures. (05marks)

(iii)Explain why latent heat of vaporization is always greater than that of fusions.

(02marks)

Solution

ii) During a hot day, the body releases sweat on the body's surface, the sweat evaporates by drawing the latent heat of evaporation from the body there by making the body cool.

UNEB 2006 Q 6

(a) (i) Define S.H.C of a substance (01mark)

(ii) State three advantages of the continuous flow method over the method of mixtures in the determination of S.H.C of a liquid (03marks)

(b) In a continuous flow experiment, a steady difference of temperature of 1.5^0C is maintained when the rate of liquid flow is $45gs^{-1}$ and the rate of electrical heating is 60.5W. On reducing the liquid flow rate to $15gs^{-1}$, 36.5W is required to maintain. Calculate the;

(i) S.H.C of the liquid (04marks)

(ii) Rate of heat loss to the surrounding (3marks)**An [$533.3Jkg^{-1}k^{-1}$, 24.5W]**

(c) (i) Describe an electrical method for the determination of the S.H.C of a metal (06marks)

(ii) State the assumptions made in the above experiment (02marks)

(iii) Comment about the accuracy of the result of the experiment in C (i) above (01mark)

Solution

C (i) assumption

- There is no heat loss to the surrounding
- The quantity of heat gained by the thermometer and the heater is negligible
- The volume of the metal is constant hence no work is done against the atmospheric pressure.

ii) Due to heat loss to the surrounding, it implies that more heat was supplied than as required to causes the observed temperature change. Hence the value of C is greater than the actual value.

UNEB 2005 QN 5

(c) The continuous flow method is used in the determination of the S.H.C of the liquid.

(i) What are the principle advantages of this method compared to the method of mixture (03marks)

(ii) In such a method, 50g of water is collected in 1minute ,the voltmeter and ammeter readings are 12.0V and 2.50A respectively. While the inflow and outflow temperatures are 20℃and 28℃ respectively. When the flow rate is reduced to $25gmin^{-1}$, the voltmeter and ammeter read 8.8V and 1.85A respectively while the temperatures remain constant. Calculate the S.H.C of water (5marks)
An[$4.116x10^3Jkg^{-1}k^{-1}$]

UNEB 2002 QN 7

(a) (i) Define S.H.C of a substance (01mark)
(ii) State how heat losses are minimized in Calorimetry

(b) (i) What is meant by a cooling correction (02marks)
(ii)Explain how the cooling correction may be estimated in the determination of the heat capacity of a poor conductor of heat by the method of mixtures. (05mks)
(iii)Explain why a small body cools faster than a larger one of the same material.

(04marks)

(c) Describe how you would determine the S.H.C of a liquid by the continuous flow method.
(07marks)

UNEB 2001 QN 7

(a) Explain why temperature remains constant during change of phase (04marks)

(b) Describe with the aid of a labeled diagram, an electrical method for determination of S.L.H of vaporization of a liquid. (07marks)

(c) Water vapour and liquid water are confirmed in a air tight vessel. The temperature of the water is raised until all the water has evaporated, draw a sketch graph to show how the pressure of the water vapour changes with temperature and account for its main features (06 marks)

Solution

a)

- During change of state from solid to liquid, the heat energy supplied is used in breaking the strong intermolecular forces in the crystalline structure. So the molecules can move closer and further apart to their potential energy increases but the kinetic energy of the molecules remain constant consequently the temperature remaining constant.
- During the change from liquid to vapour state the heat supplied is used in separating the molecules further apart i.e. it increases the potential of the molecules. Also some of the energy is used in doing work during expansion against atmospheric pressure, since kinetic energy of the liquid molecules is not altered then no temperature change occurs.

UNEB 1999 Q7

(a) Define S.H.C (01 mark)

(b) Describe an electrical method of measuring S.H.C of a metal. (06 marks)

(c) In a continuous flow calorimeter for measurement of S.H.C of a liquid, $3.6\times10^{-3}m^3$ of a liquid flows through the apparatus in 10 minutes. When electrical energy is supplied to the heating coil at the rate of 44W, a steady difference of 4k is obtained between the temperature of the out-flowing and inflowing liquid. When the flow rate is increased to $4.8\times10^{-3}m^3$ of liquid in 10 minutes, the electrical power required to maintain the temperature difference is 58W. Find the

(i) S.H.C of the liquid (06 marks)

(ii) Rate of loss of heat to the surrounding (02 marks)

[Density of liquid = $800kgm^{-3}$]

UNEB 1998 QN 5

(a) Define the S.L.H of vaporization (01mark)

(b) Describe an electrical method of determining the S.L.H of vaporization of a liquid (08marks)

(c) State any two advantages of the continuous flow method over the method of mixtures for the determination of S.H.C of liquids . (02marks)

(d) When electrical power is supplied at a rate of 12W to a boiling liquid, a mass of liquid of 8.6×10^{-3}Kg evaporates in 30minutes. On reducing the power to 7.0W, 5×10^{-3}kg of the liquid evaporates in the same time. Calculate the;

(i) S.L.H of vaporization of the liquid (04marks)

(ii) Power loss to the surrounding (02marks)

(e) Explain why evaporation causes cooling (03marks)

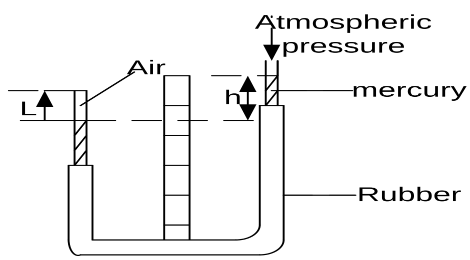
CHAPTER3: GAS LAWS

1: **Boyle's law**:

it states that the volume of fixed mass of a gas is inversely proportional to its pressure at constant temperature i.e.

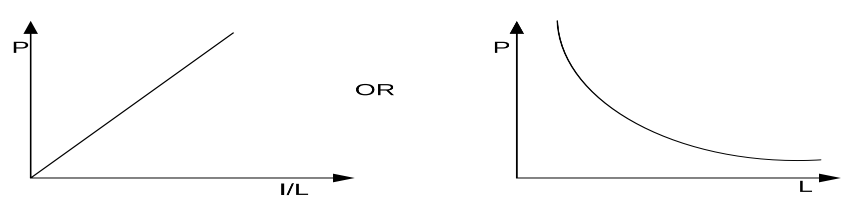
P $\propto \frac{1}{v}$ PV = constant $\boxed{P_1V_1 = P_2V_2}$

EXPERIMENT TO VARIFY BOYLES LAW



- A fixed mass of the gas is trapped inside J tube of uniform cross section using mercury.
- The pressure of the gas is varied by adding mercury to open limb
- The pressure P of air [H+h] is determined by measuring height h.
- The length L of air column is also measured using meter ruler.
- Different value of P and L are obtained and tabulated including values of $\frac{1}{L}$.
- A graph of P against $\frac{1}{L}$ is plotted and straight line graph passing through original obtained (P$\propto \frac{1}{L}$) from the graph.

Since the tube has cross section area A then P$\propto \frac{1}{AL}$ but AL=V ieP$\propto \frac{1}{V}$



This verifies Boyle's law

2:CHARLES LAW:

It states that the volume of fixed mass of gas is directly proportional to its absolute temperature at constant pressure i.e.

$V \propto T$ $\quad\quad\quad$ $\frac{V}{T}$=comstant $\quad\quad\quad$ $\boxed{\frac{V_2}{T_2} = \frac{V_1}{T_1}}$

Absolute zero temperature (0K) is the lowest temperature at which average kinetic energy of molecules is zero.

3:PRESSURE LAW/ GAY LUSSAC LAW

It states that the pressure of fixed mass of gas is directly proportional to its absolute temperature at constant volume i.e.

$P \propto T$ $\quad\quad\quad$ $\frac{P}{T}$= constant $\quad\quad\quad$ $\boxed{\frac{P_1}{T_1} = \frac{P_2}{T_2}}$

EXAMPLE

A container of a gas has a volume of $0.1m^3$ at pressure of $2.5x10^5$Pa and at temperature of 27℃.

a) Find the new pressure, if the gas is heated at constant volume to 87℃

b) Gas pressure is now reduced to $1x10^5Nm^{-2}$ at constant temperature. What is the new volume of the gas?

c) The gas is cooled to -73℃ at constant pressure. Find the new volume of the gas.

Solution

a) V_1=$0.1m^3$

$P_1 = 2.5x10^5$

$T_1 = 27^0c = [27+273]=300k$

At constant volume

$T_2 = [87+273] = 360k$

$\frac{P1}{T1} = \frac{P2}{T2}$

$\frac{2.5x10^5}{300} = \frac{P2}{360}$

$P_2 = [\frac{2.5x10^5x360}{300}]$

$P_2 = 300x10^3\ Nm^{-2}$

b) $P_2 = 1x10^5$

At constant temperature

$P_1 V_1 = P_2V_2$

$2.5x10^5x\ 0.1 = 1x10^5V_2$

$V_2 = \frac{2.5x10^5\ x\ 0.1}{1x10^5}$

$V_2 = \frac{25000}{100000}$

$V_2 = 0.25m^{+3}$

c) $\frac{V1}{T1} = \frac{V2}{T2}$

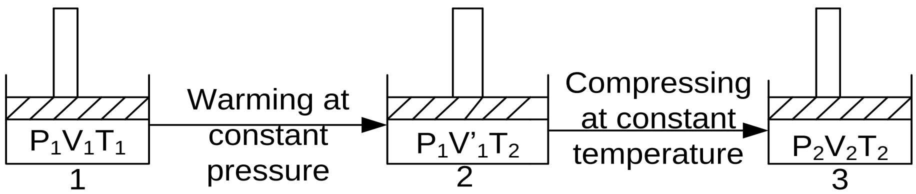
$\frac{0.1}{300} = \frac{V2}{200}$

$V_2 = \frac{200x0.1}{300}$

$V_2 = 0.066m^3$

3.1: EQUATION OF STATE

Consider fixed mass of gas with pressure $P_{1,}$ V_1 and temperature T_1 enclosed in piston cylinder system.



Moving from 1 to 2, Charles law applies

$$\frac{V_1}{T_1} = \frac{V_1'}{T_2}$$

$$V_1' = V_1 \frac{T_2}{T_1} \quad \text{……………………………………………………… (1)}$$

Moving from 2 to 3, Boyle's law apples

$$P_1 V_1' = P_2 V_2 \quad \text{……………………………………………………… (2)}$$

Putting V_1 into equation (2)

$$P_1 V_1 \frac{T_2}{T_1} = P_2 V_2$$

$$\boxed{\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}}$$

$$\frac{PV}{T} = \text{Constant}$$

PV= constant XT

$$PV = nRT$$

$$\boxed{PV = n\,R\,T}$$

This is an equation of state or ideal gas equation.

Where n= number of moles of gas

$$n = \frac{mass\ given}{relative\ molecular\ mass}$$

R= molar gas constant [$8.31Jmol^{-1}k^{-1}$]

To determine R, we consider the standard condition at $s.t.p$

Volume at $s.t.p$ = $22.4x10^{-3}$ m^3

Temperature at $s.t.p$ =273K

Pressure at $s.t.p$ = $1.01325x10^5Nm^{-2}$

Number of mole n = 1

$$PV = nRT$$

$$R = \frac{PV}{nT}$$

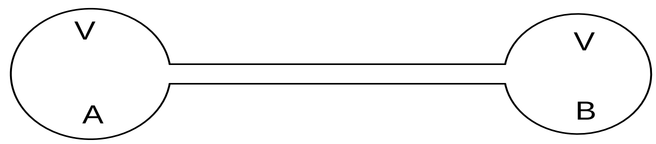
$R = \frac{1.01325x10^5 \; x \; 22.4x10^{-3}}{1X273}$ | R= 8.31$Jmol^{-1}K^{-1}$

EXAMPLES

1) Two glass bulbs of equal volume are joined by another tube and are filled with a gas at $s.t\,p$. When one of the bulbs is kept in melting ice and another place in as hot bath the new pressure is 877.6mmHg. Calculate the temperature of bath

Solution

Since they are at $s.t.p$, they all have the same initial temperature



P_A = 760mmHg P_B = 760mmHg T_A = 273k T_B = 273K

Since cylinders are enclosed, the number of moles is both cylinders before cooling will be the same after cooling (heating).

$n_A + n_B = n_A^1 + n_B^1$

$\frac{P_AV_A}{RT_A} + \frac{P_BV_B}{RT_B} = \frac{P_A'V_A'}{RT_A'} + \frac{P_B'V_B'}{RT_B'}$

$P'_A = P'_B$ = 877.6mmHg

T'_A = (0+273) = 273K

T'_B =?

$\frac{760xV}{8.31x273} + \frac{760xV}{8.31x273} = \frac{877.6xV}{8.31x273} + \frac{877.6xV}{8.31xT_B'}$

$\frac{760}{273x8.31} + \frac{760}{8.31x273} = \frac{877.6}{8.31x273} + \frac{877.6}{8.31T_B'}$

$\frac{642.4}{2268.63} = \frac{877.6}{8.31T_B'}$ T'_B =372.95K

2) Two containers A and B of volume $3x10^3cm^3$ and $6x10^3cm^3$ respectively contain helium gas at a pressure of $1.0x10^3$ pa and temperature 300k. Container A is heated to 373K while container B is cooled 273K.
Find the final pressure of the helium gas.

Solution

$V_A = 3x10^3cm^3$

$P_A = 1.0x10^3$pa

T_A = 300k

$V_B = 6x10^3cm^3$

$P_B = 1.0x10^3$pa

T_B = 300K

T'_A = 373K

T'_B = 273

$n_A + n_B = n_A^1 + n_B^1$

$\frac{P_AV_A}{RT_A} + \frac{P_BV_B}{RT_B} = \frac{P_A'V_A'}{RT_A'} + \frac{P_B'V_B'}{RT_B'}$

$\frac{1.0x10^3x3x10^3}{8.31x300} + \frac{1.0x10^3x6x10^3}{8.31x300} = \frac{P_A'x3x10^3}{8.31x313} + \frac{P_B'x6x10^3}{8.31x273}$

$P_A' = P_B'$=P

916461 = 2493 (819000 +2238000P)

916461 = $2.04x10^9 + 5.579x10^9$p

$5.579x10^9$p = 916461 – $2.04x10^9$

P = 999.3Pa

3) What volume of liquid oxygen of density 1140kgm^{-3} may be made by liquefying completely the quantity of a cylinder of gaseous oxygen containing 100litres of oxygen at 120 atmospheres at 20℃ [assume that O_2 behaves like an ideal gas]

Solution

1L = 1000cm^3

100L = 100000cm^3

V = 0.1 m^3

$1\ atm$ = 760mmHg or 1.01325 x10^3 Nm^{-2}

$120\ atm$ = 1.01325 x10^5 x 120 Nm^{-2}

P = 1.01325x10^{5}x120 Nm^{-2}

T = 20+213 = 293K

From ideal gas

n= $\frac{pv}{RT}$

n= $\frac{1.013225x10x120x1.0}{8.31x293}$

n= 499.377mole

But n= $\frac{mass}{RMM}$

From ρ = $\frac{m}{v}$

M= ρv

n= $\frac{pv}{RMM}$

V= n x RMM

V = $\frac{n\,x\,RMM}{P}$

But Rmm of oxygen = 32x10^{-3}kg

V= $\frac{499.377x32x10^{-3}}{1140}$

V= 0.014m^3

4) A gas is confined in the container of volume 0.1m^3 at pressure of 1.0x10^{5}Nm^{-2} And temperature of 300K. If the gas is found to be ideal gas, calculate the density of the gas [Rmm=32]

solution

PV= nRT

n= $\frac{pv}{RT}$

n=$\frac{1.0x10^5x0.1}{8.31x300}$

n= 4.01moles

But n=$\frac{mass}{Rmm}$

4.01= $\frac{mass}{32x10^{-3}}$

Mass = 4.01x 32x10^{-3}

Mass = 0.128kg

But ρ= $\frac{M}{V}$

ρ= $\frac{0.128}{0.1}$

ρ= 1.28kg/m^{-3}

Calculation involving loss of gas

1) Oxygen gas is contained in cylinder of volume 1x10^{-2} m^3 at temperature of 300K and pressure 2.5x0^{5}Nm^{-2}. After some oxygen is used at constant temperature, pressure falls to 1.3x10^5 Nm^{-2}. Calculate the mass of oxygen used.

Solution

$V_1 = 1x10^{-2}\ m^3$

$T_1 = 300K$, $P_1 = 2.5x10^5 Nm^{-2}$

$P_1V_1 = n_1RT_1$

$P_1V_1 = \frac{mass}{Rmm} RT_1$

$P_1V_1 = \frac{M1}{RMM} \times RT_1$

$M_1 = \frac{P1V1\ RMM}{RT1}$

$M_1 = \frac{2.5x10^5 x1x10^{-2}x32x10^{-3}}{8.31X300}$

$M_1 = \frac{80}{2493}$

$M_1 = 0.032$

$T_2 = 300K$, $P_2 = 1.3x\ 10^5 Nm^{-3}$

$V_2 = 1x\ 10^{-2} m^3$

$P_2V_2 = n_2RT_2$

$P_2V_2 = \frac{M2}{Rmm} RT_2$

$M_2 = \frac{P2V2\ RMM}{RT2}$

$M_2 = \frac{1.3X10^5 X1X10^{-2}X32X10^{-3}}{8.31X300}$

$M_2 = \frac{41.6}{2493}$

$M_2 = 0.0166$

Therefore mass of oxygen = $[M_1 - M_2]$ kg

= [0.032-0.0166] kg

= 0.0154 kg

= $15.4x\ 10^{-3}$kg

2) A cylinder of gas has mass of gas 10kg and pressure of 8 atmospheres at 27℃ when some gas is used in cold room at -3℃. The remaining gas in the cylinder at its temperature has a pressure of 6.4 atmospheres. Calculate the mass of the gas used.

Solution

$M_1 = 10kg$

$P_1 = 8atm$

$T_1 = 27+273 = 300K$

$M_2 = ?$

$P_2 = 6.4atm$

$T_2 = (-3+ 273) = 270K$

$PV = nRT$

$Pv = \frac{m}{RMM} RT$

$Rmm = \frac{MRT}{PV}$

$Rmm = \frac{M1RT1}{P1V1}$

$Rmm = \frac{10x\ 8.31x300}{8xv}$ .... (1)

$Rmm = \frac{M2RT2}{P2V2}$

$Rmm = \frac{M2x8.31x270}{6.4xv}$ .... (2)

Equating equation (1) to (2)

$\frac{10x8.31x300}{8V} = \frac{M_2x8.31x270}{6.4V}$

$\frac{24930}{8} = \frac{2243.7M_2}{6.4}$

$17949.6M_2 = 159552$

$M_2 = \frac{159552}{17949.6}$

$M_2 = 8.89kg$

Therefore mass of gas = $M_1 - M_2$

= [10-8.89] kg

= 1.11kg

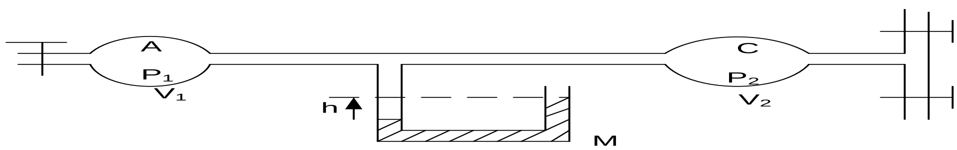
3.2: DALTONS LAW OF PARTIAL PRESSURE

It states that the pressure of a mixture of gases that do not react chemically is the sum of partial pressure of the constituents

DEFINITION

Partial pressure of gas is the pressure the gas would exert if it was to occupy the whole container alone.

3.2.1: EXPERIMENT TO DEMONSTRATE DALTON'S LAW



- The apparatus above can be used to study the pressure of mixture of gases
- A is a bulb of volume V_1 containing air at atmospheric pressure P_1
- C is another bulb of volume V_2 containing carbon dioxide at atmospheric pressure P_2
- When the bulbs are connected by opening the taps, the gases mix and reach the same pressure P

$$P = \frac{P_1 \cdot V_1}{V_1+V_2} + \frac{P_2 V_2}{V_1+V_2}$$

EXAMPLE

1) Two containers A and B of volume $3x10^3cm^3$ and $6x10^3cm^3$ respectively contain helium gas at pressure $1x10^3Pa$ and temperature 300K. Container A is heated to 373Kwhile container B is cooled to 273K. Find the final pressure of the helium gas.

Solution

$V_1=3x10^3x10^6$

$= 3x10^{-3}$

$P_1=1x10^3pa$

$P_2 = 1x10^3$ pa

$V_2 = 6x10^3x10^{-6}$

$= 6x10^{-3}$

$P= \frac{P1\ V1}{V1+V2} + \frac{P2\ V2}{V1+V2}$

$P_2 = P_1$

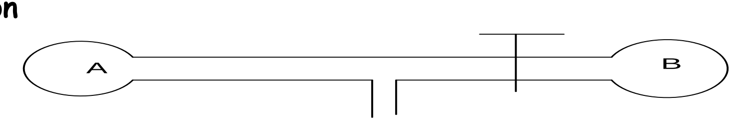
$P= \frac{1x10^3x3x10^{-3}}{3x10^{-3}+6x10^{-3}} + \frac{1x10^3x6x10^{-3}}{3x10^{-3}+6x10^{-3}}$

P= 333.33+666.67

$P= 1000Nm^{-2}$

2) Two bulbs A of volume $100cm^3$ and B of volume $50cm^3$ connected to freeway tap which enables them to be filled with gas or evacuated. Initially bulb A is filled with an ideal gas at $10^0C$to pressure of $3.0x10^5Pa$. Bulb B is filled with an I deal gas at 100℃ to a pressure of $1.0x10^5Pa$. Two bulbs and connected with A maintained at 10^0C and B at 100^0C .Calculate the pressure at equilibrium

Solution



$P_A = 3x10^5Pa$

$P_B = 1x\ 10^5Pa$

$V_A= 100cm^3$

$V_B = 50cm^3$

$T_A = (10+273) = 283K$

$T_B = (100+273) = 373K$

$P= \frac{PAVA}{VA+VB} + \frac{PBVB}{VA+VB}$

$P= \frac{3x10^5x100}{100+50} + \frac{1x10^5x50}{100+50}$

P= 200,000+ 33,333.33

P= $2.33x10^5$ pa

Alternatively

$n_A+n_B = n_A^1+n_B^1$

$\frac{P_AV_A}{RT_A} + \frac{P_BV_B}{RT_B} = \frac{P'_AV'_A}{RT'_A} + \frac{P'_BV'_B}{RT'_B}$

$(\frac{3x10^5x100}{831x283})+(\frac{1x10^5x50}{831x373}) = \frac{px100}{8.31x283} + \frac{px50}{8.31x373}$

P = $2.33x10^5$ pa

3) Two cylinder A and B of volume V and 3V respectively are separately filled with gas. The cylinders are connected with tap closed with pressure of gas A and B being P and 4P respectively. When tap is open, the common pressure becomes 60Pa. Find P

Solution

$P= \frac{PA\ VA}{VA+VB} + \frac{PB\ VB}{VA+VB}$

$P = \frac{PxV}{V+3V} + \frac{4Px3V}{V+3V}$

$60 = \frac{PV}{V+3V} + \frac{4PX\ 3V}{V+3V}$

$60 = \frac{PV}{4V} + \frac{4PX3V}{4V}$

$60 = \frac{P}{4} + \frac{12P}{4}$

$60 = \frac{13P}{4}$

13P = 60X4

P= $[\frac{60X4}{13}]$ pa

P= 18.46pa

EXERCISE5

1) Nitrogen gas under an initial pressure of $5.0x10^6$Pa at 15℃ is contained in cylinder of volume $0.040cm^3$. After a period of three years the pressure has fallen to $2.0x10^6$Pa at the since temperature because of leakage.
[Assume molar mass of nitrogen= $0.028kgmol^{-1}$]
[R= $8.3Jmol^{-1}K^{-1}$, NA= $6.0x10^{23}mol^{-1}$] Calculate;

(i) The mass of gas originally present in the cylinder.

(ii) The mass of gas which escaped from the cylinder in there years

(iii) The average number of nitrogen molecules which escaped from the cylinder per second.[Take one year the be equal to $3.2x10^7$s]

3.3.0: IDEAL GAS

It is one which obeys the gas laws and the gas equation [$Pv = nRT$] completely and at all times

3.3.1: PROPERTIES OF IDEAL GAS

- The internal energy of an ideal gas is entirely kinetic energy and depends only on its temperature and on number of atom in its molecule.

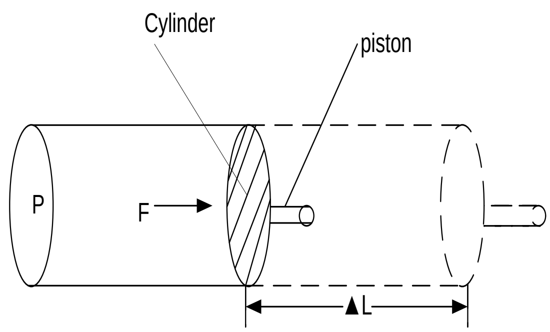
- Inter molecular forces of attraction are negligible
- The volume of molecules is negligible compared to the volume of the container
- The collision between any particles is assumed to be elastic;
- Duration of collision is negligible compared with time between collision

3.4.0: THERMO DYNAMICS

This a branch of physics which deals with heat, work and energy of gas system

3.4.1: Work done by the gas in expansion at constant pressure

Consider gas at pressure P in cylinder fitted with piston as shown above.

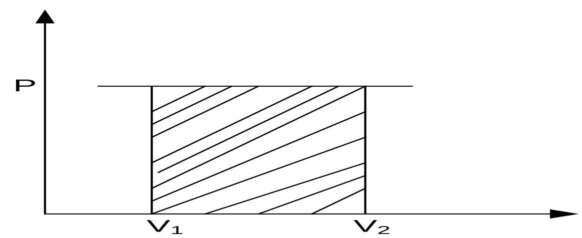


If A is the area of piston.
The force in expanding the gas is given by
F=PA……………………………….(1)
The work done in expanding gas when piston moves through distance ΔL is given by

$$W = Fx\Delta L$$

$W = PAx\Delta L$…………………………(2)
But $A\Delta L$ is the increase in volume
W=PΔV……………………………….(3)

Consider the curve below;



The total work done in changing of the volume of gas from V_1 to V_2 is

Δw = PΔV
$\int_0^w dw = \int_{v1}^{v2} pdv$
W = $\int_{v1}^{v2} pdv$ ……….. (A)
W = P $\int_{v1}^{v2} dv$
W = P$[v]_{v_1}^{v_2}$
W= P $[V_2 - V_1]$ ………(B)
Generally :The external work done in expanding gas at constant pressure

W= PΔV

NB: A piston is used such that the gas expands at constant pressure

3.4.2: THE 1ST LAW OF THERMO DYNAMICS

The **1st law states** that the total energy in a closed system is conserved.

When we warm gas so that it expands, the heat (ΔQ) appears partly as an increase in internal energy (Δu) and partly as external work done (Δw).

$$\boxed{\Delta Q = \Delta u + \Delta w}$$

But $\Delta w = P\Delta V$
$\Delta Q = \Delta u + P\Delta V$
ΔQ = heat supplied

Δu = increase in internal energy
Δw = work done

3.4.3: Internal energy U of the gas

(i) The internal of an **ideal gas** is the kinetic energy of the gas due to its thermal motion of molecules.
The magnitude of this internal energy depends on temperature and the number of atoms in its molecules
(ii) The internal energy of **a real gas** has two components;
- Kinetic energy component due to **thermal motion** of its molecules
- Potential energy component which is due to its **inter molecular forces**

3.5.0: SPECIFIC HEAT CAPACITIES OF GASES

Gases unlike solids and liquids have a number of specific principle heat capacities
- For gases a change in temperature will also produce a large change in pressure and volume.
- For solid and liquid, the change in pressure can be neglected.

In particular, there are two principle heat capacities;
(i) Specific heat capacity at constant pressure
(ii) Specific heat capacity at constant volume

a) S.H.C AT CONSTANT VOLUME

This is the amount of heat required to change temperature of 1kg mass by 1Kelvin at constant volume.
It is denoted by c_v(c-small) and it is measured in $Jkg^{-1}K^{-1}$

b) **S.H.C AT CONSTANT PRESSURE**

This is amount of heat required to change temperature of 1kg mass by 1 Kelvin at constant pressure.
It is denoted by c_p (c-small) and it is measured in $Jkg^{-1}K^{-1}$

c) MOLAR HEAT CAPACITY AT CONSTANT VOLUME

Is the amount of heat required to change the temperature of 1mole of gas by 1 Kelvin at constant volume?
It is denoted by Cv (C-capital). It is measured in $Jmol^{-1}K^{-1}$

$C_v = c_v M$ Where M= molar mass

d) MOLAR HEAT CAPACITY AT CONSTANT PRESSURE

Is the amount of heat required to change the temperature of 1 mole of gas by 1 Kelvin at constant pressure?

It is denoted by Cp (C-capital) and it is measured $Jmol^{-1}K^{-1}$

$$Cp = c_p M$$

3.5.1: DIFFERENCES BETWEEN MOLAR HEAT CAPACITIES [$Cp - Cv = R$]

- Consider 1 mole of an ideal gas heated at **constant volume**

$\Delta w = P\Delta V = 0$ ($\Delta v = 0$)

From the 1st law of thermo dynamic

$\Delta Qv = \Delta u + \Delta w$

But $\Delta w = 0$

$\Delta Qv = \Delta u$ ……………………… (1)

Where ΔQv = heat supplied at constant volume.

But $\Delta u = nC_V\Delta T$

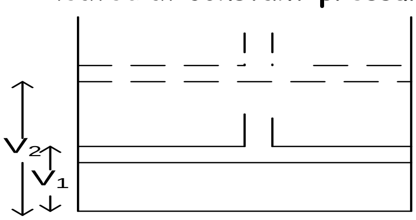
Since n= 1mole

$\Delta u = Cv\Delta T$ ……………………… (2)

Equating equation 2 and 1

$\Delta Qv = Cv\Delta T$ ………………… (3)

- Consider 1 mole of an ideal gas heated at **constant pressure**



The external work done at constant pressure

$\Delta w = P\Delta v$

$\Delta w = P(V_2 - V_1)$ ……………. (4)

Heat supplied at constant pressure is given by

$\Delta Qp = \Delta u + \Delta w$ …………… (5)

But $\Delta Qp = nCp\Delta T$ ………… (6)

Putting equation 2, 4, 6 into equation 5

$nCp\Delta T = Cv\Delta T + P(V_2 - V_1)$

But n=1

$Cp\,\Delta T = Cv\Delta T + P(V_2 - V_1)$ ……….. (7)

Consider ideal gas equation

$PV = RT$ (n =1)

$PV_1 = RT$ ………………………….. (8)

$PV_2 = R(T + \Delta T)$ ………………… (9)

Equation 9 – Equation 8, gives

$P(V_2 - V_1) = R(T + \Delta T) - RT$

$P(V_2 - V_1) = R\Delta T$ ………………. (10)

Putting equation 10 into equation 7

$Cp\Delta T = Cv\Delta T + R\Delta T$

$Cp = C_V + R$

$$\boxed{C_P - C_V = R}$$

NOTE:

- Cp is always greater than Cv because when heat is supplied at constant pressure, it is used for increasing internal energy at the same time doing external work. I.e. $\Delta Qp = \Delta u + \Delta w$.
- While when heat is supplied at constant volume. It is only used for doing internal energy. *ie* $\Delta Qv = \Delta u$ Where $\Delta u = nCv\Delta T$

Explain why $Cp - Cv = R$ is negligible for gases but not solids and liquids.

Solution

- When solid is heated at constant pressure, its change in volume is negligible i.e.$\Delta v = 0$, so the external work done $P\Delta V = 0$ (negligible)
  But since $Cp - Cv = P\Delta V$, it implies $Cp = Cv$ for solids while
- For gases, when gas is heated at constant pressure its volume change ΔV is not negligible and hence the external work done by gas ($P\Delta v$ is not negligible).
  Therefore $Cp - Cv = P\Delta V$ *is* not negligible which makes $Cp - Cv = R$ to hold for gases but not solid and liquid.

NOTE:

$\frac{C_P}{C_V} = \gamma$

γ = 1.67 for monoatomic gases such as Ne, He, Ar.

γ = 1.4 for diatomic gases e.g. O_2, H_2, N_2

γ = 1.33 for a poly atomic gases Co_2 , O_3

EXAMPLES

1) A gas in a cylinder has pressure of 2.0×10^5Pa. The piston has an area of 3×10^{-3}m^2 and it is pulled out slowly through distance of 10mm.Fi*nd the external work done by the gas*

Solution

$\Delta W = P\Delta V$

$\Delta v = A\Delta L$

$W = 2\times10^5 \times 3\times10^{-3} \times 10\times10^{-3}$

W= 6Joules

2) A gas has volume of 0.02m^3 at pressure of 2×10^5Pa and temperature of 27℃. It is heated at constant pressure until its volume increases to 0.03m^3. Calculate;

(i) The external work done

(ii) The new temperature of the gas

(iii) The increase in internal energy of gas, if its mass is 16g and molar heat capacity at constant volume is 0.8 Jmol^{-1}K^{-1} and its molar mass is 32g.

Solution

$V_1 = 0.02m^3$,$V_2 = 0.03m^3$ $\quad$ $P_1=2\times10^5$Pa, $P_2=2\times10^5$Pa $\quad$ $T_1 = 27$℃ $T_2 = ?$

i) $W = P\Delta V$

$W = P (V_2 - V_1)$

$W = 2\times10^5 (0.03 - 0.02)$

$W = 2\times10^5 \times 0.01$

W= 2000 Joules

ii) At constant pressure

$\frac{V1}{T1} = \frac{V2}{T2}$

$\frac{0.02}{300} = \frac{0.03}{T2}$

$T_2 = \frac{9}{0.02}$

T_2 = 450k

T_2 = 450-273

$T_2 = 177^0$c

iii) $u\Delta = nCv\Delta T$

$\Delta u = \frac{mass}{RMM} \cdot Cv\Delta T$

$\Delta u = \frac{16}{32} \times 0.8 \times (450-300)$

$\Delta u = 0.5 \times 0.8 \times (150)$

$\Delta u = 60$ Joules

3) A cylinder contains 4moles of oxygen gas at temperature of 27^0C. The cylinder is fitted with frictionless piston which maintains constant pressure of 1.5×10^5Pa. The gas is heated until temperature increases to 127^0C. Calculate;

(i) The amount of heat supplied to gas

(ii) What is the change in internal energy of gas?

(iii) What is the work done by the gas ($Cp = 29.4 Jmol^{-1}K^{-1}$, $R = 8.31\ Jmol^{-1}K^{-1}$)

Solution

$T_1 = 27°C = 300K$

$T_2 = 127°C = 400K$

i) $\Delta QP = nCp\Delta T$

$\Delta Qp = 4 \times 29.4 \times (400-300)$

$\Delta QP = 117.6 \times 100$

$\Delta QP = 11760$ J

ii) $\Delta u = nCv\Delta T$

$\Delta u = nCv\ (T_2 - T_1)$

But $Cv = Cp - R$

$Cv = 29.4 - 8.31$

$Cv = 21.09$

$\Delta u = 4 \times 21.09\ (400-300)$

$\Delta u = 84.36 \times 100$ $\Delta u = 8436$ J

iii) $\Delta Q = \Delta u + \Delta w$

$\Delta w = \Delta Q - \Delta u$

$\Delta w = 11760 - 8436$

$\Delta w = 3324$ J

4) 10moles of gas initially at 27°C is heated at constant pressure of 1.01×10^5Pa. As volume increases from $0.250m^3$ to $0.375m^3$. Calculate the increase in internal energy (assume Cp=28.5J/mol/K) (06marks)

Solution

$V_1 = 0.250m^3$ $V_2 = 0.375m^3$ $T_1 = (27^0+273) = 300K$, $T_2 = ?$

At constant pressure

$\frac{V1}{T1} = \frac{V2}{T2}$

$\frac{0.250}{300} = \frac{0.375}{T2}$

$T_2 = \frac{0.375x300}{0.250}$

$T_2 = 450k$

$\Delta u = nCv\Delta T$

$\Delta u = 10 \times (Cp-R)\ (T_2 - T_1)$

$\Delta u = 10 \times (285-8.31)\ (450-300)$

$\Delta u = 30285$ J

5) 1kg of water is converted to steam at temperature of 100℃ and pressure of 1.0×10^5Pa. if the density of steam is $0.58kgm^{-3}$ and S.L.H.V of water is $2.3\times10^6 Jkg^{-1}$. Calculate the;

(i) External work done (ii) The internal energy

Solution

i) $\Delta Q = MLv$

$\Delta Q = 1\times 2.3\times10^6$

$\Delta Q = 2.3\times10^6$ J

ii) $\Delta w = P\Delta v$

$\Delta w = P(\frac{Ms}{\rho_S} - \frac{Mw}{\rho_w})$.

$\Delta w = 1\times10^5 (\frac{1}{0.58} - \frac{1}{1000})$

$\Delta w = 1\times 10^5 (1.724-0.001)$

$\Delta w = 1\times10^5\times1.723$

$\Delta w = 172{,}300$Joules

$\Delta Q = \Delta u + \Delta w$

$\Delta u = \Delta Q - \Delta w$

$\Delta u = 2.3\times10^6 - 172300$

$\Delta u = 2.1277\times10^6$ J

EXERCISE6

1) Nitrogen gas is trapped in the container by movable piston. If temperature of gas is raised from 0℃ to 50℃ at constant pressure of 4.0×10^5Pa and total heat added is 3.0×10^4 J. Calculate the work done by the gas [$Cp=29.1Jmole^{-1}K^{-1}$, $\frac{C_P}{C_V}= 1.4$]

(Ans 8.57×10^3J).

2) At a temperature of 100℃and a pressure of 1.01×10^5Pa, 1.00kg of steam occupies $1.67m^3$ but the same mass of water occupies only $1.04\times10^{-3}m^3$. The S.L H of vaporization of water at 100℃ is $2.26\times10^6Jkg^{-1}$. For a system consisting of 1.00kg water changing to steam at 100℃ and 1.01×10^5Pa find;

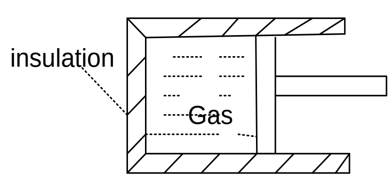
(a) The heat supplied to the system

(b) The work done by the system

(c) The increase in internal energy of the system.

An [2.26×10^6J, 1.69×10^5 J, 2.09×10^6J]

3) Some gas, assumed to behave ideally, is contained within a cylinder which is surrounded by insulation to prevent loss of heat as shown below.



Initially the volume of gas is $2.9\times10^{-4}m^3$, its pressure is 1.04×10^5Pa and its temperature is 314K.

(a) Use the equation of state for an ideal gas to find the amount in moles of gas in the cylinder.

(b) The gas is then compressed to a volume of $2.9x10^{-5}m^3$ and its temperature rises to 790K. Calculate the pressure of the gas after its compression.

(c) The work done on the gas during the compression is 91J. Use the first law of thermodynamics to find the increase in the internal energy of the gas during the compression.

(d) Explain the meaning of internal energy as applied to this system and use your result in (c) to explain why a rise in the temperature of the gas takes place during the compression.[Molar gas constant $=8.3Jk^{-1}mol^{-1}$] **An [$1.2x10^{-2}$, $2.6x10^{6}$Pa, 91J]**

4) The specific latent heat of vaporization of a particular liquid at 130^0C and a pressure of $2.60x10^5$Pa is $1.84x10^6Jkg^{-1}$. The specific volume of the liquid under these conditions is $2x10^{-3}m^3kg^{-1}$ and that of the vapour is $5.66x10^{-1}m^3kg^{-1}$. Calculate;

(a) The work done and

(b) The increase in internal energy when 1.00kg of the vapour is formed from the a liquid under these conditions. **An[$1.47x10^5$J, $1.69x10^6$J]**

5) A mass of 0.35kg of ethanol is vaporized at its boiling point of 78^0C and a pressure of $1.0x10^5$Pa. At this temperature, the specific latent heat of vaporization of ethanol is 0. $95x10^6Jkg^{-1}$ and the densities of the liquid and vapour are $790kgm^{-3}$and $1.6kgm^{-3}$ respectively. Calculate;

(i) The work done by the system.

(ii) The change in internal energy of the system. **An[$2.2x10^4$J, $3.1x10^5$ J].**

6) (a)A cylinder fitted with an apparatus which can move without friction contains 0.05moles of monatomic ideal gas at a temperature of 27^0C and a pressure of $1.0x10^5$Pa .The cylinder is calibrated to determine the boiling point of a liquid of boiling point 350K. Calculate;.

(i) The volume

(ii) The internal energy of the gas

(b) The temperature of the gas in (a) is raised to 77^0C, the pressure remaining constant. Calculate;

(i) The change in internal energy

(ii) The external work done

(iii) The total heat energy supplied (molar gas constant $8.3Jmol^{-1}k^{-1}$)

An ([$1.2x10^{-3}m^3$, $1.9x10^2$J] [31J, 21J, 52J])

7) (a)A quantity of 0.2moles of air enters a diesel engine at a pressure of $1.04 x 10^5$Pa and at a temperature of 297K. Assuming that air behaves as an ideal gas, find the volume of this quantity of air. **An[$4.75 x 10^{-3}$m^3]**

(b) The air is then compressed to one twentieth of this volume, the pressure having risened to $6.89 x 10^6$Pa. Find the new temperature. **An[984K]**

(c) Heating of the air then takes place by burning small quantity of fuel in it to supply 6150J. This is done at a constant pressure of $6.89 x 10^6$Pa and the volume of air increases and the temperature rises to 2040K. find;

(i) the molar heat capacity of air at constant pressure

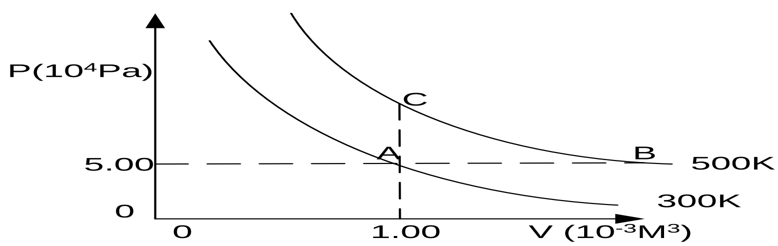
(ii) The volume of air after burning the fuel

(iii) The work done by the air during this expansion

(iv) The change in the internal energy of the air during this expansion.

(Molar gas constant = 8.31JK^{-1}mol^{-1})**An[29.1JK^{-1}mol^{-1}, $4.92 x 10^{-4}$m^3, 1.76x103J, $4.39 x 10^3$J]**

8) The diagram shows curves relating pressure, P and volume V for a fixed mass of an ideal monatomic gas at 300K and 500K. The gas is in a container fitted with a piston which can move with negligible friction.



(a) Give the equation of state for n moles of an ideal gas, defining the symbols used. Show by calculation that;

(i) The number of moles of gas in the container is $2.01 x 10^{-2}$

(ii) The volume of the gas at B on the graph is $1.67 x 10^{-3}$m^3, R= 8.31Jmol^{-1}k^{-1}

9) A steel pressure vessel of volume $2.2 x 10^{-2}$m^3 contains $4.0 x 10^{-2}$kg of a gas at a pressure of $1.0 x 10^5$Pa and temperature 300K. An explosion suddenly releases $6.48 x 10^4$J of energy, which raises the pressure instantaneously to $1.0 x 10^6$Pa. Assuming no loss of heat to the vessel, and ideal gas behavior calculate;

(a) The maximum temperature attained

(b) The two principal specific heat capacities of the gas.

(c) What is the velocity of sound in this gas at a temperature of 300K?

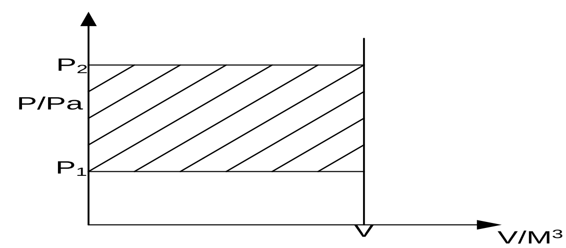
An[3000K, 600Jkg^{-1}K^{-1}, 783 Jkg^{-1}K^{-1}, 268ms^{-1}]

10) (a)A vessel of volume $1.0 \times 10^{-2} m^3$ contains an ideal gas at a temperature of 300K and pressure 1.5×10^5Pa. Calculate the mass of a gas given that the density of the gas at a temperature 285K and pressure 1.0×10^5Pa is $1.2 kgm^{-3}$.

(b) 750J of heat energy is suddenly released in the gas causing an instantaneous rise of pressure to 1.8×10^5pa. Assuming ideal gas behavior and no loss of heat to the containing vessel, calculate the temperature rise and hence the specific heat capacity at constant volume of the gas. **An[1.7×10^{-2}kg, 60K, $7.3 \times 10^2 Jkg^{-1}K^{-1}$]**

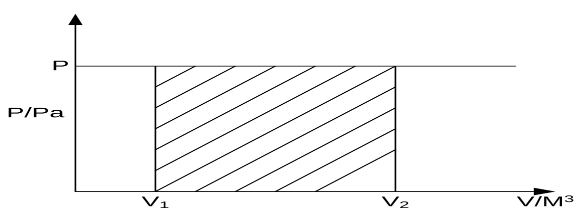
3.6.0: ISOVOLUMETRIC PROCESS [VOLUME CONSTANT]

This is the process which occurs at constant volume. The conditions for it to occur is that the gas must be contained in a sealed vessel. I.e. $\Delta w = 0$ since $\Delta v = 0$



3.6.1: ISOBARIC PROCESS (PRESSURE CONSTANT)

This is the process which occurs at constant pressure. The condition for it to occur is that the gas must be enclosed in the cylinder with frictionless movable piston. At any instant the pressure of the gas is equal to external pressure.



$\Delta w = P(V_2 - V_1)$

Δw = area under graph

3.7.0: ISOTHERMAL AND ADIABATIC PROCESS

a)ISOTHERMAL PROCESS

Is the change (expansion or compression) which occurs at constant temperature?

For an isothermal change PV = constant. Heat must be supplied at the same rate as the gas is doing its work

$$\Delta Q = \Delta u + \Delta w$$

But $\Delta u = nCv\Delta T$ but $\Delta T = 0$ $\Delta u = 0$

$$\therefore \Delta Q = \Delta w \text{ .... (x)}$$

Equation (x) above implies that in an isothermal change all heat supplied to gas must be used to do external work.

REVERSIBLE ISOTHERMAL CHANGE:

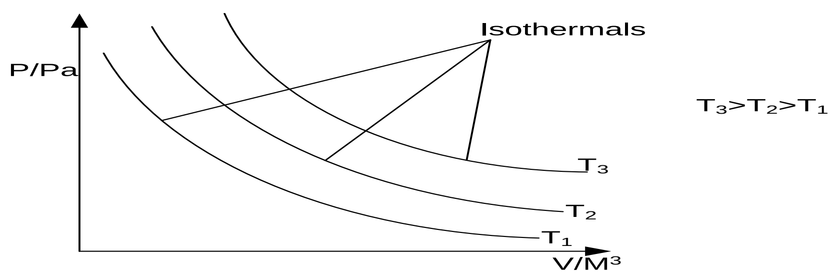
It's defined as, a change that occurs at constant temperature and can be made to go in the reverse direction by an infinitesimal change in the conditions causing it to take place

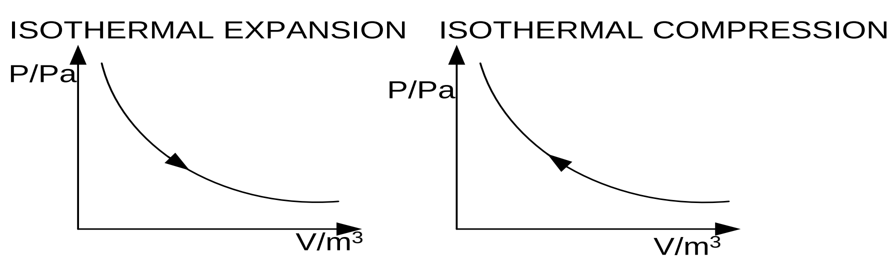
3.7.1: CONDITIONS FOR ISOTHERMAL PROCESS

- ❖ The gas must be contained in cylinder with very thin, highly conducting walls so that heat can easily be transferred to a gas.
- ❖ The gas cylinder must be surrounded by constant temperature bath
- ❖ The process must be carried slowly to allow enough time for heat transfer.

ISOTHERMALS

These are graph showing variation of pressure and volume at constant temperature.





3.7.2: EQUATION FOR AN ISOTHERMAL PROCESS

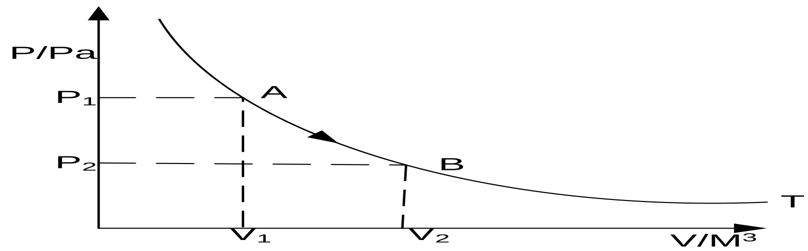
Consider an isothermal expansion of the gas from V_1 to V_2, then using the equation of state.

$PV = nRT$ $\qquad$ $i.e\ \boxed{P_1V_1 = P_2V_2}$

All isothermals obey Boyle's law

3.7.3:WORK DONE (ΔW) IN AN ISOTHERMAL EXPANSION

Consider an isothermal expansion from $V_1$to V_2



$$\Delta w = P\Delta v$$

$\int_o^w \Delta w = \int_{V1}^{V2} \mathrm{P\Delta v}$

W= $\int_{v1}^{v2} \mathrm{P\Delta v}$

But $PV = nRT$

P= $\frac{\mathrm{nRT}}{V}$

W= $\int_{V1}^{V2} \frac{\mathrm{nRT}}{V}$ dv

W= $nRT \int_{V_1}^{V_2} \frac{1}{V}$ dv

W= $nRT\ [1nV]_{V_1}^{V_2}$

W= nRT (1nV_2 -1nV_1)

$\boxed{W = n\,R\,T\,In\frac{V_2}{V_1}}$ OR $\boxed{W = P_1\,V_1 In\frac{V_2}{V_1}}$ OR $\boxed{W = P_2\,V_2 In\frac{V_2}{V_1}}$

b)ADIABATIC PROCESS (ΔQ = O)

An adiabatic process is a change (expansion or compression) in which there is no heat exchange between the gas and the surrounding.

Using the 1st law of thermal dynamics.

$\Delta Q = \Delta u + \Delta w$ But $\Delta Q = 0$

Therefor$e\ \Delta u = -\Delta w$ ………………………………………………… (xx)

- Equation (xx) shows that, in an adiabatic process the external work done in expanding the gas is at expense of internal energy and this result into cooling of the gas.

Question; explain why an adiabatic expansion results into cooling of the gas.

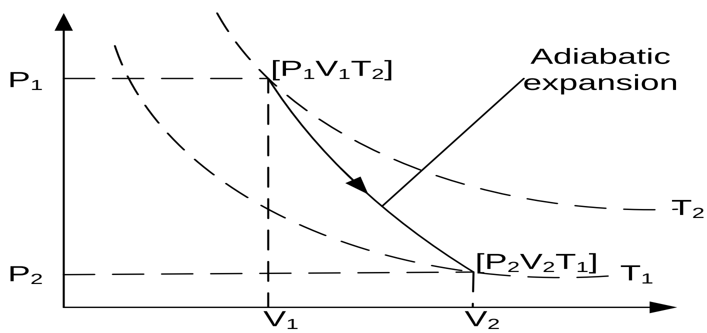
Solution

Consider gas enclosed in cylinder with frictionless movable piston. During an adiabatic expansion, the molecules of the gas bounce off the piston with the velocity less in magnitude than the incident velocity. Some of the kinetic energy is given up to the piston and this causes a decrease in average kinetic energy of molecules and consequently causes the gas to cool since the absolute temperature is proportional to mean kinetic energy of the molecules.

3.7.4: CONDITION FOR ADIABATIC PROCESS

- The gas must be contained in thick walled poorly conducting vessel
- The process must be carried out rapidly such that no heat leaves or enter system.

P- V GRAPH FOR ADIABATIC PROCESS



Reversible adiabatic change

This is a change in which there is **no** heat exchange between gas and surrounding and can be made to go in a reverse direction with an infinitesimal change in the condition causing the process .

3.7.5: EQUATION FOR ADIABATIC PROCESS

From the 1st law of thermal dynamics

ΔQ = $\Delta u + \Delta w$ ………………… (1)

But $\Delta u = Cv\Delta T$ for 1mole of gas

And $\Delta w = P\Delta V$

Putting these into equation1

$$\Delta Q = Cv\Delta T + P\Delta V$$

But for an a adiabatic process

ΔQ = 0

Therefore $Cv\Delta T + P\Delta V = 0$ …….(2)

From ideal gas equation

$Pv = RT$ for 1mole of gas

Differentiating it partially, gives

$$P\Delta V + V\Delta P = R\Delta T$$

$P\Delta V = R\Delta T - V\Delta P$ ……….(3)

Putting equation (3) into (2), gives

$Cv\Delta T + R\Delta T - V\Delta P = 0$ .....(4)

But $Cp - Cv = R$

$$Cv\Delta T + (Cp - Cv)\,\Delta T - V\Delta P = 0$$

$$Cv\Delta T + Cp\Delta T - Cv\Delta T - V\Delta P = 0$$

$$Cp\Delta T - V\Delta P = 0 \;\ldots\ldots\ldots\ldots(5)$$

From equation (2)

$$Cv\Delta T + P\Delta V = 0$$

$\Delta T = \frac{-p\Delta v}{C_V}$

Putting ΔT into equation (5)

$$Cpx\left(-\frac{P\Delta V}{Cv}\right) - V\Delta P = 0$$

$\frac{\mathrm{Cp}}{\mathrm{Cv}}$ $P\Delta V + V\Delta P = 0$

$\frac{\mathrm{Cp}}{Cv} = \gamma$

$\gamma\, P\Delta V + V\Delta P = 0$

Driving all through by PV

$\frac{\gamma\Delta V}{PV} + \frac{V\Delta P}{PV} = 0$

$\frac{\gamma\Delta V}{V} + \frac{\Delta P}{P} = 0$

Integrating all sides

$\gamma \int \frac{\Delta V}{V} + \frac{\Delta P}{P}$

γ1nV+1nP = constant

$$1nV^{\gamma} + InP = Inc$$

InPV^{γ}= Inc

PV^{γ} = Constant

$$\boxed{\boldsymbol{P_1V_1^{\gamma} = P_2V_2^{\gamma}}}$$

3.7.6: RELATIONSHIP BETWEEN TEMPERATURE 'T' AND VOLUME 'V' FOR AN ADIABATIC PROCESS

From PV^{γ} = Constant

But from ideal gas equation

PV = RT for 1 mole of gas

$\therefore$ P=$\frac{RT}{V}$

$\frac{RT}{V}.V^{\gamma}$ = Constant

RT$V^{\gamma-1}$ = Constant

T$V^{\gamma-1}$= $\frac{Constant}{R}$

T$V^{\gamma-1}$= Constant

$$\boxed{\boldsymbol{T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}}}$$

EXAMPLES

1) An ideal gas at 18^{0}C is compressed adiabatically until its volume is halved. Calculate the final temperature of gas (assume S.H.C of gas at constant pressure and volume are 2100$Jkg^{-1}\mathrm{K}^{-1}$ and 1500 $Jkg^{-1}\mathrm{K}^{-1} respectively$)

Solution

$T_1 = (18+273) = 291K$

T_2 =?

$V_1 = V$

$V_2 = \frac{V}{2}$

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

But $\gamma = \frac{CP}{CV}$

$\gamma = \frac{2100}{1500}$

$\gamma = 1.4$

$291xV^{1.4-1} = T_2\,X\left(\frac{V}{2}\right)^{1.4-1}{}_1$

$291\;X\;V^{0.4} = T_2\,x\left(\frac{V}{2}\right)^{0.4}$

$291x\;V^{0.4} = T_2\,x\frac{V^{0.4}}{2^{0.4}}$

$T_2 = 291x2^{0.4}$

$T_2 = 383.916K$

Therefore $T_2 = (383.976 - 273)^0C$

$T_2 = 110.976^0C$

2) A mass of an ideal gas of volume 200m^3 at 144K expands adiabatically to temperature of 137K. Calculate its new volume. [Take $\gamma = 1.4$]

Solution

T_1 = 144k

V_1 = 200m^3

T_2 = 137k

V_2 = ?

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$144x\,(200)^{1.4-1} = 137x\,V_2^{1.4-1}$

$144x200^{0.4} = 137XV^{0.4}$

$1198.87 = 137XV^{0.4}$

$V_2^{0.4} = \frac{1198.87}{137}$

$V_2^{0.4} = 8.75092$

$V_2 = [(8.75092)]^{\frac{1}{0.4}}$

V_2 = 226.53m^3

3) The temperature of 1 mole of helium gas at pressure of 1.0x10^{5}Pa increases from 20^{0}C to 100℃ when the gas is compressed adiabatically. Find the final pressure of gas. [$\gamma = 1.67$]

Soluion

$\therefore P_1 = 1.0x10^5$

T_1= 20+273 =293K

T_2 = 100+273=373K

P_2 = ?

$P_1V = nRT$

$P_1V_1 = nRT_1$

$1x10^5\ V_1 = 8.31x293$

V_1 = 0.0243m^3

Using

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$293x0.0243^{1.67-1} = 373V_2^{1.67-1}$

$293x0.0243^{0.67} = 373V_2^{0.67}$

$\frac{24.2786}{373} = V_2^{0.67}$

$0.0631 = V_2^{0.67}$

$V_2 = (0.0651)^{\frac{1}{0.67}}$

V_2 = 0.0169m^3

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$1x\ 10^5x\ 0.0243^{1.67} = P_2x0.0169^{1.67}$

$201.355 = P_2x0.0169^{1.67}$

P_2=1.83x10^{5}Pa

4) a)(i)What is meant by isothermal and adiabatic?

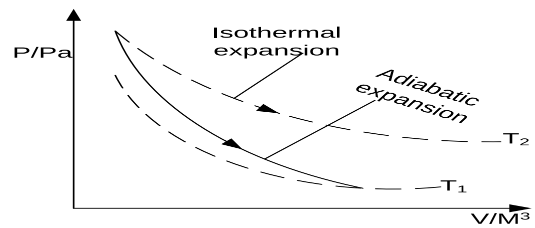
(ii)Using the same axes and starting from same point, sketch P.V diagram to illustrate changes in a)i) above.

(b)An ideal gas is trapped in cylinder by a movable piston. Initially it occupies a volume of 8x10^{-3}m^3 and exerts pressure of 108kPa. The gas undergoes an isothermal expansion until its volume is 27x10^{-3}m^3. It is then compressed adiabatically to the original volume of the gas.

(i) Calculate the final pressure of the gas

(ii) Sketch a well labeled diagram for the two stages of gas on P-V diagram. [The ratio of principal molar heat capacity of gas is 5:3]

Solution



b)(i)

$V_1 = 8x10^{-3}$, $P_1 = 108X10^3 Pa$, T_1 =?

Isothermal expansion

$V_2 = 27x10^{-3} m^3$

P_2 = ? $T_2 = T_1$

For $P_1V_1T_1 \rightarrow P_2V_2T_1$ (isothermal)

$P_1V_1 = P_2V_2$

$108x10^3x10^3 = P_2 x\ 27x10^{-3}$

$P_2 = \frac{108x103x103}{27x10-3}$

P_2 = 32000pa

ii) **Adiabatic**

$P_2\ V_2\ T_1 \rightarrow P_3\ V_1\ T_2$

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

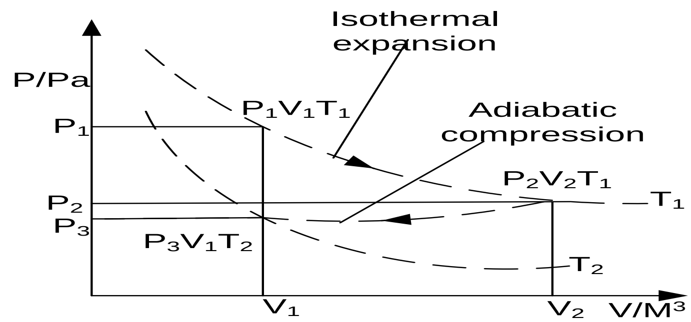
$32000x(27x10^{-3})^{1.67} = P_3\,(8x10^{-3})^{(1.67)}$

$76.829 = 0.000314891P_3$

$P_3 = \frac{76.829}{0.000314891}$

$P_3 = 2.439x10^5 Pa$

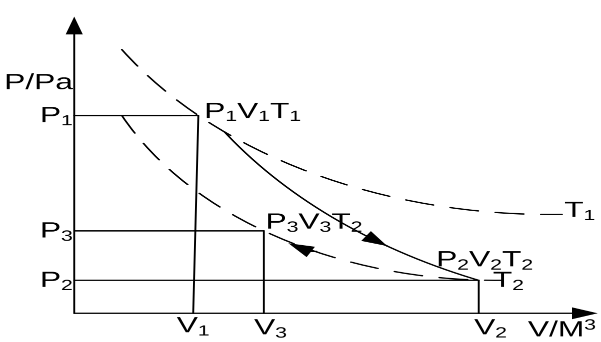
(ii)



5) Amass of air occupying initially a volume $2x10^{-3}m^3$ at a pressure of 760mmHg and temperature 20℃ is expanded a diabatically and reversibly to twice its volume and then compressed isothermally and reversibly to volume of $3x\ 10^{-3}m^3$.Find the temperature and pressure, assume that $\gamma = 1.4$

Solution

$V_1 = 2x10^3m^3$, P_1= 760mmHg, $T_1 = (20^0C+273) = 293K$, Adiabatically $V_2 = 2x2x10^{-3}m^3$



Adiabatic

$P_1\ V_1\ T_1 \rightarrow P_2V_2T_2$

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$293\ x(\ 2x10^{-3})^{1.4-1} = T_2\ x2x2x10^{-3(1.4-1)}$

$293\ x\ 0.0832 = 0.1098\ T_2$

$24.3938 = 0.1098\ T_2$

$T_2 = \frac{24.39838}{0.1098}$

$T_2 = 222.09K$

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$760\ x\ (\ 2x10^{-3})^{1.4} = P_2 x\ (\ 4x10^{-3})^{1.4}$

$0.1265 = 0.000439\ P_2$

P_2 = 287.8mmHg

Isothermal

$P_2\ V_2\ T_2 \rightarrow P_3\ V_3\ T_2$

Isothermal obey Boyles law

$P_2\ V_2 = P_3\ V_3$

$287.8 \times 4 \times 10^{3} = P_3 \times 3 \times 10^{-3}$

$1.1512 = 0.003\ P_3$

$P_3 = 383.7$mmHg

Therefore final temperature is 222K and final pressure is 383.7mmHg.

6) Show on the same graph starting on the same point P_1V_1 on P-V sketch curve for a fixed mass of an ideal gas undergoing the following process.

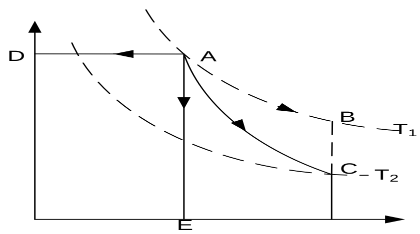
(i) Isothermal process

(ii) Adiabatic process

(iii) Isovolumetric process

(iv) Isobaric process

Solution



AB= isothermal expansion

AC = adiabatic expansion

AE = isovolumetric

AD = isobaric

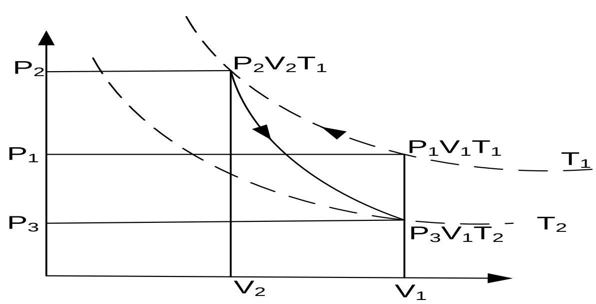
7) A vessel contains $2.5 \times 10^{-3}m^3$ of an ideal gas at pressure of $8.3 \times 10^4 Nm^2$ and temperature of 35^0C. the gas is compressed isothermally to volume of $1.0 \times 10^{-3}m^3$. It is then allowed to expand adiabatically to the original volume ($\gamma = 1.4$). Calculate ;

(i) Find temperature of the gas

(ii) Work done during isothermal compression of the gas.

Solution

$V_1 = 25 \times 10^{-3}m^3$, T_1 = 35+273 = 308K, $P_1 = 8.5 \times 10^4 Nm^{-2}$, Isothermal $V_2 = 1.0 \times 10^{-3}m^3$



i) Isothermal

$P_1\ V_1 T_1 \rightarrow P_2\ V_2\ T_1$

$P_1\ V_1 = P_2\ V_2$

$8.5 \times 10^4 \times 2.5 \times 10^{-3} = P_2 \times 1.0 \times 10^{-3}$

$P_2 = 2.125 \times 10^5$Pa

Adiabatic

$P_2\ V_2\ T_1 \rightarrow P_3\ V_1\ T_2$

$P_2 V_2^{\gamma} = P_3 V_1^{\gamma}$

$P_3 = \frac{2.125x10^5 x(10^{-3})^{1.4}}{2.5x(10^{-3})1.4}$

$P_3 = 50917.2959$

$P_3 = 50.917 \times 10^3 Pa$

$T_1 V_2^{\gamma-1} = T_2 V_1^{\gamma-1}$

$308 \times 1.0 \times 10^{-3(1.4-1)} = 2.5 \times 10^{-3(1.4-1)}\ T_2$

$19.433 = 0.091\ T_2$

$T_2 = 213.48K$

ii) $\Delta w = -\ P_1V_1 1n\frac{V1}{V2}$

Δw = - 8.5x10x2.5x10^{-3} 1n($\frac{2.3x10-3}{1.0x10-3}$)

Δw = -212.5 x1n2.5

Δw = -195Joules

The negative sign implies work done on the gas

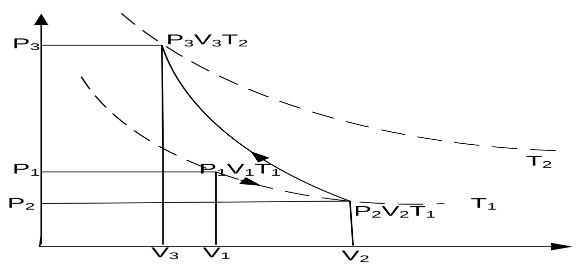
8) A gas having a temperature of 27^{0}C volume of 30000cm^3 and pressure of 80cmHg expands isothermally to double its volume. The gas is then adiabatically compressed to half its original volume.

(i) Represent these changes on P-V sketch

(ii) Calculate final pressure and temperature of gas ($\gamma = 1.4$)

Solution

(i)



(ii) T_1= 27+237 = 300K

V_1 = 3000x 10^{-6}m^3, V_1= 3x10^{-3}m^3

P_1 = 80cmHg

Isothermally $V_2 = 2V_1$ = 6x10^{-3}

Isothermal $P_1V_1T_1 \rightarrow P_2V_2T_1$

$P_1V_1 = P_2V_2$

80x3x10^{-3} = P_2x6x10^{-3}

$P_2 = \frac{80x3x10-3}{6x10-3}$

P_2 = 40cmHg

Adiabatic

$P_2V_2T_2 \rightarrow P_3 V_3T_2$

But $V_3 = \frac{1}{2}V_1$ = 1.5x 10^{-3}m^{-3}

$P2V_2^{\gamma} = P_3V_3^{\gamma}$

40x6x10$^{-3(1.4)}$ = P_3x1.5 x10$^{-3(1.4)}$

$P_3 = \frac{0.01514}{0.0000946}$

P_3 = 278.57cmHg

$T_1V_2^{\gamma-1} = T_2V_3^{\gamma-1}$

T_2 = 300 ($\frac{6x10}{1.5x10}$)

T_2 = 522.3K

Final pressure = 278.5cmHg and final temperature = 522.3K

9) A cylinder with piston containing 1 mole of gas at pressure of 1x10^{5}Pa with temperature of 300K. The gas is heated at constant pressure until its volume doubles. It is then compressed isothermally back to its original volume and finally it is cooled at constant volume to the original state.

(i) Represent the above process on P-V diagram.

(ii) Calculate the total work done in the above processes

Solution

n= 1mole V_1= 22.4x10^{-3}m^3

P_1= 1x 10 ^{3}pa T_1 = 300k

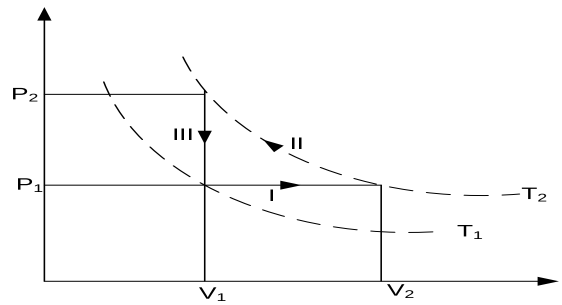
Isobaric = $P_1 = P_2$

$V_2 = 2V_1$ = 44.8x10^{-3}m^3

i)

Isothermally: temperature constant

$V_3 = V_1 = 22.4 x 10^{-3} m^3$



$P_1V_1T_1 \rightarrow P_1V_2T_2$

Isobars obey Charles law

$\frac{V1}{T1} = \frac{V2}{T2}$

$\frac{22.4x10}{300} = \frac{44.8x10^{-3}}{T2}$

$T_2 = \frac{13.44}{0.0224}$

$T_2 = 600K$

Isothermal compression

$P_1V_2T_2 \rightarrow P_2V_1T_2$

$P_1V_2 = P_2V_1$

$1x10^5 x\ 44.8x10^{-3} = P_2\ x22.4x10^{-3}$

$P = \frac{4480}{0.0224}$

$P = 2x10^5 Pa$

Isovolumetric

$V_4 = V_1$

Work done in (i)

Isobaric

$\Delta w = P_1(V_2 - V_1)$

$\Delta w = 1x10^5(44.8x10^{-3} - 22.4x10^{-3})$

$\Delta w = 10^5\ (22.4x10^{-3})$

$\Delta w = 2240 Joules$

Work done in (ii)

Isothermal

$\Delta w = -P_1V_2\ 1n\ \frac{V1}{V2}$

$\Delta w = -10^5 x 44.810^{-3}\ 1n\ (\frac{22.4x10}{44.8x10})$

$\Delta w = 3.105x10^3 Joules$

Work done in (iii)= 0

because there is no volume change

Total work done = work done in i and ii.

$= 2240+3.105x10^3$

$= 5345$ or

$= 5.345x10^3$ Joules

EXERCISE7

1) A gas is confined in the container of volume $0.1m^3$ at pressure of $1x10^5 Pa$ and temperature of 300K. if the gas is assumed to be ideal. Calculate the density of gas (RMM of the gas is 32)

2) An ideal gas at 27^0C and at pressure of $1.01x10^5 Pa$ is compressed reversibly and isothermally until its volume is halved. It is then expanded reversibly and adiabatically to twice its original volume. Calculate the final pressure and temperature of the gas if $\gamma = 1.4$

3) Air is contained in a cylinder by a frictionless gas tight piston.

(a) Find the work done by the gas as it expands from a volume of $0.015m^3$ to a volume of $0.027m^3$ at a constant pressure of $2.0x10^5Pa$

(b) Find the final pressure if, starting from the same initial conditions as in (a) and expanding by the same amount, the change that occurs

(i) Isothermally (ii) adiabatically

($\gamma of air = 1.40$) **An[$2.4x10^3J$, $1.1x10^5Pa$, $8.8x10^4Pa$]**

4) The cylinder in fig1 below holds a volume V_1 = $1000cm^3$ of air at an initial pressure P_1 = $1.10x10^5Pa$ and temperature T_1 = 300K. Assume that air behaves like an ideal gas.

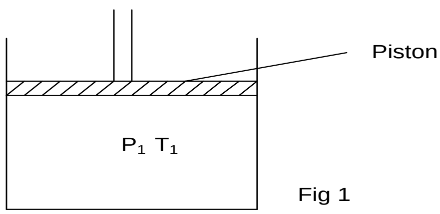


Fig 1

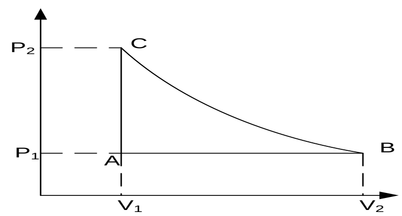


Fig 2

Fig 2 shows a sequence of changes imposed on the air in the cylinder.

AB – the air is heated to 375K at constant pressure. Calculate the new volume V_2.

BC – the air is compressed isothermally to volume V_1. Calculate the new pressure P_2.

CA – the air cools at constant volume to pressure P_1. State how a value for the work done on the air during with sequence of change may be found from the graph in fig 2 . **An1250cm^3, $1.38x10^5Pa$**

5) A vessel of volume $8.00x10^3m^3$ contains an ideal gas at a pressure of $1.14x10^5Pa$. A stop cock in the vessel is opened and the gas expands a adiabatically, expelling some of its original mass, until its pressure is equal to that outside the vessel ($1.0x10^5Pa$). The stop cock is then closed and the vessel is allowed to stand until the temperature returns to its original value in this equilibrium state, the pressure is $1.06x10^5Pa$.

(a) Explain why there was a temperature change as a result of adiabatic expansion

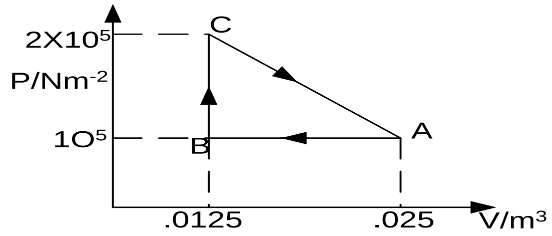
(b) Find the volume which the mass of gas finally left in the vessel occupied under the original conditions.

(c) Sketch a graph showing the way in which the pressure and volume of the mass of gas finally left in the vessel changed during the operations described above.

(d) What is the value of γ, the ratio of the principal heat capacities of the gas .

An [$7.44x10^{-3}m^3$, 1.66]

6)



The diagram represents an energy cycle where by a mole of an ideal gas is firstly cooled at constant pressure (A→B) then heated at constant volume (B→C) and then returned to its original state (C→A)

(i) Calculate the temperature of the gas at A, at B and at C
(ii) Calculate the heat given out by the gas in the process A→B
(iii) Calculate the heat absorbed in the process B→C
(iv) Calculate the net amount of work done in the cycle
(v) Calculate the net amount of heat transferred in the cycle

[R=8.3Jmol^{-1}k^{-1} Cv = $\frac{5}{2}$ R] **An[301.2K at A and C, 150.6K at B, 4375J, 3125J, 625J, 625J]**

7) A quantity of ideal gas whose ratio of principal molars heat capacities is $\frac{5}{3}$ has temperature 300K, volume $64\text{x}10^{-3}m^3$ and pressure 243kPa. It is made to undergo the following three changes in order
   A: reversible adiabatic compression to a volume $27\text{x}10^{-3}m^3$
   B: reversible isothermal expansion back to $64\text{x}10^{-3}m^3$
   C: a return to the original state

(i) calculate the pressure on completion of process A
(ii) Calculate the temperature at which process B occurs
(iii) Describe process C

UNEB 2011 Q 6

di) Distinguish between isothermal and a adiabatic changes (02marks)

ii)An ideal gas at 18^0C is compressed a adiabatically until the volume is halved. Calculate the final temperature of the gas.

(Assume specific heat capacities of the gas at constant pressure and volume are 2100Jkg^{-1}K^{-1} and 1500Jkg^{-1}K^{-1} respectively) **An[383.98k]** (4marks)

UNEB2010 Q.6

a)i)State the difference between isothermal and adiabatic expansion of a gas

ii)Using the same axes and point, sketch the graph of pressure verses volume for a fixed mass of gas undergoing isothermal and a adiabatic change (3marks)

b)Show that the work W done by a gas which expands reversibly from V_0 to V_1 is given by W= $\int_{v0}^{v1} pdv$ (4marks)

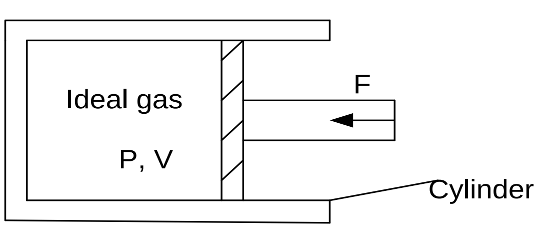
c)i)State two differences between real and ideal gases
ii)Draw labeled diagram showing P-V isothermal for a real gas above and below the critical temperature (3mark)
d)Ten moles of a gas initially at 27^0C and heated at a constant pressure $1.0x10^5$ Pa and the volume increased from $0.250cm^3$ to $0.375m^3$. Calculate the increases in internal energy [assume Cp = $28.5Jmol^{-1}K^{-1}$] (6mark) **An [$3.012x10^4$J]**

UNEB 2009 Q.6
a)i)State Boyle's law (01mark)
ii)Describe an experiment that can be used to verify Boy led law. (06mark)
c)i)What is meant by eversible process
ii)State the conditions necessary for isothermal and adiabatic process to occur
d)A mass of an ideal gas of volume $2000m^3$ at 144K expands adiabatically to a temperature of 137K. Calculate the new volume (take γ = 1.40) (3mark) **An [$226.47cm^3$]**

UNEB 2009 Q.6
a)i)State Boyle's law (01mark)
ii)Describe an experiment that can be used to verify Boy led law
c)i)What is meant by a reversible process
ii)State the conditions necessary for isothermal and adiabatic process to occur
d)A mass of an ideal gas of volume $200m^3$ at 144K expands adiabatically to a temperature of 137K. calculate the new volume (take γ = 1.40)**An[$226.47cm^3$]**

UNEB 2008 Q.6
a)Describe an experiment to verify Newton's law of cooling
c)ii)Nitrogen gas is trapped in a container by a movable piston. If the temperature of the gas raised from 0^0C to 50^0C at a constant pressure of $4x10^5$Pa and the total heat added is $3x10^4$J. Calculate the work done by the gas. **An[$8.57x10^3$]**
(Molar heat capacity of nitrogen at constant pressure is $29.1Jmol^{-1}k^{-1}$ $\frac{C_P}{C_V} = 1.4$)

UNEB 2007 Q,7
a)



A fixed mass of an ideal gas confined in a cylinder by a friction less piston of cross section area A . The piston is in equilibrium under the action of a force F as shown above. Show that the work done W by the gas when it expands from V_1 to V_2 is given by W= $\int_{v1}^{v2} pdv$
b)State the first law of thermodynamics and use it to distinguish between isothermal and adiabatic changes in a gas.

c)The temperature of one mole of helium gas at a pressure $1.0x10^5$Pa increases from 20^0C to $100^0C$when the gas is compressed adiabatically. Find the final pressure of the gas (take $\gamma = 1.67$**)** **An [1.83x10^{5}Pa**]

UNEB 2005 Q.6

a)i)What is meant by isothermal and adiabatic changes (02marks)

ii)Using the same axes, and starting from the same point, sketch a P-V diagram to illustrate the change in a(i) (02marks)

b)An ideal gas is trapped in a cylinder by a movable piston. Initially it occupies a volume of $8x10^{-3}m^3$ and exerts a pressure of 108kPa. The gas volume is $27x10^{-3}m^3$. It is then compressed adiabatically to the original volume of the gas

i)Calculate the final pressure of the gas (06marks)

ii)Sketch and label the two stages of the gas on a P-V diagram [the ratio of principal molar heat capacities of the gas = 5:3] **An [2.43x10^{3}Pa**] (02marks)

c)i)Define molar heat capacities at constant pressure. (01mark)

ii)Derive the expression C_P-Cv = R for 1mole of a gas (05mark)

iii)In what ways does a real gas differ from an ideal gas (01mark)

UNEB 2004 Q.7

b)A gas is confined in a container of volume $0.1m^3$ at a pressure of $1.0x10^5Nm^{-2}$ and a temperature of 300K. if the gas is assumed to be ideal calculate the density of the gas (05marks)[the relative molecular mass of the gas is 32]**An [7.71x10^{23}kgm^{-3}**]

c)What is meant by

i)Isothermal change (01mark)

ii)Adiabatic change (01mark)

d)A gas at a pressure of $1.0x10^6$Pa is compressed adiabatically to half its volume and then allowed to expand isothermally to its original volume. Calculate the final pressure of the gas. [assume the ratio of the principal specific heat capacities $\frac{cp}{cv} = 1.4$] (05marks)

An[1.32x10^{6}Pa]

UNEB 2003 Q.5

a)i)Define molar heat capacity of a gas at constant volume. (1mark)

ii)The S.H.C of oxygen at constant volume is $719Jkg^{-1}K^{-1}$. If the density of oxygen at S.T.P is $1.429kgm^{-3}$. Calculate the S.H.C of oxygen at constant pressure (04marks)

Solution

At S.T.P n= 1mole T= 273k

P = $1x 10^5$pa

PV=nRT

$1x10^5$v = 273R

V= $\frac{273x8.31}{1x10^5}$

V= $0.0227m^3$

$\rho = \frac{m}{v}$
m=1.429x0.0227
m=0.0324kg
But Cp –Cv =R where Cp and Cv are molar heat capacities

$mc_p - mc_v = R$ where c_p and c_v are S.H.C are constant pressure and volume respectively
0.0324cp – 0.0324x719 = 8.31
$c_p = \frac{8.31\ x6.0324x719}{0.0324}$
c_p = 975. 5 $Jkg^{-1}k^{-1}$

UNEB 2002 Q.5
d)i)What is meant by a reversible isothermal change (02marks)
ii)State the conditions for achieving a reversible isothermal change .(02marks)
e)An ideal gas at 27^0C and at a pressure of $1.0x10^5Pa$ is compressed reversibly and isothermally until its volume is halved. It is then expanded reversibly and adiabatically to twice its original volume. Calculate the final pressure and temperature of the gas if γ =1.4 **An [2.9x10^4Pa]**

UNEB 2001 Q.6
a)i)Explain what happens when a quantity of heat is applied to a fixed mass of gas (02marks)
ii)Derive the relation between the principal molar heat capacities Cp and Cv for an ideal gas (05marks)
b)i)What is an adiabatic process (1mark)
ii)A bicycle pump contains air at 290K. The piston of the pump is slowly pushed in until the volume of the air pump. The outlet is then sealed off and the piston suddenly pulled out to full extension. If no air escapes. Find its temperature immediately after pulling the piston (take $\frac{cp}{cv} = 1.4$) **An[152.3K]**

CHAPTER4: KINETIC THEORY OF GASES

The kinetic theory of gases is the theory which explains the behaviour of gases in terms of the motion of it's molecules.
The kinetic theory can be used to explain why;

- A gas occupies all the space provided to
- Temperature of gas decreases in adiabatic expansion
- The volume of fixed mass of gas kept at constant temperature decreases as pressure is increased.
- A gas exerts pressure on wall of its container.

Explain why gas fills container in which it is placed and exerts pressure on the walls using kinetic theory of gases.

- The molecules of the gas are under continuous and random motion and therefore they fill the container

- When the molecule move they collide with the walls of the container and suffer a momentum change and so they exert an impulsive force on the walls of the container thereby exerting pressure on the walls.

Explain using kinetic theory why the pressure of fixed mass of gas rises when its temperature is increased at constant volume.

- When gas temperature increases, the average kinetic energy of molecules increases. So the number of collisions made by molecules with walls of the container per second increases. The momentum change per second also increases and this leads to an increase in impulsive force exerted on walls thereby increasing pressure exerted by the gas on the walls.

4.1: DERIVATION OF EXPRESSION OF PRESSURE EXERTED ON CONTAINER BY THE GAS (P $=\frac{1}{3}\rho\overline{C^2}$)

In deriving this expression, the following assumptions are considered;

- Intermolecular force (both attractive and repulsive) are negligible
- The volume of molecules themselves is negligible compared to the volume of container.
- The duration of collision is negligible as compared with time between collisions.
- The molecules of an ideal gas behave like perfectively elastic spheres and continuously colliding with themselves and walls of container and making perfectly elastic collision.

Derivation of expression

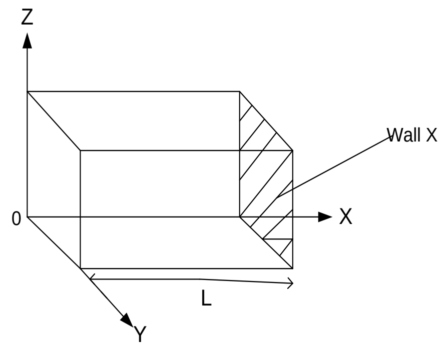
Consider cube of L containing N molecules moving randomly within cube. Let the cube contain one type of gas so that the mass of each molecule is the same.

Let $\overline{C}$ be the velocity of the molecule at some instant such that

$\overline{C} = \overline{Cx} + \overline{Cy} + \overline{Cz}$

Where

$\overline{Cx}=\overline{u}$ $\overline{Cy} = \overline{v}$ $\overline{Cz} = \overline{w}$



Change of momentum can impact = mu – (-mu) = 2mu

Time t take for molecules to move across the cube to the opposite face and back to wall X is

$$t = \frac{2L}{u}$$

$$\text{The force on X} = \frac{momentum\ change}{time} = \frac{2mu}{2L/u} = \frac{mu^2}{L}$$

$$\text{The pressure on X due to 1 molecule} = \frac{Force}{Area} = \frac{mu^2}{L}\Big/L^2 = \frac{mu^2}{L^3}$$

For N molecules the total pressure on wall X is given by;

$$P = \frac{m}{L^3}(U_1^2 + U_2^2 \ldots\ldots\ldots + U_N^2)$$

Let$\langle U^2 \rangle$ be mean value of all squares of the X-components of velocity

$$\langle U^2 \rangle = \frac{U_1^2 + U_2^2 \ldots\ldots\ldots + U_N^2}{N}$$

$$N\ \overline{U^2} = U_1^2 + U_2^2 \ldots\ldots\ldots + U_N^2$$

$$P = \frac{Nm\overline{U^2}}{L^3} = \rho\overline{U^2} \text{ since } \rho = \frac{Nm}{L^3}$$

With large number of molecules of varying speed in random motion, the molecules do not show any preferences in moving parallel to any direction.

Therefore

$$\overline{C^2} = \overline{U^2} + \overline{V^2} + \overline{W^2} \text{ and } \overline{U^2} = \overline{V^2} = \overline{W^2}$$

$$\overline{C^2} = \overline{U^2} + \overline{U^2} + \overline{U^2}$$

$$\overline{C^2} = 3\overline{U^2} \quad \therefore \quad \overline{U^2} = \frac{1}{3}\overline{C^2}$$

$$\boxed{\boldsymbol{P = \frac{1}{3}\rho\overline{C^2}}}$$

Since density, $\rho = \frac{NM}{V}$

$$P = \frac{1}{3}\frac{Nm}{V}\overline{C^2}$$

$$\boxed{PV = \frac{1}{3}Nm\overline{C^2}}$$

$$\boxed{P = \frac{1}{3}\frac{Nm}{L^3}\overline{C^2}}$$

4.1.1: RELATIONSHIP BTN MEAN KINETIC ENERGY AND ABSOLUTE TEMPERATURE

From

$PV = \frac{1}{3}Nm\overline{C^2}$…………………1

For an ideal gas

PV = nRT ………………..2

Equating equation (1) and (2) gives

$\frac{1}{3}Nm\overline{C^2}$= nRT

$m\overline{C^2}$=$\frac{3nRT}{N}$

Multiplying both side by half

$\frac{1}{2}m\overline{C^2}$ = $\frac{1}{2}$ X $\frac{3nRT}{N}$

But $\frac{1}{2}M\overline{C^2}$ = mean kinetic energy

Mean $K.e$ = $\frac{3}{2}\frac{nRT}{N}$

But for 1mole of gas N = N_A

$$\boxed{Mean\, K.e = \frac{3}{2}\frac{RT}{N_A}}$$

OR $\frac{1}{2}m\overline{C^2}$ =$\frac{3}{2}\frac{RT}{N_A}$

OR $\frac{1}{2}m\overline{C^2} = \frac{3}{2}kT$

where k is Boltzmann constant

From above equation

Mean kinetic energy ∝ temperature

Mean $K.e$ ∝ T

$i.e$ $\overline{C^2}$ ∝ T

$\sqrt{\overline{C^2}} \propto \sqrt{T}$

N.B: The number of molecules N is

$\boxed{N = nNA}$

The mass of molecules $M = mN$

Where m is mass of one molecule

OR $M = mn$N$_A$

For 1mole, $M = N_A$

EXAMPLES

1) Calculate the rms of the gas molecules and the speed of sound in the atmosphere of Jupiter given that the speed of sound in the gas is 0.682ms^{-1} , and the atmosphere of Jupiter contains mainly methane gas.
(Temperature of Jupiter atmosphere is -130℃) molecular weight of methane 16.04gmol^{-1} and the gas constant R = 8.31Jmol^{-1}K^{-1}).

Solution

T = -130 + 273 = 143K

$\frac{1}{2}m\overline{C^2}$ =$\frac{3}{2}\frac{RT}{N_A}$

$\overline{C^2}$ =$\frac{3}{M}\frac{RT}{N_A}$

But mN_A = $16.4x10^{-3}$kgmol^{-1}

$\overline{C^2}$ = $\frac{3X8.31X143}{16.04X10^{-3}}$

$\sqrt{\overline{C^2}} = \sqrt{222.256X10^3}$

$\overline{C}$ =4.71X10^2ms^{-1}

speed of sound in atmosphere =

= $0.682x4.71x10^2$

= 321.2ms^{-1}

2) Calculate the rms speed of molecule of an ideal gas at 130°C, given that the density of the gas at pressure of 1.0 x $10^5 Nm^{-2}$ and temperature of 0°C is 1.43kgm^{-3}

Solution

$C_1^{\bar{2}}$= ?, P_1 = 1.0 x 10^3, T_1 = 273K, ρ = 1.43kgm^{-3}, $C_2^{\bar{2}}$= ? T_2 = 400k

$P_1 = \frac{1}{3}\rho C_1{}^{\bar{2}}$

$C_1^{\bar{2}} = \frac{3P_1}{\rho_1}$

$C_1^{\bar{2}} = \frac{3X1.0X10^5}{1.43}$

$C_1^{\bar{2}} = 209.79X10^3 m/s$

$\sqrt{C^{\bar{2}}{}_1} \propto \sqrt{T_1}$

$\sqrt{C^{\bar{2}}{}_1} = \sqrt{T_1}$.........1

$\sqrt{C_2^{\bar{2}}} = \sqrt{T_2}$...............2

Dividing equation 1 and 2

$\frac{\sqrt{C^{\bar{2}}{}_1}}{\sqrt{C_2^{\bar{2}}}} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$

$\frac{\sqrt{209.79x10^3}}{\sqrt{C_2^{\bar{2}}}} = \frac{\sqrt{273}}{\sqrt{423}}$

$\sqrt{C_2^{\bar{2}}}$ **=556.4878m/s**

$\bar{C}_2$ **= 556.4878m/s**

3) Calculate the root mean square speed of the molecules of hydrogen at 27°C given that the density of hydrogen at pressure of 1.0 x $10^5 Nm^{-2}$ and a temperature of 0°C is $0.09kgm^{-3}$.

Solution

$C_1^{\bar{2}}$ = ?, P_1 = 1.0 x 10^5, T_1 = 273K, $C_2^{\bar{2}}$ = ? T_2 = 27 + 273 = 300K, ρ = $0.09kgm^{-3}$

$P_1 = ⅓\rho C_1^{\bar{2}}$

$C_1^{\bar{2}} = \frac{3P_1}{\rho}$

$C_1^{\bar{2}} = \frac{3X1X10^5}{0.09}$

$C_1^{\bar{2}}$ = 3.333 x $10^6 ms^{-1}$

$\sqrt{C^{\bar{2}}{}_1} \propto \sqrt{T_1}$

$\sqrt{C^{\bar{2}}{}_1} = \sqrt{T_1}$.........1

$\sqrt{C_2^{\bar{2}}} = \sqrt{T_2}$...............2

$\frac{\sqrt{C^{\bar{2}}{}_1}}{\sqrt{C_2^{\bar{2}}}} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$

$\frac{\sqrt{3.333X10^6}}{\sqrt{C_2^{\bar{2}}}} = \frac{\sqrt{273}}{\sqrt{300}}$

$C_2^{\bar{2}} = \mathbf{1.91389X10^3 m/s}$

EXERCISE8

1) A mole of an ideal gas at 300K is subjected to a pressure of 10^5Pa and it's volume is 0.025m^3 calculate

(a) the molar gas constant R

(b) the Boltzmann constant k

(c) the average translational kinetic energy of a molecule of the gas

(N_A = 6.0 x $10^{23} mol^{-1}$) **An (8.3$Jk^{-1}mol^{-1}$, 1.4 x $10^{-23} JK^{-1}$, 6.3 x 10^{-21}J)**

2) A vessel of volume 1.0 x $10^{-3} m^3$ contains helium gas at a pressure of 2.0 x 10^5Pa when the temperature is 300K.

(a) What is the mass of helium in the vessel

(b) How many helium atoms are there is the vessel

(c) Calculate the $r.m.s$ speed of the helium atoms.
(Relative atomic mass of helium = 4, the Avogadro constant = $6.0 \times 10^{23} mol^{-1}$ the molar gas constant R = $8.3 Jmol\ K^{-1}$) **An(0.32g, 4.8×10^{22}, $1.4 \times 10^{3} ms^{-1}$)**

3) What would be the total kinetic energy of the atoms of 1kg of neon gas at a pressure of 10^5Pa and temperature 293K, given that the density of neon under these conditions is $828gm^{-3.}$ What would be the total kinetic energy of the atoms of 1kg of neon gas at 300K. Hence determine the specific heat capacity of neon at constant volume. **An[$1.81(2) \times 10^5$J, $1.85(5) \times 10^5$J $6.1 \times 10^2 Jkg^{-1}K^{-1}$]**

4) **S**ome helium (molar mass = $0.004kgmol^{-1}$) is contained in a vessel of volume $8 \times 10^{-4} m^3$ at a temperature of 300K. The pressure of the gas is 200kPa. Calculate
(i) The mass of helium present
(ii) the internal energy (the translational kinetic energy of the gas molecules)
(molar gas constant = $8.3 JK^{-1}mol^{-1}$) **An [2.57×10^{-4}kg 240J]**

5) **A** cubical container of volume$0.10m^3$ contains Uranium hexafluoride gas at a pressure of 1.0×10^6Pa and a temperature of 300K. Assuming that the gas is ideal determine;
(i) the number of moles of gas present given that universal gas constant R = $8.3 JK^{-1}mol^{-1}$.
(ii) the mass of gas present, given that it's relative molecular mass is 352.
(iii) the density of the gas
(iv) the $r.m.s$ speed of the molecules **An (40.2, 14.1kg, $141kgm^{-3}$, $146ms^{-1}$)**

6) Helium gas is contained in a cylinder by a gas tight piston which can be assumed to move without friction. The gas occupies a volume of $1.0 \times 10^{-3} m^3$ at a temperature of 300K and a pressure of 1.0×10^5Pa
(a) calculate;
(i) the number of helium a toms in the container
(ii) the total kinetic energy of the helium atoms. **An(2.4×10^{22}, 150J)**
(b) Energy is now supplied to the gas in such away that the gas expands and the temperature remains constant at 300K. State and explain what changes, if any will have occurred in the following quantities
(i) the internal energy of the gas

(ii) the $r.m.s$ speed of the helium atoms
(iii) the density of the gas (the Boltzmann constant = $1.4 \times 10^{-23} JK^{-1}$)

7) Use the following data to calculate the root mean square speed of helium molecules at 2000°C
Mass of one mole of helium = 4g
Molar gas constant = $8.31 Jmol^{-1}K^{-1}$ **An[$3.76 \times 10^{3} ms^{-1}$]**

8) A cylinder of volume $0.080m^3$ contains oxygen at a temperature of 280K and a pressure of 90kPa. Calculate
(i) the mass of oxygen in the cylinder
(ii) the number of oxygen molecules in the cylinder.
(iii) the R.M.S speed of the oxygen molecules
(NA = $6.0 \times 10^{23} mol^{-1}$ R = $8.3 JK^{-1}mol^{-1}$ and molar mass of oxygen M = $0.032 kgmol^{-1}$)
An (9.9×10^{-2}kg, 1.9×10^{24}, $4.7 \times 10^{2} ms^{-1}$)

9) Helium gas occupies a volume of $0.04m^3$ at a pressure of 2×10^5 Pa and temp of 300K. Calculate;
(i) the mass of helium
(ii) the rms speed of its molecules
(iii) the rms at 432K, when the gas is heated at constant pressure to this temperature
(iv) the rms of hydrogen molecule at 432k (Rmm of helium and hydrogen, 4 and 2 respectively R = $8.31 Jmol^{-1}K^{-1}$) An[**12.8359g, 1368ms^{-1}**, **1643ms^{-1}, 2324ms^{-1}**]

10) 1g of hydrogen at $s.t.p$ has it's volume halfed by an adiabatic change. Calculate the change in internal energy of the gas.[R = $8.31 J\ mol^{-1}K^{-1}$, γ= 1.4]. **An [905.79J]**

4.2: DEDUCTIONS OF KINETIC THEORY

1.Boyles law:

It states that for a fixed mass of gas, the volume is inversely proportioned to pressure at constant temperature. i.e

PV = a constant
From kinetic theory
PV = ⅓ N$m\overline{C^2}$

Multiply both sides by ½
½ PV = ⅓N x ½$m\overline{C^2}$
But ½ $m\overline{C^2} \propto$ T

For a fixed mass of gas N is constant | If T is constant, Therefore PV = constant

2. Charles's law:

It states that the volume of fixed mass of a gas is directly proportional to absolute temperature at constant pressure.

From kinetic theory

$PV = \frac{1}{3}Nm\overline{C^2}$

Multiplying both sides by $\frac{1}{2}$

$\frac{1}{2}PV = \frac{1}{3}N \times \frac{1}{2}m\overline{C^2}$

Making V the subject

$V = \frac{2}{3}\frac{N}{P} X \frac{1}{2}m\overline{C^2}$

But $\frac{1}{2}m\overline{C^2} \propto T$

For a fixed mass of a gas N is constant,

Hence $V \propto T$

4. Daltons law of partial pressure:

It states that partial pressure of a mixture of gases which do not react chemicall is the sum of the partial pressure of component gases.

Note: partial pressure of gas, is the pressure the gas would have if it is to occupy the whole container alone.

Gas 1, P_1 V T N_1 m_1 $\overline{C_1^2}$: $P_1V = \frac{1}{3}N_1 m\overline{C_1^2}$

Multiplying both side by $\frac{1}{2}$

$\frac{1}{2}P_1V = \frac{1}{3}N_1 \times \frac{1}{2}m_1\overline{C_1^2}$

$\frac{3/2\ P1V}{½m1\overline{C_1^2}} = N_1$ ……………………………(1)

Where $\frac{1}{2}m_1\overline{C_1^2}$= mean K.e

Gas 2, P_2 V T N_2 m_2 $\overline{C_1^2}$: $\frac{3/2\ P2V}{½m2\overline{C_2^2}} = N_2$………………………… (2)

If the two gases are now mixed isothermally

Gas mixture P V T N m $\overline{C^2}$: $\frac{3/2\ PV}{½m\overline{C^2}} = N$………………………………(3)

Since gases are mixed

$N = N_1 + N_2$

$$\frac{3/2\ PV}{½m\overline{C^2}} = \frac{3/2\ P1V}{½m1\overline{C_1^2}} + \frac{3/2\ P2V}{½m2\overline{C_2^2}}$$

Since the temperature of the mixture is the same as that of the individual gas, then

$\frac{1}{2}m\overline{C^2} = \frac{1}{2}m_1\overline{C_1^2} = \frac{1}{2}m_2\overline{C_2^2}$

Hence $P = P_1 + P_2$

4.2: REAL GASES

Real gases obey ideal gas equation $ie\ PV = nRT$ only when they are at very low pressure and at temperature well above critical temperatures.

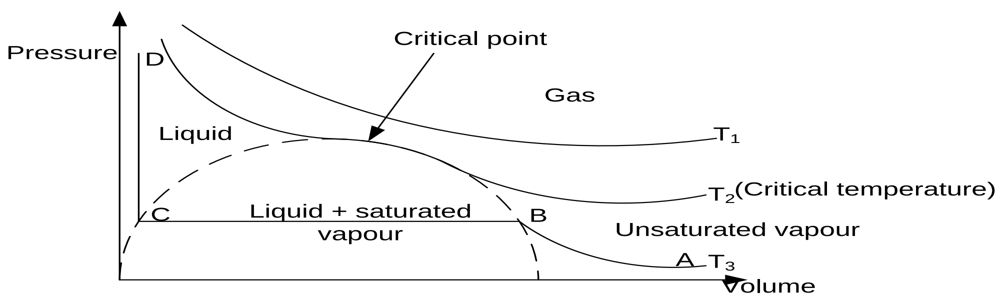
Note

Critical temperature of gas is the temperature above which the gas can not be liquefied by mere compression.

4.2.1: PROPERTIES OF REAL GASES

- Intermolecular forces of attraction and repulsion are not negligible
- Volumes of molecules are not negligible compared to volume of container
- The collision in real gases are inelastic
- They do not obey gas laws and equations

4.2.2: ANDREW'S EXPT ON COMPRESSION OF CARBONDIOXIDE AT DIFFERENT TEMPERATURES



- In this expt Andrew discovered that there exist a temperature above which the gas can not be liquefied by compression. This temperature is called critical temperature of the gas.
- At temperature above critical point the gas obey Boyle's law and no change of gas to liquid occurs.
- At temperatures below critical temperature, the point is reached during compression when the volume will change with out change in pressure ie BC. Here the liquid is seen to be formed.
- As compression continues there comes a time when pressure rises suddenly with out change in volume i.e CD. This is when all vapour has become liquid.

4.2.3: VANDER-WAAL EQUATION

Vander Waal modified the ideal gas equation by taking into account two of assumption made by kinetic theory to be valid.

The two assumptions include;

- The volume of molecules may not be negligible as compared to volume V occupied by the gas.
- The intermolecular forces of attraction between molecules may not be negligible.

1. The volume of the molecules may not be negligible in relation to volume V occupied by the gas. Molecules have particular volume because repulsive forces occur when they approach each other closely and hence can not be compressed indefinitely. Surrounding each molecule, there is definite volume called **co-volume** which can not be occupied by any other molecule. Therefore the free movement of molecules is not volume V of the container but **(V-b)** where **b** is the factor which depends on **co-volume** of the molecules.

2. In real gas the attractive forces between molecules are not negligible. Any molecule approaching the walls of the container would experience an attractive force from bulk molecules. This would reduce the momentum of bombarding molecule thereby reducing pressure exerted on walls. The pressure exerted on wall is less than the pressure exerted in bulk by $\frac{a}{V^2}$. Therefore the pressure experienced is **$(P + \frac{a}{V^2})$**

Therefore Vander Waal's equation is given by

$$\boxed{\left(\mathbf{P} + \frac{\mathbf{a}}{\mathbf{V^2}}\right)(\mathbf{V} - \mathbf{b}) = \mathbf{nRT}}$$

4.3: VAPOURS

A vapour is gaseous state of substance below its critical temperature. A vapour can either be saturated or unsaturated

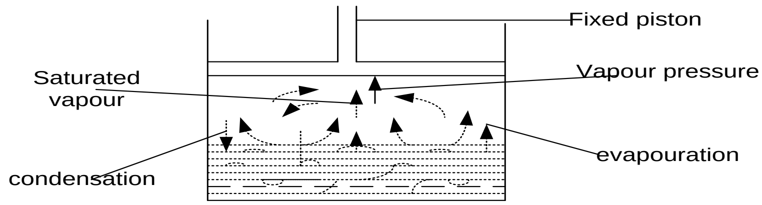
A gas is a gaseous state of substance above it's critical temperature

4.3.1: SATURATED AND UNSATURATED VAPOUR

- A saturated vapour is one which is in dynamic equilibrium with it's own liquid.
- Unsaturated vapour is one which is not in dynamic equilibrium with it's own liquid.

4.3.2: SATURATED VAPOUR PRESSURE (S.V.P)

1:Explanation of occurrence of S.V.P using kinetic theory



- Consider a liquid confined in the container with fixed piston, the liquid is in contact with its vapour. The liquid molecules are moving randomly with mean kinetic energy determined by liquid temperature. The most energetic molecules have sufficient K.e to overcome the attraction by other molecules and leave the surface of liquid to become vapour molecules. The process is called **evaporation** and it rate is determined by the **liquid temperature**.
- The molecules of the vapour are also moving randomly with a mean kinetic. The vapour molecule collide with walls of the vessel giving rise to vapour pressure and others bombard the surface of the liquid and re-enter the liquid. The process is called **condensation**. The process of condensation and vapour pressure depend on **density of the vapour.**
- A state of dynamic equilibrium is attained when the rate of condensation equals to rate of evapouration. At this point the density of vapour and hence vapour pressure is maximum at that temperature of the vapour and this is called S.V.P.

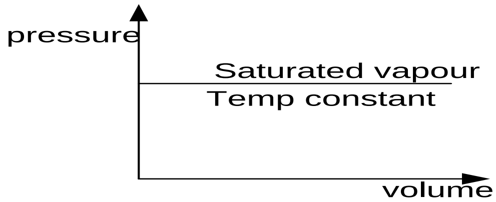
NB:

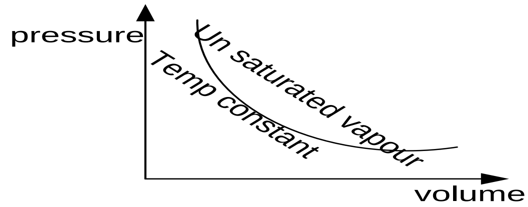
- The rate of evapouration depend on temperature of the liquid
- The rate of condensation depends on density of vapour
- Vapour pressure depends on density of the vapour
- Saturated vapour pressure depends on density of the vapour
- S.V.P is the maximum vapour pressure at any temperature

2:Effect of volume change on S.V.P at constant temperature

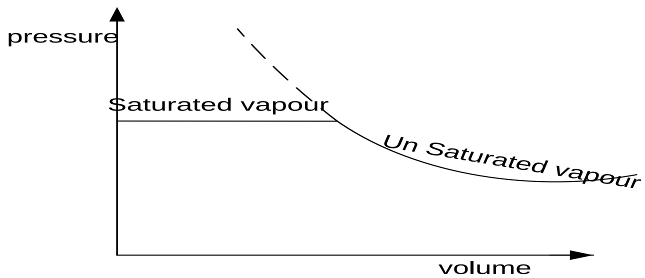
- When the volume of saturated vapour is decreased at constant temperature, the density of vapour, the rate of condensation and S.V.P **momentarily increases** but the rate of evapouration remains **constant**.
- An increase in the rate of condensation with out any increase in rate of evapouration. Causes the vapour density, the rate condensation and SVP to decrease therefore dynamic equilibrium is retained at **original value**.

NB: Volume change at constant temperature has no effect on SVP



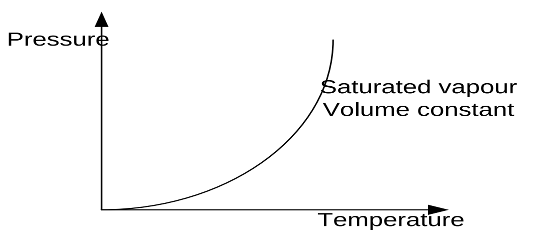


Saturated vapours do not obey Boyle's law, unsaturated vapour obey Boyle's law

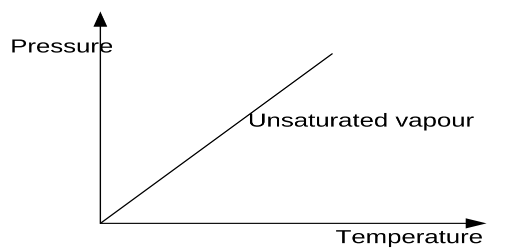


3:Effects of increasing temperature on SVP at constant volume

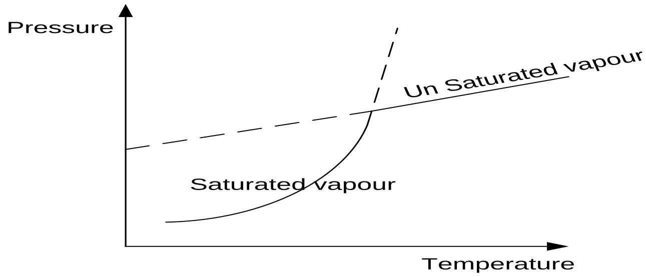
An increase in temperature increases the average kinetic energy of molecules and therefore evapouration rate increases. This causes an increase in vapour density which in turn increases the rate of condensation and eventually equilibrium and saturation are re-obtained at higher saturated vapour pressure than before.



Saturated vapour increases almost exponentialy with temperature.
Saturated vapour donot obey Pressure law



Pressure increases linearly with temperature.
Unsaturated vapours obey pressure law



Note:

- Unlike gas and unsaturated vapours, saturated vapours do not obey Boyle's law. So never apply Boyle's law to saturated vapours.
- Never apply pressure law to saturated vapour however it can be applied to unsaturated vapours.
- A combination of Boyles law and pressure law gives equation of state i.e

  $\frac{PV}{T}$= constant

  Since saturate vapours do not obey either Boyles law or Pressure law they can not obey equation of state.

  Never apply ($\frac{PV}{T}$ = constant) to saturated vapours however, it can be applied to unsaturated vapours

  During numerical calculation, first identify whether a problem involves a mixture of gas and unsaturated vapour or mixture of gas and saturated vapour.

- If it is a mixture of gas and unsaturated vapour, apply the equation of state to the mixture.
- If it is a mixture of gas and saturated vapour, apply Dalton law of partial pressure to separate the pressure of the gas (Pg) from SVP (P_s) and apply the equation of state to gas alone
- Pressure of a mixture of gases (P) = pressure of gas (Pg) + SVP (P_s)

  P = Pg + Ps

  $\boxed{Pg = P - Ps}$

EXAMPLES

1) A closed vessel contains air saturated with water at 77°C. The total pressure in vessel is 1000mmHg. Calculate the new pressure in the vessel if the temperature is reduced to 27°C. [SVP of water at 77°C = 314mmHg, SVP of water at 27°C = 27mmHg]

Solution

T_1 = 77 + 213 = 350K

T_2 = 27 + 273 = 300K

P_1 = 1000mmHg,

P_{s1} = 314mmHg

P_{g1} = ?

$P_{g1} = P_1 - P_s$

P_{g1} = 1000 – 314

P_{g1} = 686mmHg

P_2 = ? Pg_2 = ? P_{s2} = 27mmHg

$Pg_2 = (P_2 - 27)$

$\frac{Pg1}{T1} = \frac{Pg2}{T2}$

Using pressure law

$\frac{686}{350} = \frac{P_2 - 27}{300}$

$350P_2 - 9450 = 205,800$

$350P_2 = 215,250$

P_2 = 615mmHg

2) A closed vessel of fixed volume contain air and water, the pressure in vessel are 20°C and 75°C are 737.5mmHg and 1144mmHg respectively. Some of the water remains a liquid at 75°C. If SVP of water are 20°C is 11.5mmHg. Find it's value at 75°C.

Solution

$T_1 = 20 + 273 = 293K$

$P_1 = 737.5mmHg$

$P_{s1} = 17.5mmHg$

$Pg_1 = P_1 - Ps_1$

$Pg_1 = (737.5 - 17.5)$

$Pg_1 = 720mmHg$

$T_2 = 75 + 273 = 348K$

$P_2 = 1144$

$P_{s2} = ?$

$Pg_2 = P_2 - Ps_2$

$Pg_2 = 1144 - Ps_2$

Using pressure law

$\frac{Pg1}{T1} = \frac{Pg2}{T2}$

$\frac{720}{293} = \frac{1144 - Ps2}{348}$

$293\,(1144 - Ps_2) = 250{,}560$

$335192 - 293Ps_2 = 250{,}560$

$293P_{s2} = 335192 - 250560$

$P_{s2} \quad = 288.8mmHg$

3) A narrow tube of uniform bore closed at the end has air trapped by small drop of water. If the atmospheric pressure 760mmHg and saturated vapour pressure of air at 10°C and 30°C are 10mmHg and 40mmHg respectively. Calculate the length of column of air at 30°C, if it is 10cm at 10°C.

Solution

$T_1 = 10 + 273 = 283k$

$P_1 = 760mmHg$

$Ps_1 = 10mmHg$

$Pg_1 = 760 - 10$

$Pg_1 = 750mmHg$

$L_1 = 10cm$

$T_2 = 30 + 273 = 303K$

$P_2 = 760mmHg$

$Ps_2 = 40mmHg$

$Pg_2 = (760\text{-}40)mmHg$

$Pg_2 = 720mmHg$

$L_2 = ?$

$\frac{P_{g1}\,V_1}{T_1} = \frac{P_{g2}\,V_2}{T_2}$

$V_1 = L_1A$

$V_2 = L_2A$

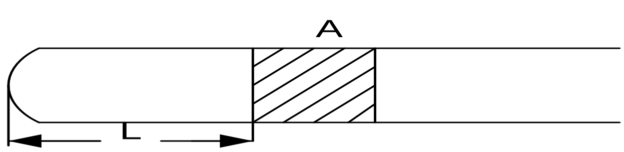
$\frac{P_{g1}L_1\,A}{T_1} = \frac{P_{g2}\,L_2A}{T_2}$

$\frac{750X10}{283} = \frac{720XL_2}{303}$

$L_2 = 11.15cm$

EXERCISE 9

1) State the relation between pressure and volume at constant temperature for
(a) an ideal gas (b) a saturated vapour



A long uniform horizontal capillary tube sealed at one end and open to the air at the other contains air trapped behind a short column of water A. The length L of the trapped air column at temperature 300K and 360K is 10cm and 30cm respectively. Given that the vapour pressure of water at the same temperature are 4kPa and 62kPa respectively. Calculate the atmospheric pressure.
An(1.01×10^5Pa)

2) A sealed vessel contains a mixture of air and water vapour in contact with water. The total pressure in the vessel at 27°C and 60°C are respectively 1.0×10^5Pa and 1.3×10^5Pa. If the saturated vapour pressure of water at 60°C is 2.0×10^4Pa what is it's value at 27°C (1Pa = $1Nm^{-2}$). **An[9×10^2Pa]**

3) The saturation vapour pressure of water is $6 \times 10^4Nm^{-2}$ at a temperature 360K and $0.3 \times 10^4Nm^{-2}$ at temperature 300K. A vessel contains only water vapour at a temperature of 360K and pressure $2 \times 10^4Nm^{-2}$. It may be assumed that unsaturated water vapour behaves like an ideal gas. If the vapour were to remain unsaturated what would be the pressure in the vessel at 300K. What is the actual pressure at this temperature and what fraction, if any of the vapour has condensed. **An[$1.7\times10^4Nm^{-2}$, $3.0\times10^3Nm^{-2}$,82%]**

4) A horizontal tube of uniform bore closed at one end has some air trapped by small quantity of water. The length of air column is 20cm at 12°C. Find stating any assumption made the length of air column when the temperature is increased to 38°C. [SVP of H_2O at 12°C and 38°C are 105mmHg and 49.5mmHg respectively, atmospheric pressure = 75.0cmHg]. **An (23.04)**

4.4.1: EVAPOURATION

This is the process by which a liquid become a vapour.
It can take place at all temperatures and only at the surface but it is greatest when the liquid is at it's boiling point.

4.4.2: Explanation using kinetic theory

- In kinetic theory, the molecules of liquid are in continuous motion and make frequent collision with each other. As they continue colliding with each other, they gain kinetic energy and those which have acquired sufficient kinetic energy move up to the surface of the liquid and escape as vapour.
- The molecules that remain are those with low kinetic energy. Since mean kinetic energy of the molecules is directly proportional to absolute temperature, the liquid cools

4.4.3: Ways of increasing evapouration

- Increasing surface area of liquid
- Increasing temperature of the liquid
- Reducing air pressure above the liquid
- Causing a drought to remove vapour molecule before they have any chance to retain the liquid.

4.5.1: BOILING

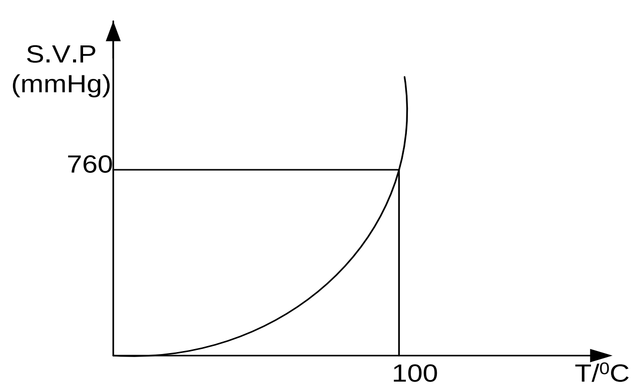
Boiling point of liquid is the temperature at which liquid vapour pressure is equal to external atmospheric pressure.

A liquid boils when its temperature is such that bubble of vapour form through out it's volume.

4.5.2: Explanation of boiling using kinetic theory

- In kinetic theory, the molecules of a liquid are in continuous motion and make frequent with each other. As they continue colliding with each other they gain kinetic energy and those which have acquired sufficient kinetic energy move up to the surface of the liquid and overcome the attractive forces holding them together at boiling.
- At boiling point the saturated vapour pressure of the liquid is equal to the external pressure (atmospheric pressure plus hydrostatic pressure plus the pressure due to surface tension). The bubbles grows and rise to the surface where they burst and give off the vapour to the atmosphere.

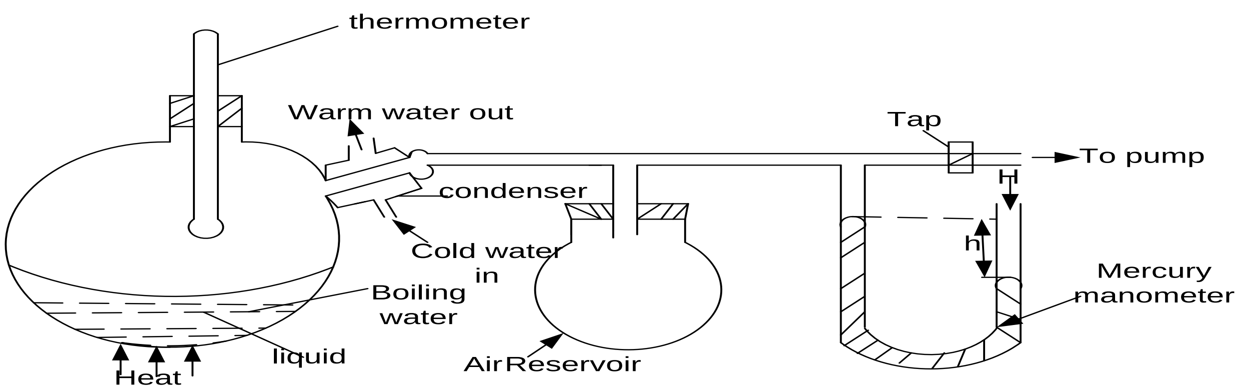
4.5.3: A graph of SVP Against temperature



4.5.4: Differences between evapouration and boiling

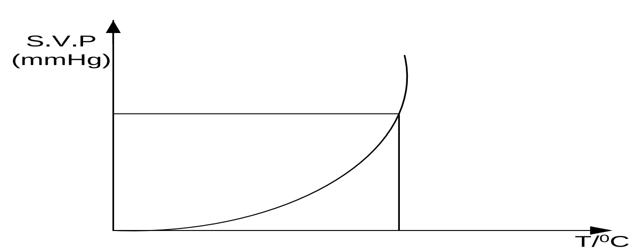
- Boiling occurs through out the volume of the liquid while evapouration occurs at the surface.
- A liquid boils at single temp for any given external pressure whereas evaporation occurs at any temperature.

4.5.6: EXPERIMENT TO VERIFY VARATION OF SVP WITH TEMPERATURE



- The pressure above the water is set to any desired value (below or above) atmospheric pressure using a vacuum pump.
- The tap is closed and the liquid is heated until it boils.
- The temperature θ of the vapour is determined using a thermometer and noted.
- The difference, h in mercury levels is noted from the manometer.
- The pressure, p of the vapou; $P = H \pm\ h$ where H is barometric height
- The procedure is repeated for various value of pressure P and corresponding temperature θ noted.
- A graph of p against θ is plotted and SVP of the liquid at a particular temperature can be obtained

A graph of SVP against temperature is plotted



From the graph it can be concluded that SVP increase with temperature

Question: Explain why at a given external pressure a liquid boils at a constant temp.
Solution

A liquid boils when it's saturated vapour pressure is equal to the external pressure. But since the saturated vapour pressure is dependent on the temp of the liquid, then it implies that for a given external pressure the boiling will occur at a constant temperature.

Question: Explain why the temperature of a liquid does not change when the liquid is boiling.
Solution

At boiling point all the heat supplied is used in increasing the potential energy of the molecules by increasing their distances of separation as they change from liquid phase to the gaseous phase and also in doing work against external atmospheric pressure during expansion. So there is no change in the kinetic energy of the liquid molecules therefore the temperature of the liquid remains constant.

UNEB 2013 Q.6

(a) The pressure, P, of an ideal gas is given by $\boldsymbol{P = \frac{1}{3}\rho \overline{C^2}}$, where ρ is the density of the ideal gas and $\overline{C^2}$ it's mean square speed.

(i) Show clearly the steps taken to derive this expression. (06marks)

(ii) State the assumptions made in deriving this expression. (02marks)

(b) Sketch the pressure versus volume curve for a real gas for temperatures above and below the critical temperature. (03marks)

(c) For one mole of a real gas, the equation of state is

$$\mathbf{(P + \frac{a}{V^2})\,(V - b) = RT}$$

Explain the significance of the terms $\frac{a}{V^2}$ and b (02marks)

(d) A balloon of volume $5.5x\ 10^{-2} m^3$ is filled with helium to a pressure of $1.10x\ 10^5$ N m^{-2} at a temperature of 20°C. Calculate the;

(i) Number of helium atoms in the balloon **A n[$\mathbf{1.496x10^{24}}$]** (03marks)

(ii) Net force acting on the square metre of material of the balloon if the atmospheric pressure is $1.01x\ 10^5\ N\ m^{-2}$ **An(0.09N)** (04marks)

UNEB 2012 Q. 6

(a) (i) Define saturated vapour pressure (01mark) (ii)Describe with the aid of a diagram, how saturated vapour pressure of a liquid can be determined at a given temperature. (06marks)

(b) Use the kinetic theory to explain the following observations.

(i) Saturated vapour pressure of a liquid increases with temperature (03marks)

(ii) Saturated vapour pressure is not affected by a decrease in volume at constant pressure

(c) When hydrogen gas is collected over water the pressure in the tube at 15°C and 75°C are 65.5cm and 105.6cm of mercury respectively. If the saturated vapour pressure at 15°C is 1.42cm of mercury, find the value at 75°C.

(d) Explain why the molar heat capacity of an ideal gas at constant pressure differs from the molar heat capacity at constant volume. (03marks).

Solution

d) From $\boxed{Pg = P - Ps}$

$T_1 = 15 + 273 = 288K$

$T_2 = 75°C + 273 = 348K$

$Ps_1 = 1.42cmHg$

$Ps_2 = ?$

$P_{g1} = 65.5 - 1.42$

$P_{g1} = 64.08cmHg$

$P_{g2} = 105.6 - P_{s2}$

$\frac{Pg1}{T1} = \frac{Pg2}{T2}$

$\frac{64.08}{288} = \frac{105.6 - Ps2}{348}$

$P_{s2} = 28.17cmHg$

The pressure of saturated vapour at 75 is 28.17cmHg

UNEB 2009 Q.6)

(b) Explain the following observations using the kinetic theory

(i) A gas fills any container in which it is placed and exerts a pressure on it's walls

(ii) The pressure of a fixed mass of a gas rises when it's temperature is increased at a constant volume. (02marks)

UNEB 2008 Q.6

(b) (i) Distinguish between a real and an ideal gas (03marks)

(ii) Derive the expression P = $\frac{1}{3}\rho \overline{C^2}$ for the pressure of an ideal gas of density ρ and mean square speed $\overline{C^2}$ (06marks)

(c) (i) Explain why the pressure of a fixed mass of gas in a closed container increases when temperature of the container is raised. (02marks)

UNEB 2007 Q.7

(d) With the aid of a P-V diagram, explain what happens when a real gas is compressed at different temperatures. (04marks)

(e) The root mean square speed of the molecules of a gas is $44.72ms^{-1}$. Find the temperature of the gas, if it's density is $9 \times 10^{-2}kgms^{-1}$ and the volume is $42m^3$

An (T = $\frac{303.2K}{n}$)   n is no of moles

UNEB 2006 Q.5

(a) Define saturated vapour pressure (SVP) (01marks)

(b) Use the kinetic theory of matter to explain the following observations

(i) Saturated vapour pressure of a liquid increases with temperature. (03marks)

(ii) saturated vapour pressure is not affected by a decrease in volume at constant temperature
(03marks)

(c) Describe how the saturated vapour pressure of a liquid at various temperatures can be determined
(07marks)

(d) (i) State Dalton's law of partial pressure.
(01marks)

(ii) A horizontal tube of uniform bore, closed at one end, has some air trapped by a small quantity of water. The length of the enclosed air column is 20cm at 12°C. Find stating any assumption made the length of the air column when the temperature is raised to 38°C. (SVP of water at 12°C and 38°C are 105mmHg and 49.5mmHg respectively, atmospheric pressure = 75cmHg)**An (23.04cm)** (5marks)

UNEB 2003 Q.7

(a) (i) What is meant by kinetic theory of gases
(03marks)

(ii) Define an ideal gas
(01marks)

(iii) State and explain the condition under which real gases behave as ideal gases

(b) (i) Describe an experiment to show that a liquid boils only when it's saturated vapour pressure is equal to the external pressure
(05marks)

(ii) Explain how cooking at a pressure 76cm of mercury and a temperature of 100°C may be achieved on top of high mountains (03marks)

(c) (i) Define root – mean – square speed of molecules of a gas (01marks)

(ii) The masses of hydrogen and oxygen atoms are $1.66\ x\ 10^{-27}kg$ and $2.66\ x\ 10^{-26}$kg respectively. What is the ratio of the root mean square speed of hydrogen to that of oxygen molecules at the same temperature. **An (4:1)**

Solution

a) (iii)

At high temperature and low pressure real gases behave like ideal gases.

- ❖ At high temperature the average kinetic energy of the molecules is high and this minimizes the effect of intermolecular forces of attraction so these forces become negligible and thus the gas behaves like an ideal gas.
- ❖ At low pressure for a given temperature the number of molecules in a container is low. So the molecules will be spaced and consequently their volumes will be negligible in relation to the volume of the containing vessel. Hence the gas will behave like an ideal one.

b) (ii)

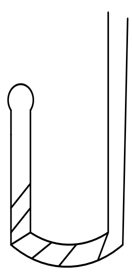


Fig 1

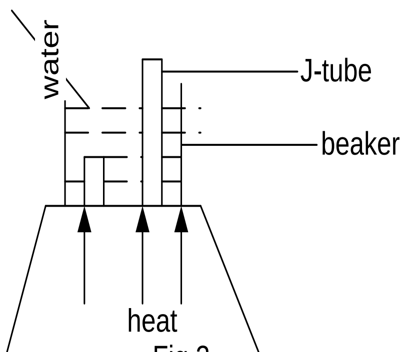


Fig 2

At high altitude you can cook at a pressure of 76cmHg and a temperature of 100°C by use of a pressure cooker that has a safety valve that opens when the saturated vapour pressure in side the cooker is 76cmHg. This valve ensures that the saturated vapour pressure can not exceed 76cmHg and consequently the temperature of the contents being boiled can not exceed 100°C.

UNEB 2003 Q.5

(b) Indicate the different states of a real gas at different temperature on a pressure versus volume sketch graph. (03marks)

(c) (i) In deriving the expression P = ⅓$\rho \overline{C^2}$ for the pressure of an ideal gas, two of the assumptions made are not valid for a real gas. State the assumptions. (2mk)

(ii) the equation of state of one mole of a real gas is $\left(\mathbf{P} + \frac{\mathbf{a}}{\mathbf{V^2}}\right)(\mathbf{V} - \mathbf{b}) = \mathbf{nRT}$

Account for the terms $\frac{\mathbf{a}}{\mathbf{V^2}}$ and **b**

(02marks)

(d) Use the expression P = ⅓$\rho \overline{C^2}$for the pressure of an ideal gas to denve Dalton's law of partial pressures. (04marks)

(e) Explain with the aid of a volume versus temperature sketch graph, what happen to a gas cooled at constant pressure from room temperature to zero Kelvin (4mk)

UNEB 2002 Q.2

(a) State the assumptions made in the derivation of the expression P = $\frac{1}{3}\rho \overline{C^2}$ for pressure of an ideal gas (02marks)

(b) Use the expression in (a) above to deduce Dalton's law of partial pressure

(03marks)

(c) Describe an experiment to determine the saturation vapour pressure of a liquid

(06marks)

CHAPTER5:HEAT TRANSFER

There are 3 ways of heat transfer namely;

Conduction Radiation Convection

5.1: CONDUCTION

This is the transfer of heat from one place to another through a medium without the movement of a substance as whole due to a temperature difference.

5.1.2: MECHANISMS OF HEAT CONDUCTION

a) IN GASES;

When a gas is heated the molecule gain energy, this increases the vibration of molecules. Fast moving molecules pass on kinetic energy to low moving molecules when they collide with them. In this way heat is slowly conducted through the gas.

b) IN METALS;

- When heat is supplied to one end of a metal, the atoms at that part gain kinetic energy which increase the rate of vibration. As they vibrate they pass on heat to their cold neighbours through inter atomic bonds.
- In additional the free electrons in the metallic lattice at the heated end gain high kinetic energy and move faster to the colder areas where they pass on kinetic energy to the fixed positive ions in lattice due to collision.

c) IN NON METALLIC SOLIDS AND LIQUIDS.

When heat is supplied to one part of non metallic solid or liquid, the atoms at that part do vibrate more vigorously about their mean position But since they are coupled by inter atomic bond to the colder molecules, they pass on heat energy to the neighbours and heat is transferred from one place to another.

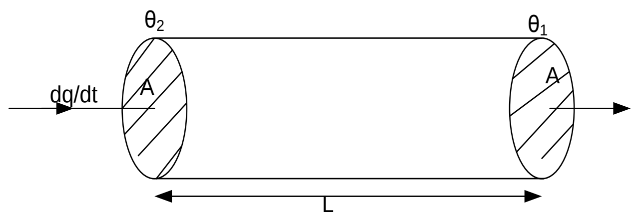
Question: Explain why metals are better conductor than non metallic solids.

Solution.

In metals heat is carried by inter atomic vibration just like in non- metallic solid. But in addition to this, metals have free electrons in their lactice that move with very high velocity when heated since they are light. So they pass on their thermo energy due to collision with the positive fixed ions in metallic lattice and this occurs at faster rate

5.1.3: THERMAL CONDUCTIVITY (K)

Consider a conductor of thickness L, Cross sectional area A, Having θ_1 and θ_2 at its end. ($\theta_2 > \theta_1$)



The rate of heat flow per second is directly proportional to the cross sectional area and the temperature difference but inversely proportional to thickness i.e

$$\frac{dQ}{dt} \propto \frac{A(\theta_2 - \theta_1)}{L} \qquad \frac{dQ}{dt} = \frac{KA(\theta_2 - \theta_1)}{L}$$

K is called co efficient of thermal conductivity of given material which depends on nature of material.

5.1.4: FACTORS ON WHICH RATE OF HEAT FLOW ($\frac{dQ}{dt}$) DEPENDS.

- It depends on cross sectional area A
- It depends on temperature difference between faces $(\theta_2 - \theta_1)$
- It depends on nature of material (thermal conductivity K)
- It depends on inverse of thickness L between faces.

Definition

- Thermal conductivity is the rate of heat flow at right angles to the opposite faces of $1m^3$ of material when temperature difference across the faces is 1 Kelvin,

OR

- Is the rate of heat flow through material per unit area in region of unit temperature gradient?

5.1.5: UNITS OF THERMAL CONDUCTIVITY (K)

K = $\frac{L\frac{dQ}{dt}}{A(\theta_{2-}\ \theta_1)}$

Unit of K = $\frac{mJs^{-1}}{m^2K}$

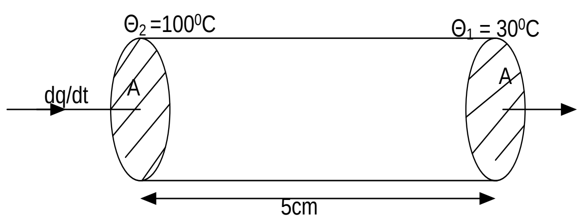
Unit of K = $\boxed{J\,S^{-1}m^{-1}K^{-1}}$ OR

Unit of K = $\boxed{W\,m^{-1}k^{-1}}$

Question.

An aluminum plate of cross section area 300cm^2 and thickness 5cm has one side maintained at 100°C by steam and another side by 30°C .The energy passes through the plate at a rate of 9kW.Calculate the coefficient of thermal conductivity of aluminum.

Solution



A = 300 x 10^{-4}m

$\frac{dQ}{dt}$ = 9 X 1000 W

L = 5 x 10^{-2}m

$(\theta_{2-}\ \theta_1)$ = (100 – 30) °C

= 70°C

K = $\frac{L\frac{dQ}{dt}}{A(\theta_{2-}\ \theta_1)}$

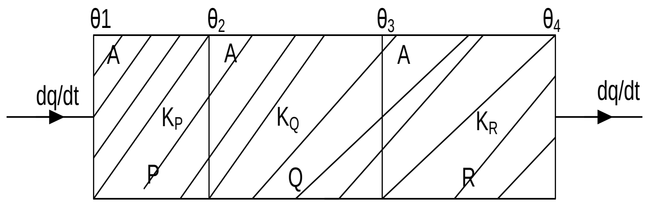
K = $\frac{5x10^{-2}x9000}{300x10^{-4}x70}$

K = 214.29Wm^{-1}K^{-1}

5.1.6: HEAT FLOW THROUGH SEVERAL SURFACES

i)Surface in Series

Consider 3 plates PQR of thermal conductivity K_P, K_Q , K_R respectively whose ends are maintained at θ_1 to $\theta_2$end their junction having temperature θ_3 and θ_4



For surface P, $\frac{dQ}{dt} = \frac{K_P A(\theta_1 - \theta_2)}{L_P}$..............................1

For surface Q, $\frac{dQ}{dt} = \frac{K_Q A(\theta_2 - \theta_3)}{L_Q}$..........................2

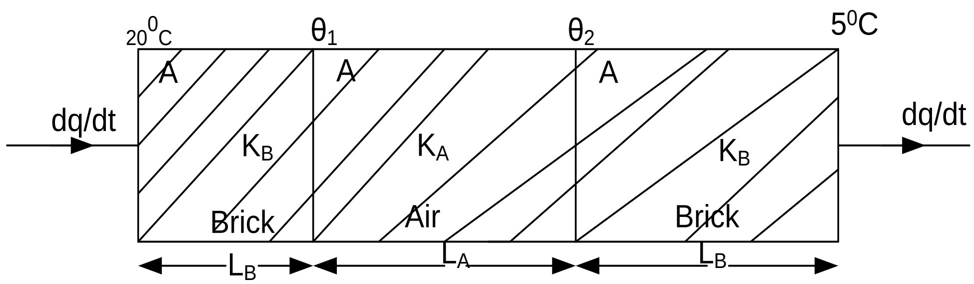
For surface R, $\frac{dQ}{dt} = \frac{K_R A(\theta_3 - \theta_4)}{L_R}$..................3

$$\frac{dQ}{dt} == \frac{K_P A(\theta_1 - \theta_2)}{L_P} = \frac{K_Q A(\theta_2 - \theta_3)}{L_Q} = \frac{K_R A(\theta_3 - \theta_4)}{L_R}$$

EXAMPLES

1) Two brick walls each of thickness 10cm are separated by air gap of thickness10cm, the outer faces of brick walls are maintained at 20° C and 5°C respectively. Calculate temperature of inner surface of walls. Compare the rate of heat loss through the layer of air with heat through a single brick wall (thermal conductivity of air 0.02Wm^{-1}K^{-1} and that of bricks 0.6W m^{-1} K^{-1})

Solution



$\frac{dQ}{dt} = \frac{K_b A(20-\theta_1)}{L_b} = \frac{K_a A(\theta_1 - \theta_2)}{L_a} = \frac{K_b A(\theta_2 - 5)}{L_b}$

$\frac{K_b A(20-\theta_1)}{L_b} = \frac{K_a A(\theta_1 - \theta_2)}{L_a}$

$\frac{0.6(20-\theta_1)}{10x10^{-2}} = \frac{0.02x(\theta_1 - \theta_2)}{10x10^{-2}}$

$0.62\theta_1 - 0.02\theta_2$ =.......... 1

$\frac{K_a A(\theta_1 - \theta_2)}{L_a} = \frac{K_b A(\theta_2 - 5)}{L_b}$

$\frac{0.02(\theta_1 - \theta_2)}{10x10^{-2}} = \frac{0.6x(\theta_2 - 5)}{10x10^{-2}}$

$0.02\theta_1 - 0.62\theta_2 = -3$ 2

Solving expression (1) and (2) simultaneously.

θ_1 = 19.5°C

θ_2 = 5.5°C

(ii) $\frac{dQ}{dt} = \frac{K_b A(20-\theta_1)}{L_b}$

$= \frac{0.6(20-19.5)A}{10x10^{-2}}$

$= 3A$

$$\frac{dQ}{dt} = \frac{K_a A(\theta_1 - \theta_2)}{L_a}$$

$=\frac{0.02(19.5-5.5)A}{10x10^{-2}}$

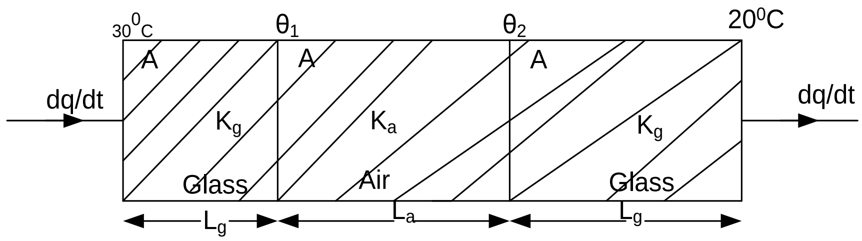
=2.8A

$=\frac{3A}{2.8A}$

=3:2.8

2) A window of height 1m and width 1.5m contain double glazed unit of two single glass plates each of thickness 4.0mm separated by air gap of 2.0mm.Calulate the rate at which heat is conducted through the window if the temperature of external surface of glass are 20°C and 30°C respectively.(Thermal conductivity of glass and air are $0.72Wm^{-1}K^{-1}$ and $0.025Wm^{-1}K^{-1}$ respectively).

Solution



$$\frac{dQ}{dt}=\frac{K_gA(30-\theta_1)}{L_g}=\frac{K_aA(\theta_1-\theta_2)}{L_a}=\frac{K_gA(\theta_2-20)}{L_g}$$

$$\frac{K_gA(30-\theta_1)}{L_g}=\frac{K_aA(\theta_1-\theta_2)}{L_a}$$

$$\frac{0.72A(30-\theta_1)}{4x10^{-3}}=\frac{0.025A(\theta_1-\theta_2)}{2x10^{-3}}$$

$12.5\theta_2 - 192.5\theta_1 = -5400$ ……… 1

$$\frac{K_gA(30-\theta_1)}{L_g}=\frac{K_gA(\theta_2-20)}{L_g}$$

$30 - \theta_1 = \theta_2 - 20$

$\theta_1 + \theta_2 = 50$ …………2

Solving expression 2 and 1 simultaneously

$\theta_1 = 29.4°C$, $\theta_2 = 20.61°C$

$$\frac{dQ}{dt}=\frac{K_gA(30-\theta_1)}{L_g}$$

$$\frac{dQ}{dt}=\frac{0.72x1.5(30-29.4)}{4x10^{-3}}$$

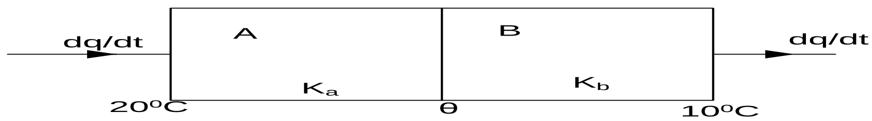
$\frac{dQ}{dt}$ = <u>$164.7JS^{-1}$</u>

3) A wall 6m by 3m consists of two layers A and B of thermal conductivities $0.6Wm^{-1}K^{-1}$ and $0.5Wm^{-1}K^{-1}$ respectively. The thickness of layer is 15.0cm.The inner surface of layer A is at temperature of 20°C while outer layer B is at temperature of 10°C.Calculate

(i) The temperature of interface of A and B.

(ii) The rate of heat through wall.

Solution



i)

$$\frac{dQ}{dt} = \frac{K_a A(30-\theta)}{L_a} = \frac{K_b A(\theta-10)}{L_b}$$

$$\frac{0.6A(20-\theta)}{15x10^{-2}} = \frac{0.025A(\theta-10)}{15x10^{-2}}$$

$0.6(20-\theta)$ = $0.5(\theta$ -10)

1.1 θ = 17

$\theta = 15.45$°C

ii) $\frac{dQ}{dt} = \frac{K_a A(30-\theta)}{L_a}$

$=\frac{0.6A(20-\theta)}{15x10^{-2}}$

$=\frac{0.6x6x3(20-15.45)}{15x10^{-2}}$

$\frac{dQ}{dt}$ = 327.6Js^{-1}

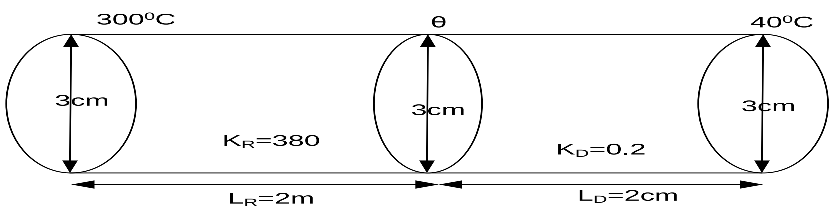
4) A copper rod 2m long and of diameter 3cm is lagged. One end is maintained at 300°C, the other end is placed against 3cm thick card board disk of same diameter as the rod. The free end of disk is maintained at 40°C .Calculate;

(i) Steady state temperature at copper card board junction.

(ii) Quantity of heat flowing against junction in 10 minutes.

(Thermal conductivity of copper and card board are 380 and 0.2Wm^{-1}K^{-1}respectively).

Solution



$$\frac{dQ}{dt} = \frac{K_R A(300-\theta)}{L_R} = \frac{K_b A(\theta-40)}{L_b}$$

$$\frac{380(300-\theta)}{2} = \frac{0.2(\theta-40)}{3X10^{-2}}$$

57000-190θ = 6.667θ - 266.667

θ = 287°C

$$\frac{dQ}{dt} = \frac{K_R A(300-\theta)}{L_R}$$

Area = $\frac{\pi d^2}{4}$

Area = $\frac{\frac{22}{7}(3x10^{-2})^2}{4}$

Area = 7.07x10^{-4}m^2

$$\frac{dQ}{dt} = \frac{380x\ 7.07\text{x}10^{-4}(300-287)}{2}$$

$\frac{dQ}{dt}$ =1.75Js^{-1}

$\frac{Q}{t}$= 1.75

Q =1.75xt

Q = 1.75x10x60

Q = 1047.9J

EXERCISE10

1) A well lagged composite metal bar of uniform cross section area $2cm^2$ is made by joining 40cm rod of copper to 25cm rod of Aluminium. The extreme ends of the bar are maintained respectively at 100° C and 0°C. Calculate;

(i) The temperature of junction of two rods.

(ii) Rate of heat flow

(Thermal conductivity of copper and Aluminum is 386 and 210 $Wm^{-1}k^{-1}$ respectively).

An (i) 53.5℃ (ii) 2696J

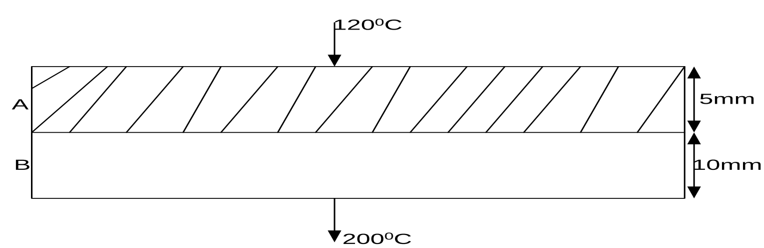
2) A rectangular room 12m by 10m has vertical walls 4m high to support the roof. The walls and a roof are 25cm thick and are made of material 0f thermal conductivity $0.25Wm^{-1}K^{-1}$.Tthe door and window covers area $16m^2$ and are made of glass of thickness 5mm and thermal conductivity $1.2Wm^{-1}K^{-1}$,If the room is maintained at constant temperature above that of its surrounding. Calculate the percentage heat loss by conduction through the doors and window. Heat losses through the floor may be neglected. **An(93.7%)**

3) A concrete floor of a hall has dimensions of 10.0m by 8.0m.It is covered with carpet of thickness 2.0cm.The temperature inside the hall is 22°C while that of the surrounding just below the concrete is 12°C.Thermal conductivity of concrete and carpet are 1 and $0.05Wm^{-1}k^{-1}$ respectively and thickness of concrete is 10cm.Calculate

(i) Temperature at the interface of concrete and Carpet

(ii) The rate at which flow through the floor. **An(14° C , 1600W)**

4)



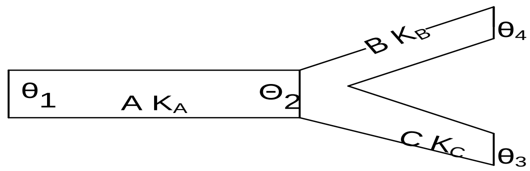
The metal conductors A and B each of radius 20cm and thickness 5mm and 10mm respectively are placed in contact as shown above. The upper surface of A and lower surface of B are maintained at temperature of 120°C and 200° C respectively. Calculate;

(i) Temperature of interface

(ii) Rate of hard flow through A **An(138.9°C, 99.75 x 10^{-3}w)**

(Thermal conductivities of A and B are 210 and 130 $Wm^{-1}K^{-1}$ respectively)

ii)Surface not in series (Y shaped)



$$\frac{dQ}{dt} = \frac{K_A A(\theta\,1 - \theta_2)}{L_A} = \frac{K_B A(\theta_{2-}\,\theta_3)}{L_B} + \frac{K_C A(\theta_{2-}\,\theta\,4)}{L_C}$$

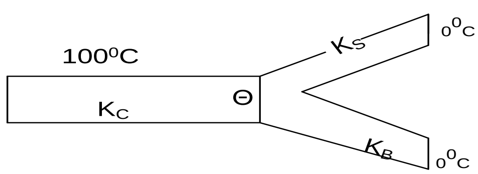
EXAMPLE

Rods of copper, brass and steel are welded together to form Y-Shaped figure. The cross sectional area of each rod is $2cm^2$ .The end of copper rod maintained at 100°C and the ends of brass and steel rod at 0°C, assume that there is not heat loss from surface of rod and that length of rods are 46cm, 13cm and 12cm respectively. Calculate the;

(i) temperature of junction.

(ii) heat current in the copper rod (thermal conductivities of copper, brass and steel are respectively $385Wm^{-I}K^{-I}$, $109Wm^{-1}K^{-1}$ and $50.2Wm^{-1}K^{-1}$

Solution



$$\frac{dQ}{dt} = \frac{K_C A(100-\theta)}{L_C} = \frac{K_B A(\theta-0)}{L_B} + \frac{K_S A(\theta-0)}{L_S}$$

$$\frac{385(100-\theta)}{0.46} = \frac{109(\theta-0)}{0.13} + \frac{50.2(\theta-0)}{0.12}$$

8369565–836.9565 θ=418.33 θ + 838.46 θ

θ = 39.97°C

$$\frac{dQ}{dt} = \frac{K_C A(100-\theta)}{L_C}$$

$$= \frac{3852X10^{-4}(100-39.97)}{0.46}$$

$\frac{dQ}{dt}$ = 10.05 JS^{-1}

5.1.7: RELATIONSHIP BETWEEMN RATE OF HEAT FLOW ($\frac{dQ}{dt}$) AND LATENT HEAT OF VAPOURISATION.

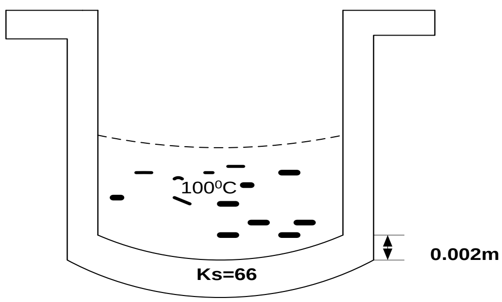
$\frac{dQ}{dt}$ = ML_V. Where M= Mass per unit time. Lv= Latent heat of Vapourisation

.

EXAMPLE

1) An Iron saucepan containing water which boils steadily at 100°C stands on a hot plate and heat is conducted through the base of the pan of area 4m^2 and uniform thickness 2 x 10^{-3}m. If water evaporate at a rate of 0.09 kg/min. Calculate the surface temperature of out side surface of the pan.(Thermal conductivity of Iron = 66Wm^{-1}K^{-1} and Lv = 2.2 x 10^6 JK^{-1})

Solution



$\frac{dQ}{dt}$= MLv

= $\frac{0.09}{6o}$ x 2.2 X10^6

$\frac{dQ}{dt}$= <u>3300JS^{-1}</u>

$\frac{dQ}{dt}$= $\frac{K_S A(\theta - 100)}{L_S}$

3300 = $\frac{66x0.04(\theta - 100)}{2x10^{-3}}$

6.6 = 66 x 0.04 (θ – 100)

6.6 = 2.64(θ – 100)

6.6 2.64θ – 264

θ = $\frac{6.64264}{2.64}$

θ = <u>102.5°C</u>

2) A copper kettle has Circular base of radius 10cm and thickness 3mm, the upper surface of base is covered with a uniform layer of soot 1mm thick. Kettle contains water which is boiled to boiling point by an electrical heat. In steady state 5g of steam are produced each minute. What is the temperature of the lower surface of the base assuming that heat conduction from the side of the kettle can be ignored (thermal conductivity of copper and soot respectively are 390Wm^{-1}K^{-1} and 13.0Wm^{-1}K^{-1} and Lv= 2.26 x 10^{6}Jkg^{-1}.)

Solution

$\frac{dQ}{dt}$ = ML_V.

= $\frac{5x10^{-3}}{60}$ x 2.26 x 10^{+6}

$\frac{dQ}{dt}$ =188.333

$$\frac{dQ}{dt} = \frac{K_S A(\theta_2 - \theta_1)}{L_S} = \frac{K_k A(\theta_{1-} 100)}{L_k}$$

188.333 = $\frac{13\pi x(10x10^{-2})^2(\theta_2 - \theta_1)}{1x10^{-3}}$

0.188333 = 0.4084$\theta_2 - 0.4084\theta_1$ …….1

Also:

188.333 = $\frac{390\pi x(10x10^{-2})^2(\theta_1 - 100)}{3x10^{-3}}$

0.564999 = 12.2522θ_1 - 12.2522x100.....2

$\theta_1 = 100.046$

Put into eqn1

0.188333 = 0.4084$\theta_2 - 0.4084x100.046$

θ_2 = 100.5^{0}C

EXERCISE11

1) Water contained in an aluminum kettle on a stove steadily boiling away at 100° C at a rate of 3.68 x 10^{-4}kgs$^{-1.}$ The base has an area of 6.0 x 10mm and thickness 4mm.Calculate;

(i) The rate of heat flow through the base **An (882J)**

(ii) The temperature of lower surface of the base. **An (102.6°C)**

[Thermal conductivity of Aluminum 210 Wm^{-1}K$^{-1,}$ $S.L.v$ of H_2O= 2.26 X 10^6Jkg^{-1})

2) A layer of boiler scale deposits on the inside of boiler, in order to maintain same rate of heat flow. What will be the temperature difference between the exposed surface of the boiler If the deposit is 5mm thick (SLv .H of water 2.27 x 10^6 Jkg^{-1}, thermal conductivity of boiler scale 4.7 Wm^{-1}K^{-1} **AN. (359.6°C)**

5.1.8: METHODS OF DETERMINING COEFFICIENT OF THERMAL CONDUCTIVITY K

In measurement of thermal conductivity k of materials, use is made of the equation

$$\frac{dQ}{dt} = \frac{KA(\theta_2 - \theta_1)}{L}$$

θ_1 and θ_2 are temperature at ends.

A is the area of Cross section,

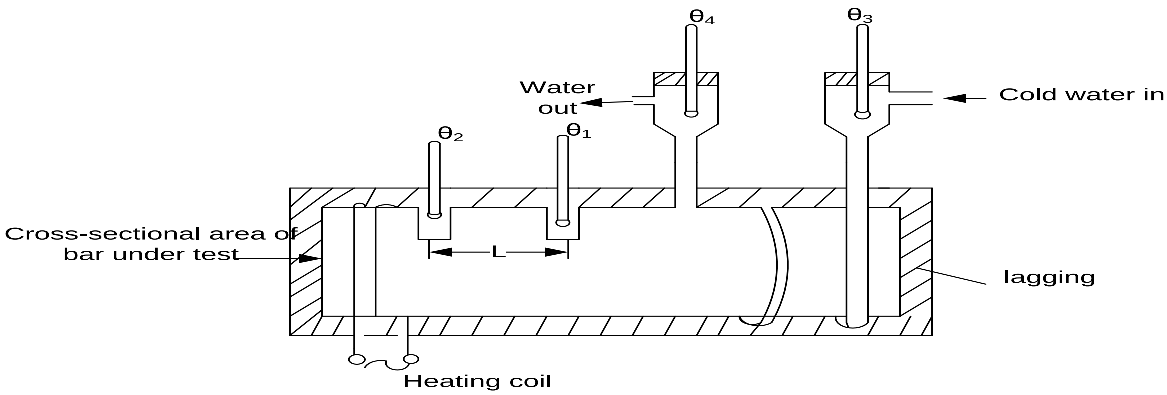
L is the length of thickness

and $\frac{dQ}{dt}$ is the rate of heat flow

a) DETERMINATION OF THERMAL CONDUCTIVITY K OF A GOOD CONDUCTOR OF HEAT E.G AMETAL LIKE COPPER

USING SEARLE'S METHOD

Searle's is best suited for a good conductor because it achieves measurable temperature gradient and measurable heat flow and this can be obtained by good conductor.



- The specimen conductor is cut into a long bar of small cross sectional area to obtain measurable temperature gradient and inadequate heat flow rate.
- Its cross sectional area A is determined along with the length between thermometer T_1 and T_2.
- The bar is polished highly (using Vaseline) to ensure good thermal contact and lagged to minimize lateral heat loss.
- The bar is heated using a heating coil to a steady state where all temperatures θ_1, θ_2, θ_3 and θ_4 are steady and so they are noted.
- The mass rate of flow (m/t) of water which is cooling the bar are noted.

Thus at steady state

$$\frac{dQ}{dt} = \frac{KA(\theta_2 - \theta_1)}{L}$$

Also all the heat which flows in the bar is used to increase the temperature of water

$$\frac{dQ}{dt} = MC(\theta_4 - \theta_3)$$

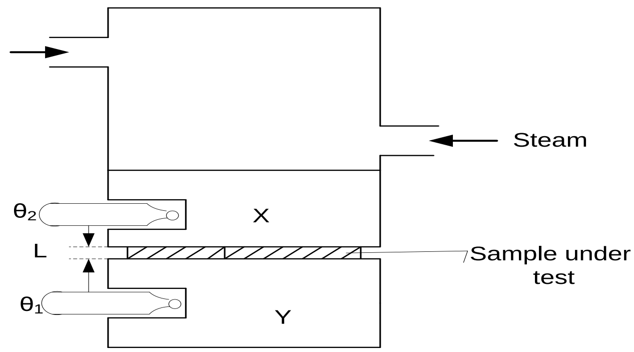
$$\frac{KA(\theta_2 - \theta_1)}{L} = MC(\theta_4 - \theta_3)$$

$$\boxed{K = \frac{MCL(\theta_4 - \theta_3)}{A(\theta_2 - \theta_1)}}$$

b) DETERMINATION OF THERMAL CONDUCTIVITY (K) OF A POOR CONDUCTOR E.G RUBBER, GLASS USING

CHEST OR LEE DISK METHOD.

For a poor conductor, the material has to be made thin so that a measurable temperature gradient can be obtained

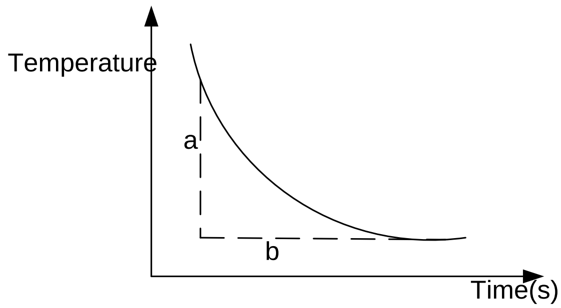


- The specimen is shaped into disc of small thickness Land large cross sectional area A in order to obtain an adequate rate of heat flow and a measurable temperature gradient.
- The thin disc is sand wicked between the thick base X of a steam chest and thick that Y, with each containing a thermometer and to ensure good thermal contact, X and Y are polished using Vaseline.
- Steam is passed through the chest and the apparatus is left to reach steady state and temperature θ_2 and θ_1 noted.
- At steadily state .The rate at which heat is flowing through the sample is equal to the rate at which Y is losing heat to the surrounding.

$$\frac{dQ}{dt} = \frac{KA(\theta_2 - \theta_1)}{L} \text{.....................} 1$$

- At this stage, the sample is removed so that Y comes into direct contact with X and is heated by it. When the temperature of Y has risen by about 10°C X is removed and the sample is put back on top of Y. Since X is no longer present Y cools and temperature θ is noted at one minute internal until it has dropped to about 10°C below steady state temperature θ_1

A graph of temperature against time is plotted.



Rate of temperature fall at Y is

$$\frac{d\theta}{dt} = \frac{a}{b}$$

$$\frac{dQ}{dt} = \frac{MCd\theta}{dt}$$

$$\frac{dQ}{dt} = MC\frac{a}{b}$$

$$\frac{KA(\theta_2 - \theta_1)}{L} = MC\frac{a}{b}$$

$$\boxed{K = \frac{MCLa}{bA(\theta_2 - \theta_1)}}$$

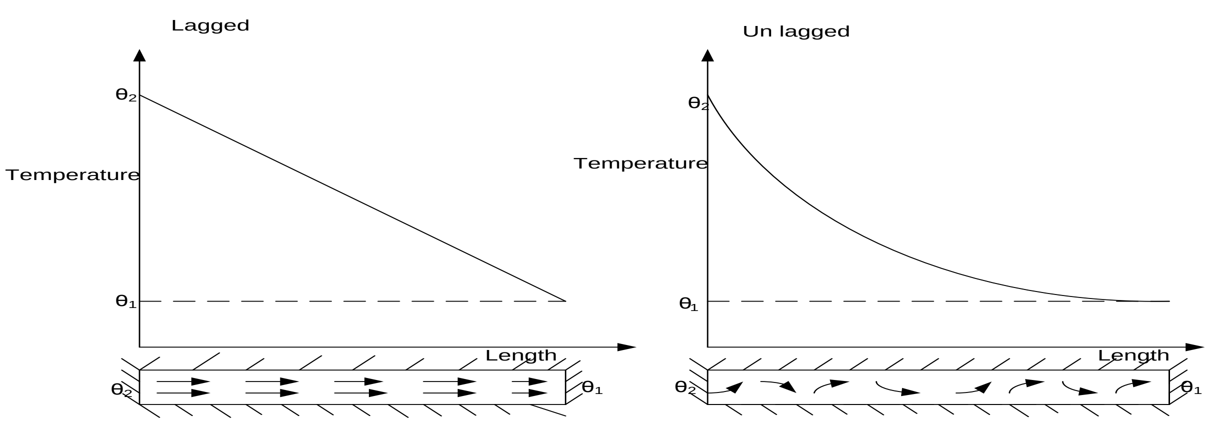
5.1.9: VARIATION OF TEMPERATURE ALONG A BAR WHICH IS :

1. Lagged.

For a lagged conductor the heat that escape to the surroundings is negligible and therefore the rate of heat flow is uniform a long the length of the bar.

2.Un lagged

For un lagged bar, the rate of heat flow decreases as you move from hot end to colder end since heat is lost to the surrounding.



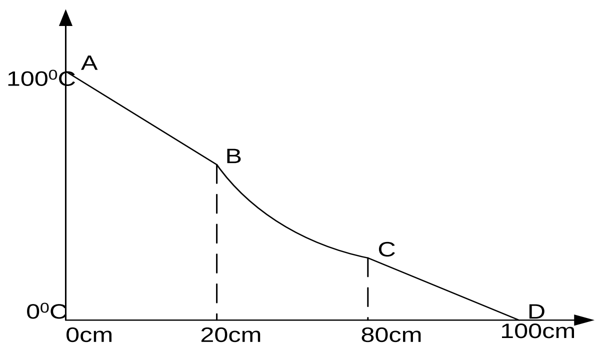
EXAMPLE

1) Two end of metal bar of length 1m are perfectly lagged up to 20cm from either end. The end of the bar is maintained at 100 and 0°C respectively.

(i) Sketch a graph temp vs distance along the bar

(ii) Explain the features of graph in (i) above

Solution



- Along BC, the bar is un lagged and therefore the heat is lost to the surroundings hence the rate of heat flow decreases as you move from B to C.
- Along CD, the bar is lagged and therefore the rate of heat flow is uniform along this section. However is it at lower rate that at AB since the heat was lost to surrounding along BC.
- Along AB, the bar is lagged and therefore the rate of heat flow s form along this section.

EXERCISE 12

1) An ideally lagged composite bar 25cm long consists of a copper bar 15cm long joined to an aluminum bar 10cm long and of equal cross sectional area. The free end of the copper is maintained at 100^0C and the free end of aluminum at 0^0C. Calculate the temperature gradient in each bar when steady state conditions have been reached. (thermal conductivity of copper=$390Wm^{-1}\ {}^0C^{-1}$, thermal conductivity of aluminum=$210Wm^{-1 0}C^{-1}$).

   An [copper =$3x10^2\ {}^0Cm^{-1}$, aluminum=$5.5x10^2\ {}^0Cm^{-1}$]

2) If a copper kettle has a base of thickness 2.0mm and area $3.0x10^{-2}m^2$, estimate the steady difference in temperature between inner and outer surface of the base which must be maintained to enable enough heat to pass through so that the temperature of 1kg of water rises at the rate of $0.25Ks^{-1}$. Assume that there are no heat losses, the thermal conductivity of copper = $3.8x10^2Wm^{-1}K^{-1}$ and the specific heat capacity of water =$4.2x10^3Jkg^{-1}K^{-1}$. After reaching the temperature of 373K, the water is allowed to boil under the same conditions for 120 seconds and

the mass of water remaining in the kettle is 0.948kg. Deduce a value for the S.L.H of vaporization of water.

An[0.2^0C, $2.4x10^6Jkg^{-1}$]

3) A cubical container full of hot water at a temperature of 90^0C is completely lagged with an insulating material of thermal conductivity $6.4x10^{-2}$ Wm^{-1} $^0C^{-1}$. The edge of the container are 1.0m long and the thickness of the lagging is 1.0cm. estimate the rate of flow of heat through the lagging if the external temperature of the lagging is 40^0C. Mention any assumptions you make in deriving your result.
Discuss qualitatively how your result will be affected if the thickness of the lagging is increased considerably assuming that the temperature of the surrounding air is 18^0C.

An [$1.9x10^3W$]

4) A thin walled hot water tank, having a total surface area $5m^2$, contains $0.8m^3$ of water at temperature of 350K. it is lagged with a 50mm thick layer of material of thermal conductivity $4x10^{-2}Wm^{-1}K^{-1}$. The temperature of the outside surface of the lagging is 290K. What electrical power must be supplied to an immersion heater to maintain the temperature of the water at 350K. (Assume the thickness of the copper walls of the tank to be negligible). What is the justification for the assumption that the thickness of the copper walls of the tank may be neglected? (Thermal conductivity of copper=400 $Wm^{-1}K^{-1}$)
If the heater were switched off, how long would it take for the temperature of the hot water to fall 1K. (Density of water =$1000kgm^{-3}$, specific heat capacity of water =4170 $Jkg^{-1}K^{-1}$) **An [240W, 232min]**

5) A small green house consists of $34m^2$ of glass of thickness 3.0mm and $9.0m^2$ of concrete wall of thickness 0.080m. On a sunny day, the interior of the green house receives a steady 25kW of solar radiation. Estimate the difference in temperature between inside and outside of the green house. The temperature inside and outside may be assumed uniform and heat transfers downwards into the ground inside the green house may be neglected.(Thermal conductivity of glass and concrete are 0.85 $Wm^{-1}K^{-1}$, 1.5 $Wm^{-1}K^{-1}$ respectively)
An[2.6^0C]

6) A window pane consists of a sheet of glass of area $2.0m^2$ and thickness 5.0mm. if the surface temperature are maintained at 0^0C and 20^0C. Calculate the rate of flow of heat through the pane assuming a steady state is maintained. The window is now double glazed by adding a similar sheet of glass so that a layer of air 10mm thick is trapped between the two panes. Assuming that the air is still, calculate the ratio of the rate of flow of heat through the window in the first case to that in the second (conductivity of glass =0.80 $Wm^{-1}K^{-1}$, conductivity of air =0.025 $Wm^{-1}K^{-1}$)An**[6400W, 66:1].**

7) An iron pan containing water boiling steadily at 100^0C and stands on a hot-plate and heat conducted through the base of the pan evaporates 0.09kg of water per minute. If the base of the pan has an area of $0.04m^2$ and a uniform thickness of $2x10^{-3}$m, calculate the surface temperature of the underside of the pan. [Thermal conductivity of iron
=66 $Wm^{-1}K^{-1}$ and S.L.H of evaporation of water at 100^oC =2.2 $x10^6Jkg^{-1}$]
An[102.5^{0}C]

8) (a) A sheet of glass has an area of $2.0m^2$ and a thickness of $8.0x10^{-3}$m. the glass has a thermal conductivity of 0.80 $Wm^{-1}K^{-1}$. Calculate the rate of heat transfer through the glass when there is temperature difference of 20K between its faces.
An[4.0kW]
(b) A room in a house is heated to a temperature 20K above that outside. The room has $2m^2$ of windows of glass similar to the type used in(a). Suggest why the rate of heat transfer through the glass is much less than the value calculated above.
(c) Explain why two sheets of similar glass insulate much more effectively when separated by a thin layer of air than when they are is contact.

9) Outline an experiment to measure the thermal conductivity of a solid which is a poor conductor, showing how the results is calculated from the measurements
Calculate the theoretical percentage change in heat loss by conduction achieved by replacing a single glass window by a double glass separated by 10mm of air. In each case the glass is 2mm thick (The ratio of the thermal conductivities of glass to air is 3:1)
Suggest why, in practice the change would be much less than that calculated.
An[94%]

10) A double glazed window consists of two panes of glass each 4mm thick separated by a 10mm layer of air. Assuming the thermal conductivity of glass to be 50 times greater than that of air, calculate the ratios:

(a) Temperature gradient in the glass to that in air gap

(b) Temperature difference across one pane of the glass to temperature difference across the air gap.

An[0.02, 0.008]

11) Describe how you would measure the thermal conductivity of a metal.

The two ends of an iron bar of uniform cross section are maintained at temperature of 100^0Cand 20^0C, respectively. Sketch and explain the variation of temperature along the bar.

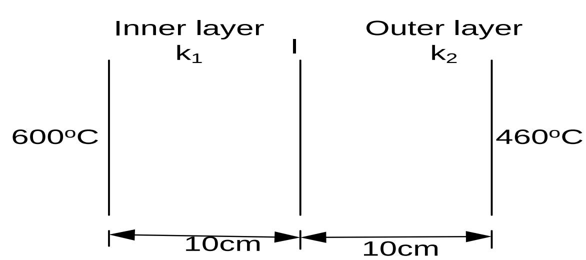
(a) If it is lagged so that no heat can escape through, the side

(b) If it is not lagged

12) (a) The thermal conductivities of glass and copper when measured at room temperature are found to be 0.6 $Wm^{-1}K^{-1}$ and 400 $Wm^{-1}K^{-1}$ respectively. Account for the large difference in these values in terms of the mechanism of thermal conduction.

(b) You have been asked to design some apparatuses to determine the thermal conductivity of copper. Describe the shape you would choose for the specimen of copper, giving reasons for your choice.

13) The diagram shows a furnace wall which is constructed of two types of brick. The temperatures of the inner and outer surfaces of the wall are 600 °C and 460 °C respectively, as shown in the diagram. The value of the thermal conductivity, k_1, for the inner layer of the furnace wall is $0.8Wm^{-1}K^{-1}$ and that of the outer layer, k_2, is $1.6Wm^{-1}K^{-1}$



(i) Explain why, in steady state, the rate of thermal energy transfer must be the same in both layers
(ii) Determine the temperature at the interface, I, between the layers. **An(507 °C)**
(iii) Sketch and label a graph which shows the variation of temperature with distance a cross the wall

5.2: RADIATION

Thermal radiation is electromagnetic radiation emitted by a body mainly due to its temperature.
At lower temperature, the body emits mainly infrared and at high temperatures the body emitts ultraviolet, visible in addition to infrared

5.2.1:Infrared radiations

Infrared is part of electromagnetic spectrum extending from 0.7μm to about 1mm

5.2.2:Properties of infrared radiation

- Move at a speed of light ($3x10^{8}ms^{-1}$)
- It can be reflected and refracted just like light
- Cause an increase in temperature when absorbed by matter
- It can cause photo electric emission surface
- It affects special types of photographic plates and it enables pictures to be taken in dark
- It is absorbed by glass but is transmitted by rock salt and quartz

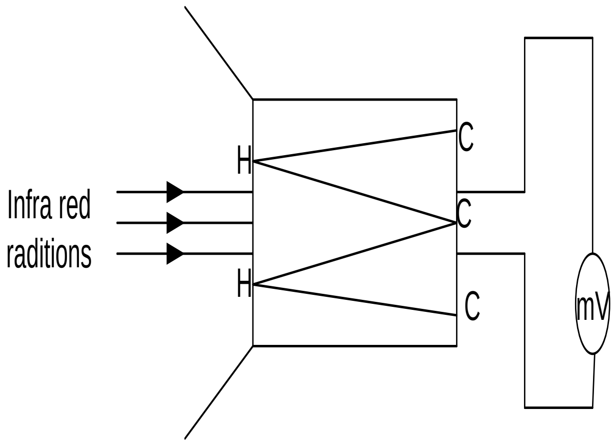
5.2.3:Detection of infrared radiations

Infrared radiations can be detected using 3 methods

- Thermopile
- Ether thermoscope
- Beam of light

Thermopile

Thermopile consists of many thermocouples connected in series

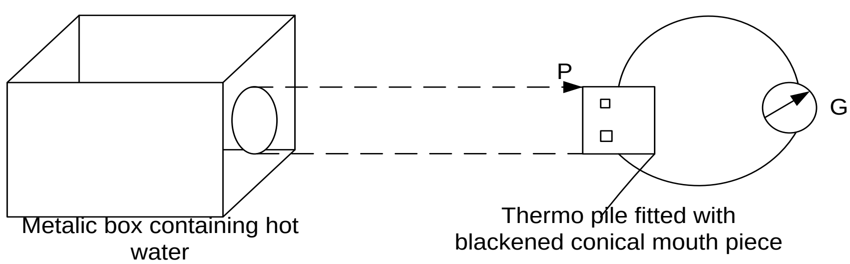


- The hot junction of thermopile H are blackened to make it good absorber of radiation. While the cold junction C are shielded junction.
- Highly polished metal core concentrates the radiation on the exposed junctions. Infrared radiations fall on blackened junction H of thermopile and warms the junction and sets up an $E.m.f$ that can be measured using mill voltmeter connected directly to the thermopile.

5.2.4: PREVOST'S THEORY OF HEAT EXCHANGE

- **Prevost's theory states** that a body emits radiation at a rate which depends on its temperature and nature of its surface and it absorbs heat at rate that depends on the nature of its surface and the temperature of surrounding.
  A body at constant temperature is in state of thermodynamic equilibrium with surrounding. Its rate of emission of radiation to the surrounding in equal the rate of absorption of radiation from its surrounding.
  It is concluded in provosts theory that a good absorber of radiation, must also be a good emitter otherwise its temperature would rise above that of its surrounding.

5.2.5: EXPERIMENT TO DETERMINE WHICH SURFACE ARE GOOD ABSORBERS AND POOR ABSOBERS OF HEAT RADIATION



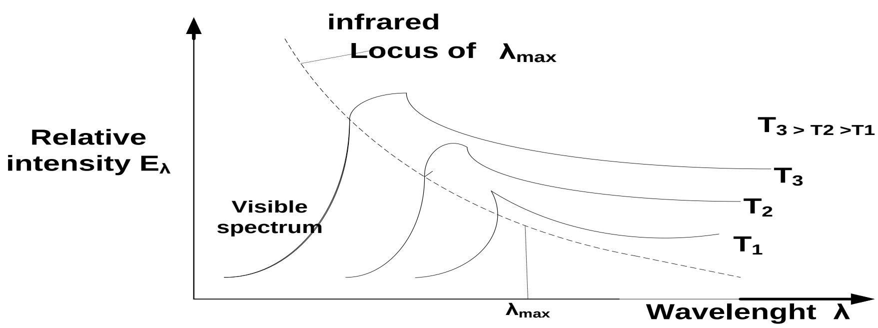
- The above metallic box was painted with a variety of colours which included dull black and highly polished silver surface.
- The box containing boiling water was connected to the galvanometer as shown above
- The galvanometer deflection was greatest when the thermopile faced a dull black surface of the box and least when the thermopile faced a highly polished silver surface

- ❖ This means that a highly polished surface is a poor radiator and the dull black surface is the best radiator.

5.2.6:BLACK BODY RADIATION

A black body radiation is the radiation whose quality (wave length) depends only on the temperature of the body.

Spectral curves for black body radiation



<u>Special features of the curve</u>

- ❖ At each temperature t, the energy radiated is a max, for a certain wave length $\lambda_{maximum}$ which decreases with rising temperature i.e $\lambda_{max} \alpha \frac{1}{T}$ or $\boxed{\lambda max\ T\ =\ constant}$

  This statement is called **Wein's displacement law**

  wein's displacement constant =$2.9\text{x}10^{-3}$mK
- ❖ As the temperature increases, the relative intensity (energy) at each wave length increases (the body becomes brighter) but the increase is much the rapid for shorter wave length. (the colour of the body changes).The appearance of the body depends on the position of λ
- ❖ The body changes its colour when cold (λ_{max} in the red region of visible spectrum) to yellow hot, white hot (λ_{max} in the middle spectrum visible) and eventually to blue hot (λ_{max} in blue region)
- ❖ The area under each spectral curve = intensity E or energy emitted per second per meter squared or power per meter squared.

Question

1) Explain how Wein's displacement law is used to explain colour change of hot metal object us to temperature raises.
2) Explain why the centre of fire appears white

3) State wein's displacement law.
4) State black body radiation laws.

Solution

Weins displacement law and Stefan-Boltzmann's law

5.2.7:Relative intensity E_λ, is the energy radiated per meter square per second per unit wave length interval.

Intensity E, is the energy emitted per second per meter squared or power emitted per meter squared.

5.2.8:BLACK BODY

When radiation falls on surface three thinks may happen on it

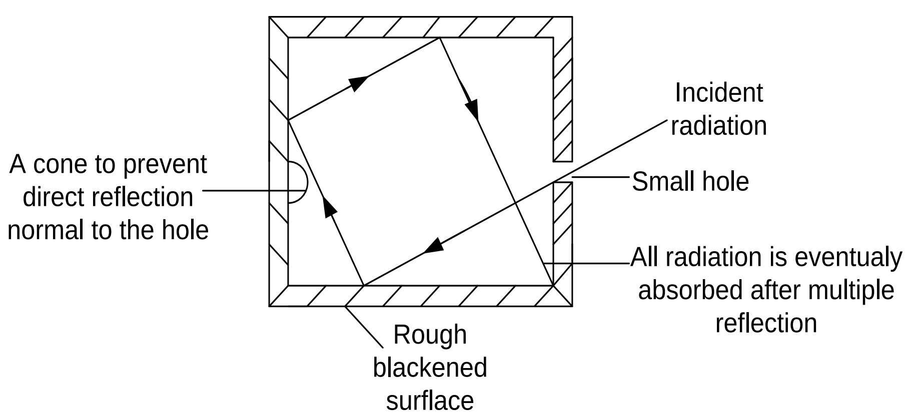
- A certain amounts will be reflected
- A certain amount A will be absorbed
- A certain amount T will be transmitted

Incident energy =R+A+T

Definition

A black body is one which absorbs all radiations falling on it and it reflects non or transmits none.

5.2.9: Approximation of a black body OR realization of black body



- ❖ The absorber which approximates a black body can be made by punching small hole in an enclosure (cavity) whose inside walls are dull and black.
- ❖ After each reflection inside the cavity, a high percentage of radiation is absorbed.
- ❖ All the radiations are eventually absorbed after multiple reflections .

A black body radiator (cavity radiator)

The amount of radiation emitted by a black body depends on three things

- Surface area of the body
- The type of the surface
- The temperature of the body

Definition: A black body radiator is one which emits radiation which is characteristic of its temperature and does not depend on the nature of its surface.

5.3: STEFAN'S LAW (STEFAN- BOLTZMAN'S LAW)

- It states that "the total power per meter squared of a black body is directly proportional to the fourth power of its absolute temperature" ($P\alpha T^4$)

OR

- Total energy radiated by a blackbody per unit area per unit time is directly proportional to the fourth power of its absolute temperature (t). ($E\alpha T^4$)

$$\frac{energy}{A\,reax\,Time} \propto T^4$$

$$\frac{energy}{A\,reax\,Time} = \sigma T^4$$

$$\sigma = \frac{energy}{A\,reax\,Time\,T^4}$$

$$\sigma = \frac{J}{m^2 S k^4}$$

$$\sigma = Js^{-1}m^{-2}Sk^4$$

$$\boldsymbol{\sigma = Wm^{-2}Sk^4}$$

$$\sigma = 5.67x10^{-8}m^{-2}Sk^4$$

5.3.1:Expression for power radiated by black body

From Stefan's law

$$\frac{energy}{surface\ area\ x\ Time} = \sigma T^4$$

$$\frac{I.Vt}{S.t} = \sigma T^4$$

$$\boxed{P = S\,\sigma T^4}$$

Example

1) A cylinder has radius 10^{-2}m and height 0.75mm. Calculate the temperature of cylinder if it is assumed to be lamp of power 1kW.

Solution

P=S σT^4
P=1kW= 1x1000W
S=2πrh
σ=5.67x$x10^{-8}Wm^{-2}SK^4$
T=?

1000=5.67x$x10^{-8}$x2πx10^{-2}x7.5x10^{-4}xT^4
1000=2.67192x10^{-12} T^4
T^4= (3.74262x10^{14})
T = $(3.74262x10^{14})^{\frac{1}{4}}$
T=4398.435K

5.3.2:Expression for net power for a body in the surrounding

If a black body of surface area S is at absolute temperature T_0 placed in an environment which is at lower temperature T.
The body emits heat to the surrounding environment by radiation and also receive heat from surrounding environment such that in both cases heat exchange is by radiation.

$$\mathrm{P_{net}} = P_{absorbed} - P_{emitted}$$

$$\mathrm{P_{net}} = \mathbf{S\sigma T_0^4 - S\sigma T^4}$$

$$\boxed{P_{net} = S\,\sigma(T_0^4 - T^4)}$$

For T_0>T

Example

1) The element of 1kW electric fire lamp is 30cm long and 1cm diameter if the temperature surrounding in 20^{0}C. Estimate the working temperature of element

Solution

V=πr^3h
S=2πrh
P=1x1000
P=1000W
T=20+273=293K
P= S σ(T$_0^4$- T^4)

1000=2$x3.14$x0.5x10^{-2}x30x10^{-2}{ T$_0^4$-293^4} σ
1000=9.42x10^{-3}{ T$_0^4$-293^4} x5.67x10^{-8}
1000= 5.34114x10^{-10}(T$_0^4$-293^4)
1000= 5.34114x10^{-10} T$_0^4$-3.93645
T^4=1.87963x10^{12}
T$_0$=1170.8K

2) A small blackened solid copper sphere of radius 2cm is placed in evacuated enclosure those walls are kept at 100^{0}C. find the rate at which energy must be supplied to sphere to keep its temperature constant at 127^{0}C.

Solution

T$_0$=100^{0}C = 373K

$T=127^0c \quad = \quad 400K$

$S = 4\pi r^2$

$P_{net} = P_R - P_A$

$P_{net} = S\sigma T_0^4 - S\sigma T^4$

$P_{net} = S\sigma(T_0^4 - T^4)$

$P_{net} = 4\pi x(2x10^{-2})^2 x5.67x10^{-8}(400^4 - 373^4)$

$P_{net} = 1.425x10^{-8}(2.56x10^{10} - 1.9356x10^{10})$

$P_{net} = 1.779W$

3) A solid copper sphere of diameter 10mm and temperature of 150K is placed in an enclosure maintained at temperature of 290K. Calculate stating any assumption made the initial rate of rise of temperature of sphere. ($\rho$of copper $8.95x10^3 kg/m^3$, S.H.C of copper $3.7x10^2 Jkg^{-1}K^{-1}$)

Assumption

- The sphere behaves like by a black body
- All heat exchange by radiation

Solution

$\rho_e = 8.75x10^{-3}kgm^{-3}$

S.H.C = $3.7x10^2 Jkg^{-1}K^{-1}$

T=150K

T_0 = 210K

$P_{net} = P_A - P_R$

$P_{net} = S\sigma T_0^4 - S\sigma T^4$

$P_{net} = S\sigma(T_0^4 - T^4)$

$S = 4\pi r^2$

$P_{net} = 4\pi r^2 x5.67x10^{-8}(290^4 - 150^4)$

$\frac{MC\Delta\theta}{t} = 4\pi r^2 x5.67x10^{-8}(290^4 - 150^4)$

$\frac{\Delta\theta}{t} = \frac{4\pi r^2}{mc} x5.67x10^{-8}(290^4 - 150^4)$

M=vol$x\rho$

$\frac{\Delta\theta}{t} = \frac{4\pi r^2 x5.67x10^{-8}(290^4 - 150^4)}{\frac{4}{3}\pi r^3 x8.95x10^{-8}x3.7x10^{+2}}$

$= \frac{3x5.67x10^{-8}(290^4 - 150^4)}{(5x10^{-3})x8.95x10^{-8}x3.7x10^{+2}}$

=0.067K/s

4) A solid metal sphere is placed in an enclosure at temperature of 27^0C when temperature of the metal is 227^0C, it cools at rate of 3.2^0C per minute. What is the rate of cooling when solid sphere of same metal but twice the radius at 127^0C is placed in the same enclosure

Solution

$P_{net} = S\sigma(T_0^4 - T^4)$

$\frac{MC\Delta\theta}{t} = S\sigma(T_0^4 - T^4)$

Let y= rate of cooling[$\frac{\Delta\theta}{t}$]

M=vol$x\ \rho$

$MCy = S\sigma(400^4 - 300^4)$

$\frac{4\pi(2r)^3\rho Cy}{3} = S\sigma(400^4 - 300^4)$...1

$\frac{4\pi(r)^3\rho C3.2}{3} = S\sigma(500^4 - 300^4)$...2

Eqn1 divide by Eqn2

$\frac{8\,y}{3.2} = \frac{4\pi(2r)^2\sigma\,(400^4 - 300^4)}{4\pi r^2\sigma\,(500^4 - 300^4)}$

$y = \frac{3(400^4 - 300^4)}{2\,(\,500^4 - 300^4)}$

$y = 0.48^0Cmin^{-1}$

Exercise13

A copper of diameter 20mm is cooled to temperature of 500K and then placed in an enclosure maintained at 300K. Assuming that all heat exchange is by radiation, calculate the initial rate of loss of temperature of sphere assumed as a black body.

(ρ of copper $8.39 x 10^3 kg/m^3$, S.H.C of copper $370 J/kg^{-1}$) **An (0.28W)**

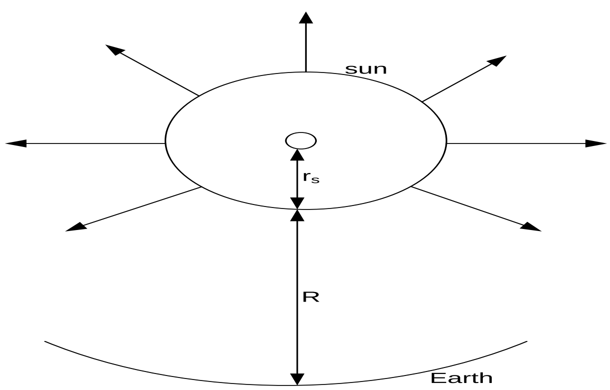
5.3.3:Expression for solar constant

Assuming the sun to be a black body and spherical. The power radiated by the sun.

Ps = $S\sigma T^4$

Where S is its surface area of sun ($4\pi r_s^2$)

r_s is The radius of the sun



Ps =$4\pi r_s^2 \sigma T^4$

The total area of the earth receiving radiation (solar constant/solar power)

$=\frac{power\ of\ the\ sun}{surface\ area\ of\ the\ earth}$

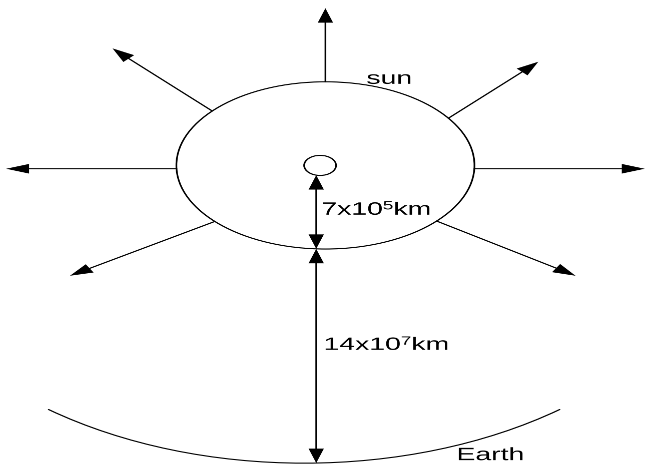
$$\boxed{\boldsymbol{Solar\ power = \frac{4\pi r_s^2 \sigma T^4}{4\pi R^2}}}$$

A solar power is the amount of energy received from the sun per second per meter squared.

Example

1) The energy intensity received by a spherical planet from star is $1.4 \times 10^3 Wm^{-2}$. The star is of radius $7.0 x 10^5$km and $14.0 x 10^7$km from the planet. Calculate the surface temperature of star and state any assumptions made.

Solution



Solar power =$\frac{4\pi r_S^2 \sigma T^4}{4\pi R^2}$

$1.4x10^3 = \frac{(7.0x10^5x1000)^2 x5.67x10^{-8}xT^4}{(14.0x10^7x1000)^2}$

T=5605.976K

Assumption

-The star behaves as a black body
-The star is a perfect sphere
-There is no heat loss to the surrounding

2) The sun's radius is $7.0x10^8$m.It's distance from the earth is $1.52x10^{11}$m and the solar constant is 1400Wm^{-2}. Calculate the surface temperature of the sun.

Solution

Solar power =$\frac{4\pi r_S^2 \sigma T^4}{4\pi R^2}$

$1400 = \frac{r_3^2 \sigma T^4}{R^2}$

$1400 = \frac{(7.0x10^8)^2 x5.67x10^{-8}xT^4}{(1.52x10^{11})^2}$

T=584.129K

5.3.4: RADIACTIVE EQUILIBRIUM OF THE SUN AND THE EARTH

The power radiated by the sun is given by

$P_S = 4\pi r_s^2 \sigma T_S^4$

Where T_s=surface temperature of sun , r_s= radius of sun

The solar power = $\frac{4\pi r_3^2 \sigma T_s^4}{4\pi R^2}$

The power received by effective area of the earth =solar power x area of earth

= solar power x πr_E^2

Where r_E – radius of earth

power received by the earth = $\frac{4\pi r_3^2 \sigma T_s^4}{4\pi R^2} \times \pi r_E^2$ ---------------------------------- [1]

Earth also behaves like a black body, then the power radiated by the earth is

$P_E = 4\pi r_E^2 \sigma T_E^4$ --- [2]

$4\pi r_E^2 \sigma T_E^4 = \frac{4\pi r_3^2 \sigma T_s^4}{4\pi R^2} \times \pi r_E^2$

$T_E^4 = \frac{r_s^2}{4R^2} T_s^4$

$$\boxed{T_E^4 = \left(\frac{r_s}{2R}\right)^2 T_s^4}$$

Example

1) Consider the sun to be the sphere of radius $7.0x10^8$m where surface temperature is 5900K.

(i) Find the solar power incident on an area of $1m^2$ on the top of earth's atmosphere if it's at a distance of $1.5x10^{11}$m from the sun. Assume that the sun radiates as a black body.

(ii) Explains why solar power incident on $1m^2$ of earth surface is less than the calculated value in (i) above.

Solution

solar power = $\frac{4\pi r_3^2 \sigma T_S^4}{4\pi R^2}$

$=\frac{(7.0x10^8)^2 x5.67x10^{-8}x5900^4}{(1.5x10^{11})^2}$

$= 1.496x10^8 Wm^{-2}$

(ii)

- Some of the energy is absorbed by the particles in atmosphere
- Some of the energy is scattered by particles in atmosphere

2) Assume that the sun is sphere of radius $7.0x10^8$m at temperatures 6000k. Estimate the temperature of surface of mars if its distance from the sun $2.28x10^{11}$m.

Solution

solar power = $\frac{4\pi r_S^2 \sigma T_S^4}{4\pi R^2}$

power received by mars = $\frac{4\pi r_S^2 \sigma T_S^4}{4\pi R^2} x\ \pi r^2{}_m$

power radiated by mars $=4\pi r_m^2 \sigma T_m^4$

at equilibrium

power radiated =power received

$4\pi r_m^2 \sigma T_m^4 = \frac{4\pi r_S^2 \sigma T_S^4}{4\pi R^2} x\ \pi r_m^2$

$T_m^4 = \frac{r_S^2}{4R^2} T_S^4$

$T_m^4 = \frac{r_S^2}{4R^2} T_S^4$

$T_m^4 = \left(\frac{r_s}{4R^2}\right)^{\frac{1}{4}} T_S^4$

$T_m = \left\{\frac{(7x10^8)^2 x6000^4}{4(2.28x10^{11})^2}\right\}^{\frac{1}{4}}$

$T_m = 235.08K$

3) The average distance of plants is about 40 times to that of earth from the sun. If the sun radiates as black body at 6000k and is $1.5x10^{11}$m from the earth. Calculate the surface temperature of Pluto.

Solution

Distance of Pluto = 40xdistance from earth

$R=40x1.5x10^{11}$

solar power = $\frac{4\pi r_S^2 \sigma T_S^4}{4\pi R^2}$

Power received by mars = $\frac{4\pi r_S^2 \sigma T_S^4}{4\pi R^2} x\ \pi r^2 p$

Power radiated by mars $=4\pi r_p^2 \sigma T_p^4$

At equilibrium

Power radiated =power received

$4\pi r_p^2 \sigma T_p^4 = \frac{4\pi r_S^2 \sigma T_S^4}{4\pi R^2} x\ \pi r_p^2$

$T_p^4 = \frac{r_S^2}{4R^2} T_S^4$

$T_p^4 = \left(\frac{r_s}{2R^2}\right)^2 T_s^4$

$T_m = \left\{\frac{(7x10^8)^2 x6000^4}{(2x60x10^{11})^2}\right\}^{\frac{1}{4}}$

T_m= 45.8K

Exercise 13

1) The element of an electric fire, with an output of 1.0kW, is a cylinder 25cm long and 1.5cm in diameter. Calculate its temperature when in use , if it behaves as a blackbody. (Stefan constant = $5.7x10^{-8}Wm^{-2}K^{-4}$)
   An(1105K)

2) Solid copper sphere of diameter 10mm is cooled to atmosphere of 150K and is then placed in an enclosure ,maintained at 290K. Assuming that all interchanges of heat is by radiation. Calculate the initial rate of rise of temperature of the sphere. The sphere may be treated as a black body.
   Density of copper = $8.93x10^3kgm^{-3}$,S.H.C of copper=$3.70x10^2Jkg^{-1}K^{-1}$, Stefan constant
   = $5.7x10^{-8}Wm^{-2}K^{-4}$) **An($6.78x10^{-2}Ks^{-1}$)**

3) The silica cylinder of a radiant wall heater is 0.6m long and has a radius 5mm. if it is rated at 1.5kW. Estimate its temperature when operating. State two assumptions you have made in making your estimate. Stefan's constant, $\sigma=6x10^8Wm^{-2}K^{-4}$)
   An(1073K)

4) A blackened metal sphere of diameter 10mm is placed at the focus of a concave mirror of diameter 0.5m directed towards the sun. if the solar power incident on the mirror is $1600Wm^{-2}$. Calculate the maximum temperature which the sphere can attain. State the assumptions you make (Stefan's constant =$6x10^{-8}Wm^{-2}K^{-4}$)
   An(2021K)

5) An unlagged, thin walled copper pipe of diameter 2.0cm carries water at a temperature of 40K above that of the surrounding air. Estimate the power loss per unit length of the pipe if the temperature of the surroundings is 300K and Stefan constant, $\sigma = 5.7x10^{-8}wm^{-2}K^{-4}$. State two important assumption you have made.
   An($19Wm^{-1}$)

6) The solar radiation falling normally on the surface of the earth has an intensity $1.40kWm^{-2}$. If this radiation fell normally on one side of a thin freely suspended blackened metal plate and the temperature of the surrounding was 300K, calculate the equilibrium temperature of the plate. Assume that all heat interchange is by radiation. (Stefan's constant $=5.67x10^{-8}Wm^{-2}K^{-4}$)
An(378K)

7) Estimate the surface temperature of the earth assuming that it is radioactive equilibrium with the sun. (radius of sun $7.0x10^{8}m$, surface temp of sun 6000K, distance from the earth to the sun $1.5x10^{11}m$, $\sigma =5.7x10^{-8}Wm^{-2}K^{-4}$) **An [289K]**

5.3.6:GREEN HOUSE EFFECT

- The sun behaves like a black body which emits radiations of short wavelength. These radiations pass through the green house gases such as water vapour, carbon dioxide in the atmosphere. The gases are absorbed by the earth which causes it to warm.
- The earth then behaves as a black body and emits these radiations but with a longer wave length .These radiations can now be absorbed by particles in the atmosphere and hence causing global warming.

5.3.7:THERMAL CONVECTION

Is the transfer of heat from one point to another by bulk movement of particles due to temperature difference

5.3.8:SEA BREEZE

This occurs day time when the earth warms faster than the sea, since water has high specific heat capacity than the land. Warm air (less dense) at the surface of earth rises above leaving a vacuum below and cool air (dense air) from the sea therefore moves to occupy the vacuum left at the surface of the land.

5.3.9:LAND BREEZE

This occurs at night when the land cools faster than the sea, since the sea is heated to a greater depth during the day than the land. Warm air at the surface of the sea rises above leaving a vacuum and therefore cool air from the land moves in to occupy the vacuum left at the surface of the sea.

UNEB 2013 Q.7

(a) (i) Define **thermal conductivity** of a material (01marks)

(ii) Describe an experiment to determine the thermal conductivity of copper

(b) (i) what is meant by **a black body**

(ii) Describe how infrared radiations can be detected using a bolometer (3mks)

(iii) Give one characteristic property of infrared radiation (01mark)

(c) (i) A spherical black body of radius 2.0cm at $-73°C$ is suspended in an evacuated enclosure whose walls are maintained at $27°C$. If the rate of exchange of thermal energy is equal to $1.85\, Js^{-1}$, find the value of Stefan's constant, σ. (05marks)

(ii) Calculate the wavelength at which the radiation emitted by the enclosure has maximum intensity (03marks)

Solution

(c) (ii)

$\lambda\text{max T} = \text{constant}$

Wein's constant = $2.9x10^{-3}$mK

$\lambda\text{max T} = 2.9x10^{-3}$

$T = 27 + 273K = 300K$

$\lambda_{max} = \frac{2.9x10^{-3}}{300}$

$\lambda_{max} = 9.67x10^{-6}$m

UNEB 2012 Qn7

(a) (i) Define **thermal conductivity**

(01marks)

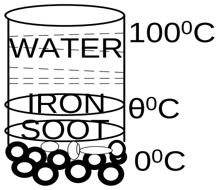
(ii) Compare the mechanism of heat transfer in **poor** and **good solid** conductors

(05marks)

(b) Describe, with the aid of a diagram how you would measure the thermal conductivity of a poor conductor, stating the necessary precautions.(08marks)

(c) A cylindrical iron vessel with a base of diameter 15cm and thickness 0.30cm has it's base coated with a thin film of soot of thickness 0.1cm.It is then filled with water at 100^0C and placed on a large block of ice at 0^0C. Calculate the initial rate at which the ice will melt.[The conductivity of soot=$0.12Wm^{-1}K^{-1}$, conductivity of iron=$75Wm^{-1}K^{-1}$]

Solution



$$\frac{dQ}{dt} = \frac{KA(\Delta\theta)}{L}$$

$$\frac{dQ}{dt} = \frac{75A(100-\theta)}{0.3X10^{-2}} = \frac{0.12A(\theta-0)}{0.1X10^{-2}}$$

$$\frac{75A(100-\theta)}{0.3X10^{-2}} = \frac{0.12A(\theta-0)}{0.1X10^{-2}}$$

$$7.5(100-\theta) = 0.36(\theta)$$

θ =99.52^0C

But $\frac{dQ}{dt} = \frac{0.12A(\theta-0)}{0.1X10^{-2}}$

$$= \frac{0.12x\frac{22}{7}x\frac{(15x10^{-2})^2}{4}(99.52-0)}{0.1X10^{-2}}$$

$\frac{dQ}{dt}$ =211.04Js^{-1}

$\frac{dm}{dt} = \frac{dQ}{dt}$xL

$$\frac{dm}{dt} = 211.04x15x10^{-2}$$

$\frac{dm}{dt}$=31.65 Jms^{-1}

UNEB 2011 Q7

(a) State **Stefan's law** of black body radiation (01marks)

(b) Briefly describe how a thermopile can be used to detect thermal radiation (05marks)

(c) Explain the temperature distribution along

(i) A perfectly lagged metal bar (02marks)

(ii) An un lagged metal bar (02marks)

(d) The wall of a furnace is constructed with two layers. The inner layer is made of bricks of thickness 10.0cm and thermal conductivity 0.8Wm^{-1}K^{-1} and the outer layer is made of a material of thickness 10.0cm and thermal conductivity 1.6Wm^{-1}K^{-1}

(i) Explain why in steady state, the rate of thermal energy transfer must be the same in both layers (01marks)

(ii) Calculate the rate of heat flow per square meter through the wall (05marks)

An(1066.64Jm^{-2}s^{-1})

(e) Explain the green house effect and how it is related to global warming (04marks)

UNEB2010 Q7

(a) What is meant by the following;

(i) Conduction

(ii) Convection

(iii)Green house effect

(06marks)

(b) One end of a long copper bar is in steam chest and the other end is kept cool by a current of circulating water. Explain with the aid of a sketch graphs, the variation of temperature along the bar, when steady state has been attained if the bar is;

(i) Lagged
(02marks)

(ii) Exposed to the surrounding
(02marks)

(c) (i) what is meant by a black body (01marks)
(ii) describe how a black body can be approximated in practice (04marks)

(d) (i) State Prevost theory of heat exchanges
(01marks)
(ii) Sketch the variation with wavelength of the intensity of radiation emitted by a black body at two different temperatures (01marks)

(e) A cube of side 1cm has a grey surface that emit 50% of radiation emitted by black body at the same temperature. If the cube's temperature is 700^0C, calculate the power radiated by the cube. (03marks) **An(4.08W)**

UNEB2009Q7

(a) State **thermal conductivity**
(01marks)

(b) (i)Explain the mechanisms of thermal conduction in non-metallic solids (03marks)
(ii) Why are metals better thermal conductors than non metallic solids (02marks)

(c) With the aid of a diagram, describe an experiment to determine the thermal conductivity of a poor conductor.
(06marks)

(d) (i) What is meant by a **black body**.
(01marks)
(ii) Sketch curves showing the spectral distribution of energy radiated by a black body at different temperatures
(02marks)

(e) A small blackened solid copper sphere of radius 2cm is placed in an evacuated enclosure whose walls are kept at 100^0C.Find the rate at which energy must be supplied to the sphere to keep it at constant temperature of 127^0C. **An(1.78W)**

UNEB2008Q7

(a) (i) State **laws of black body** radiation (02marks)

(ii) Sketch the variation of intensity with wavelength in a black body for three different temperatures. (03marks)

(b) (i) What is a **perfectly black body?**
(01mark)

(ii) How can a perfectly black body be approximated in reality. (04marks)

(c) The energy intensity received by a spherical planet from a star is $1.4x10^3Wm^{-2}$.The star is of radius $7.0x10^5$km and $14.0x10^7$km from the planet.

(i) calculate the surface temperature of the star. **An(5605.98K)**
(04marks)

(ii) state any assumption you have made in (c)(i) above. (01mark)

(d) (i) What is convection (01mark)

(ii) Explain the occurrence of land and sea breeze (04marks)

UNEB2006Q7

(a) (i) Define **thermal conductivity**
(01mark)

(ii) Explain the mechanism of heat transfer in metals
(03marks)

(b) Two brick walls each of thickness 10cm are separated by an air gap of thickness 10cm. The outer faces of the brick walls are maintained at 20^0C and 5^0C respectively.

(i) Calculate the temperature of the inner surfaces of the walls (06marks)

(ii) Compare the rate of heat loss through the layer of air with that through a single brick wall [Thermal conductivity of air is $0.02Wm^{-1}K^{-1}$ and that of the brick is $0.6WWm^{-1}K^{-1}$](03marks)

An(5.5^0C, 19.5^0C, 1:32.1)

(c) (i) State **Stefan's law of black body** radiation
(01marks)

(ii) The average distance of Pluto from the sun is a bout 40times that of earth from the sun. If the sun radiates as a black body at 6000K, and is $1.5x10^{11}$m from the earth, calculate the surface temperature of Pluto. **An(45.8K)** (06marks)

UNEB2005Q7

(a) (i) Define **thermal conductivity**
(01mark)

(ii) State two factors which determine the rate of heat transfer through a material(02marks)

(b) (i) Describe with the aid of a labeled diagram an experiment to measure the thermal conductivity of glass. (08marks)

(ii) Briefly discuss the advantages of the apparatus in b(i) above (02marks)

(c) Metal rods of copper, brass and steel are welded together to form a Y-shaped figure. The cross sectional area of each rod is $2cm^2$. The free ends of copper rod is maintained at 100^0C, while the free ends of brass and steel rods are maintained at 0^0C .If there is no heat loss from the surfaces of the rods and the lengths of the rods are 0.46m, 0.13m, and 0.12m respectively.

(i) Calculate the temperature at the junction (05marks)

(ii) find the heat current in the copper rod (02marks)

[Thermal conductivities of copper , brass, and steel are $385Wm^{-1}K^{-1}$, $109\ Wm^{-1}K^{-1}$, and $50.2\ Wm^{-1}K^{-1}$ respectively]

An(39.97K,1.01X10^{1}J)

UNEB2004 Q6

(a) Define **thermal conductivity** of a material and state it's units (02marks)

(b) Describe with the aid of a diagram, how to determine the thermal conductivity of a poor conductor. (07marks)

(c) A cooking sauce pan made of iron has a base area of $0.05m^2$ and thickness of 2.5mm. It has a thin layer of soot of average thickness 0.5mm on it's bottom surface. Water in the sauce pan is heated until it boils at 100^0C. The water boils a way at a rate of 0.60kg per minute and the side of the soot nearest to the heat source is at1500C. Find the thermal conductivity of soot. [Thermal conductivity of iron =$66Wm^{-1}K^{-1}$ and specific latent heat of vapourization=$2200KgJK^{-1}$] **An($3.3Wm^{-1}K^{-1}$)**

(d) (i) What is a black body (01mark)

(ii) Sketch the spectral distribution of a black body radiation for different temperatures and describe their main features (04marks)

SECTIONC: MODERN PHYSICS

CHAPTER1: CATHODE RAYS& POSITIVE RAYS

1.0: GASES AS CONDUCTORS OF ELECTRICITY

When a gas is free from external influences it has no free charges to act as carriers of electricity but can be a conductor of electricity in various ways;

In all these ways the molecules are made to be ionized by detachment of one or more electrons. E.g. reducing the pressure of a gas and applying a high $p.d$ across the container having the gas.

Energy must be supplied to molecules in order to ionize them, gas molecules become ionized as a result of collision with negative charges moving rapidly. The pressure of the gas is reduced to enable ions produced to have means free path long enough to allow them gain free energy for ionization.

1.1:CATHODE RAYS

These are streams of fast moving electrons that travel from cathode to anode when a $p.d$ is connected across the plate.

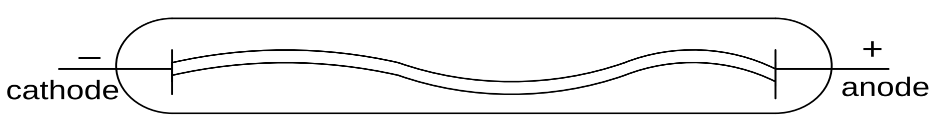
DISCHARGE TUBE

Properties of cathode rays can be studied in a discharge tube in which electricity is passed through a gas at low pressure. The electrons are enclosed in a glass tube which is connected to a pressure gauge and a vacuum pump.

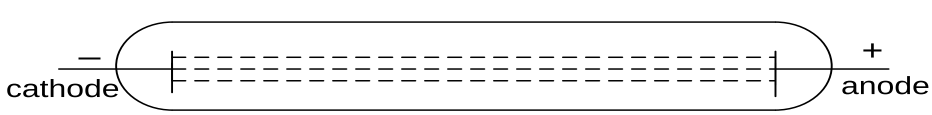
A discharge tube contains 2 metal electrodes approximately 30cm apart between which a $p.d$ of about 1000V is applied.

Steps leading to the production of a cathode ray.

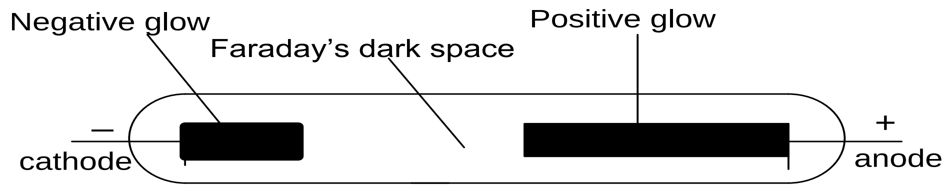
- At a pressure of $\approx$10 mmHg, a discharge of blue violet streamers pass between cathode and anode



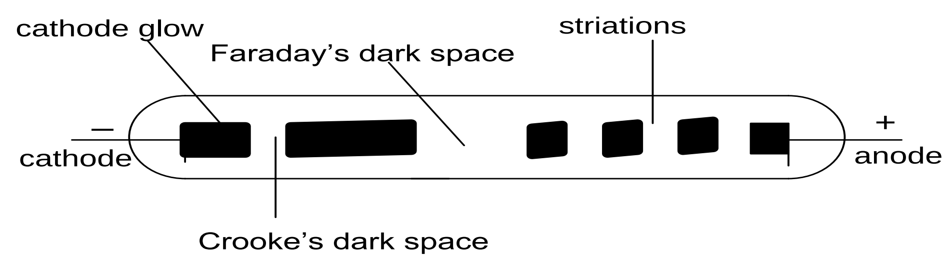
- At a pressure of $\approx$2 mmHg, long luminous positive column appears from anode to cathode



- At a pressure ≈1 mmHg, a pink positive column and a negative glow appear near the cathode. These two regions are separated by Faraday's dark space



- At a pressure of 0.1 mmHg, the positive column becomes striated. The negative glow moves away from the cathode, Crookes' dark space appears and the cathode glow appear round the cathode

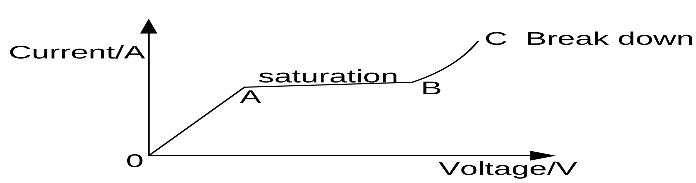


- At pressure of about 0.01mmHg, Crookes' dark space fills the glass tube and the tube flouresces due to electron.

Limitations of the discharge tube method

- When cathode rays strike the anode they may produce x-rays which are dangerous
- A very high $p.d$ is needed across the electrodes which can be hazardous to handle
- The gas is needed at appropriate low gas pressure which is difficult to attain

Characteristics of a discharge tube



- ✓ In region OA, as the applied voltage increases the number of electrons reaching the anode increases leading to the increase of the current.
- ✓ In region AB, at saturation the electrons released by collision reach the anode roughly at the same time so that the current through the tube appears constant.

- ✓ In region BC, the number of electrons due to ionization increases rapidly and not all the electrons reach the anode at the same time so the current increases gradually.
- ✓ At break down the number of electrons reaching the anode is so large and current rises sharply and this may damage tube. It can be prevented by connecting a resistance in series with the tube.

Applications of a discharge tube

(1) lighting fluorescent tubes

(2) In advertisement sign tube, neon signs

(3) Making Flood lights

(4) Making mercury lamps, sodium lamps

1.1.1:THERMIONIC EMISSION

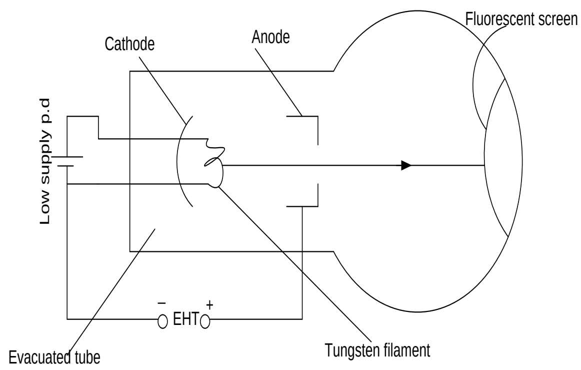
Definition

- ❖ **Thermionic emission** is a process by which electrons are emitted from a hot metal surface.
- ❖ **Work function** is the minimum energy required to release an electron from the metal surface. The work function of a metal is expressed in electron volts [eV]
- ❖ **eV** is defined as the kinetic energy gained by an electron in being accelerated by a potential difference of one volt.

1.1.2: MECHANISM OF THERMIONIC EMISSION

- ❖ Metals contain free electrons in their lattice that are loosely bound to their parent nuclei.
- ❖ As the temperature of the metal is raised, velocities of the electrons increase, some of the surface electrons acquire sufficient kinetic energy to overcome the electrostatics attraction force of the atomic nucleic and consequently escape from the metal surface.

1.1.3: MODERN CATHODE RAY TUBE [PRODUCTION OF CATHODE RAYS]



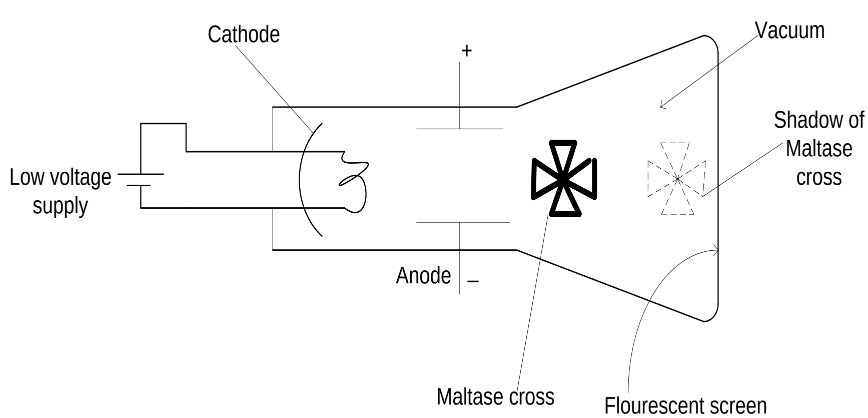
- The electrons are focused by the cathode and accelerated by EHT to the fluorescent screen which gives a glow when they strike the screen.
- It is the beam of fast moving electrons from the cathode which constitute the cathode rays.
- The cathode is heated by a low p.d and produces electrons by thermionic emissions.

1.1.4: PROPERTIES OF CATHODE RAYS

- They travel from cathode to anode in a straight line
- They are electrons and carry a negative charge
- They can be deflected in an electric field towards the positive plate
- They can be deflected in a magnetic field towards the North Pole according to Flemings left hand rule.
- They cause certain substances to fluorescence when they collide with them
- They posses kinetic energy which is changed to heat when they are brought to rest
- They can produce x-rays if they are of sufficiently high energy

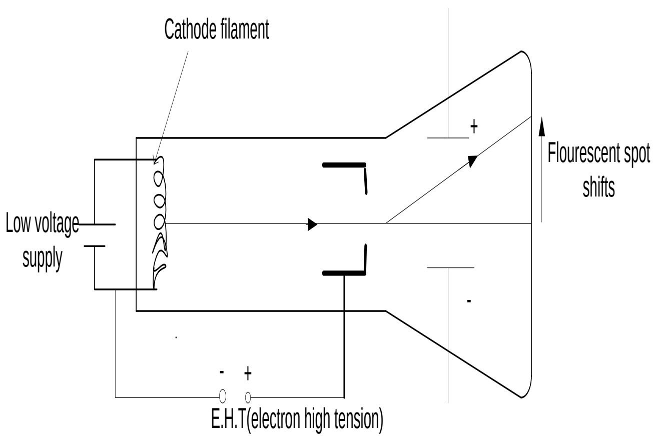
1.1.5:TO STUDY PROPERTIES OF CATHODE RAYS

1: Straight line movement



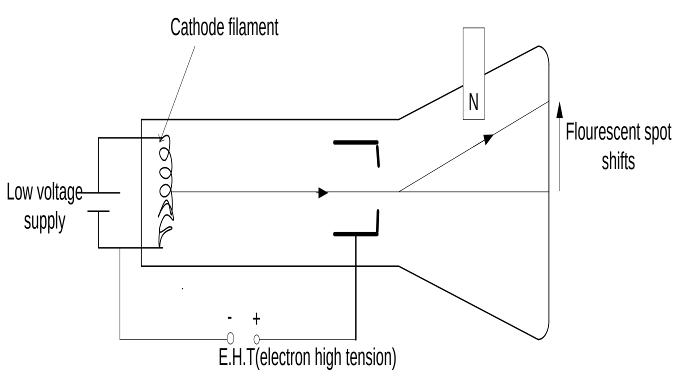
- Electrons emitted from a heated cathode, are accelerated by the anode and directed towards a maltase cross placed in the center of the a glass tube.
- A sharp shadow of the maltose cross is cast on a screen at the end of the tube. This shows that cathode rays travel in a straight line

2: Carry a negative charge



- The cathode ray tube is modified to include parallel plates connected to the terminals of a battery as shown below;
- When cathode rays are produced thermionically and passed through the plate, the fluorescent spot is seen to shift upwards from its initial position before the plates were applied. The spot moves towards the positive plate and away from the negative plate, this shows that cathode rays carry a negative charge.

3: Deflection in the magnetic field



The magnetic field is provide by the magnet. The application of the magnetic field makes the fluorescent spot to shift in the direction which depends on the field i.e. the pole nearer the beam.
The beam bends towards the North Pole.

1.1.6:ELECTRODE DYNAMICS

1:Motion in an Electric field

a) Speed of an electron

Suppose an electron of charge e and mass m is emitted from a hot cathode and **accelerated** by an electric field of potential V_a towards the anode, then;

Kinetic energy gained by the electron =work done on an electron by the accelerating p.d V_a

$\frac{1}{2}$ mu^2=e V_a

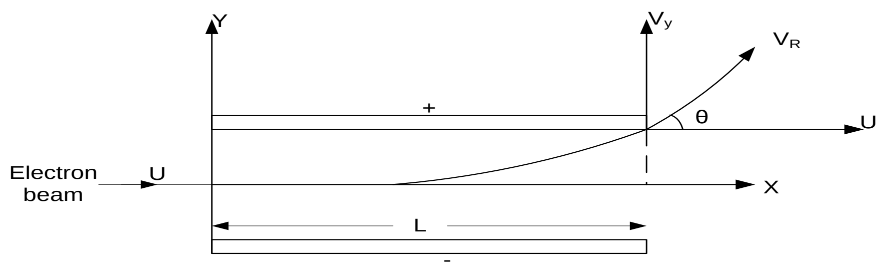
Or

$$\boxed{u = \sqrt{\frac{2eV_a}{m}}}$$

Note V_a must be accelerating $p.d$ and not $p.d$ between the plates

b) **Electron path in an electric field**

Consider an electron of charge e and mass m entering an electric field horizontally with a speed u.



Since electric field intensity is force per unit charge, the force on the electron is therefore given by

F= Ee-----------------------------[1] Where E is electric field intensity

But also F=ma ----------------------------------- [2]

Equating 1 and 2 Ma= Ee

a= $\frac{Ee}{m}$ ------------------------------------ [3]

if the $p.d$ between the plates is V and their distance is d, then

E= $\frac{V}{d}$

Put into equation 3

a= $\frac{Ve}{md}$ -- [4]

For vertical motion [u=0m/s], y= $\frac{1}{2}$ $\frac{Ve}{md}$ t^2-- [5]

for horizontal motion [a=0m/s, constant velocity], x= ut

t= $\frac{x}{u}$ --- [6]

put into equation 5

$y = \frac{1}{2}\frac{Ve}{md}\left(\frac{x}{u}\right)^2$

$$\boxed{\boldsymbol{y = \left(\frac{Ve}{2mdu^2}\right)x^2}}$$

Note

- When an electron moves horizontally into a uniform electric field, it describes a parabolic path. This parabolic motion is brought by the electric force [F=Ee] experienced by electrons in the direction of that of the field.
- The horizontal motion of the electron is not affected by the field. A charge gains energy when it moves in the direction of an electric field

Formula when the electron just leaves the plate

After leaving the plates the electron moves in a straight line. Just after passing through the plates, the distance x moved through is equal to the length of the plate . Therefore $x=l$

$$\boxed{y = \left(\frac{Vel^2}{2mdu^2}\right)} \quad \text{OR} \quad \boxed{y = \left(\frac{Ee}{2mu^2}\right)l^2}$$

For the velocity along the vertical (u=0m/s)

$$v = u + at$$

$$V_y = at$$

$$V_y = \frac{Ve}{md}\frac{l}{u} \qquad \text{where } t=\frac{l}{u}$$

$$\boxed{V_y = \frac{Ve}{md}\frac{l}{u}} \quad \text{OR} \quad \boxed{V_y = \frac{Eel}{mu}}$$

The resultant velocity V_R with which the electron leaves the field is given by

$V_R^2 = v_y^2 + u^2$

$$\boxed{V_R = \sqrt{v_y^2 + u^2}}$$

The direction with which the electron emerges from the field

$\tan\theta = \frac{V_y}{u}$

$$\boxed{\boldsymbol{\theta = \tan^{-1}\left(\frac{V_y}{u}\right)}}$$

Example

1. An electron is accelerated from rest through a p.d of 1000V. what is;

(a) Its kinetic energy in eV

(b) Its kinetic energy in joules

(c) Its speed

Solution

a) Va=1000V

$k.e$ =1000eV

b) $k.e$ =eVa

=1.6x10^{-19}x1000

$k.e$ =1.6x10^{-16}J

c) $k.e$ =eVa

½ mu^2=1.6x10^{-19}x1000

u=$\sqrt{\frac{2x1.6x10^{-19}x1000}{9.11x10^{-31}}}$

m$_e$=$9.11x10^{-31}$, m$_e$ is mass of the electron

u=1.874x10^{7}m/s

2. Calculate the speed of a proton which has been accelerated through the $p.d$ of 400V [mass of proton =1.67x10^{-27}kg, e=1.6x10^{-19}C]

Solution

$k.e\ = eVa$

½ mu^2=eVa

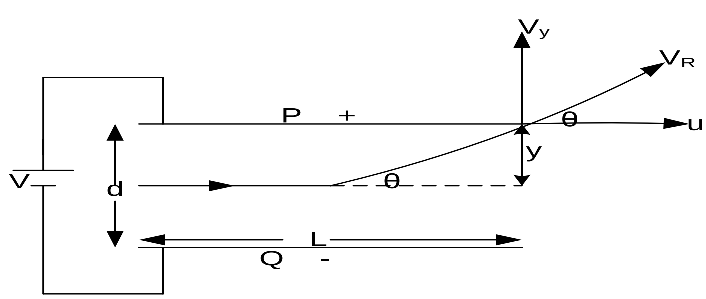
U=$\sqrt{\frac{2eVa}{m}}$

U=$\sqrt{\frac{2x1.6x10^{-19}x400}{1.67x10^{-27}}}$

U=$2.769x10^5 m/s$

3. A beam of electrons, moving with velocity of 1.0x10^{7}ms^{-1}, enters midway between two horizontal parallel plates P, Q in a direction parallel to the plates. P and Q are 5cm long, 2cm apart and have a $p.d$ V applied between them. Calculate V if the beam is deflected so that it just grazes the edge of the upper plate P

Solution



L=$\frac{5}{100}m$

d=$\frac{2}{100}m$

u=1x10^{-1}ms^{-1}

y=$\frac{1}{100}m$ since the beam is directed midway

from s=ut+ ½ at^2

y= ½ at^2 (u=0m/s)

but a=$\frac{ve}{md}$

y=$\frac{vet^2}{2md}$

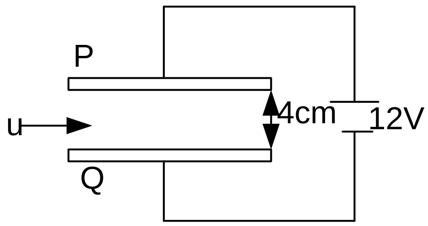
t=$\frac{l}{u}$

y=$\frac{Vel^2}{2mdu^2}$ m=9.11x10^{-37}kg mass of an electron

$\frac{1}{100}$=$\frac{vx1.6x10^{-19}x}{2x9.11x10^{-31}\left(\frac{2}{100}\right)}\left(\frac{5}{100}\right)^2x(1x10^7)^2$

V=91.1V

4.



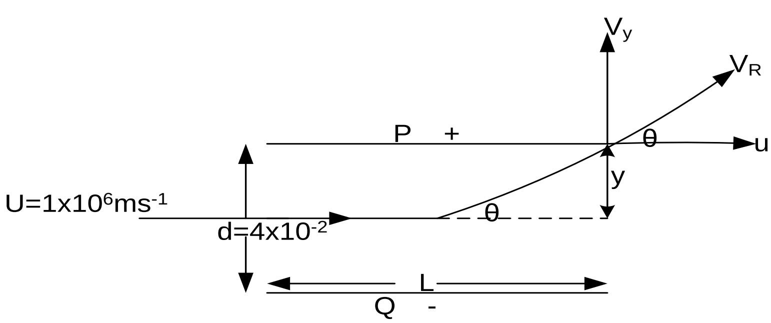
The figure above shows 2 parallel metal plates P and Q each of length 4cm and separately by a distance 4cm. A p.d of 12V is applied between P and Q and abeam of electrons of speed

u $=1x10^{6}ms^{-1}$ is directed midway between P and Q at right angles to the electric field between the plates, calculate;

(i) The angle to the initial direction of the beam at which the beam emerges from the space between plates P and Q

(ii) The velocity at which electron leaves the plates.

Solution



d=$4x10^{-2}$m, u=$1x10^{6}ms^{-1}$, V=12V, L=$4x10^{-2}$m

V_y=at

$V_y= \frac{ve}{md}\frac{l}{u}$

$V_y=\frac{12x1.6x10^{-19}x4x10^{-2}}{9.11x10^{-31}x4x10^{-2}x1x10^{6}}$

V_y=$2.11x10^{6}ms^{-1}$

θ=$\tan^{-1}\frac{v_y}{u}$

θ=$\tan^{-1}\frac{2.11x10^6}{10^6}$

θ=64.6^0

ii) V_y=$2.11x10^{6}ms^{-1}$

u=$1x10^{6}$

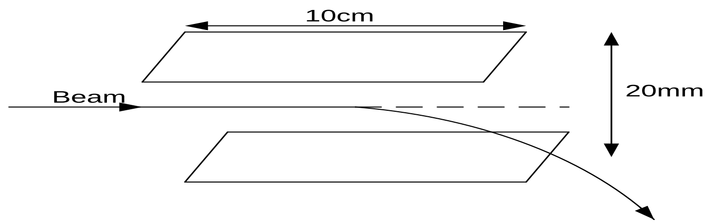
$V_R=\sqrt{v_y^2+u^2}$

$V_R=\sqrt{(2.11x10^6)^2+(1x10^6)^2}$

V_R=$2.33x10^{6}$m/s

The electron leaves the plates with a velocity of $2.33x10^{6}$m/s

5. Two parallel metal sheep's of length 10cm are separated by 20mm in a vacuum. A narrow beam of electrons enters symmetrically between them as shown.

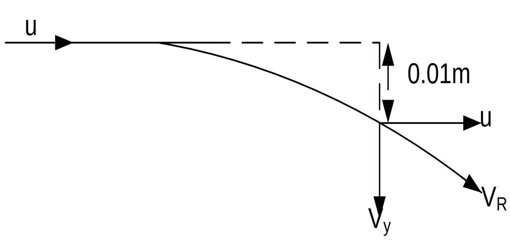


When a $p.d$ of 1000V is applied between the plates the electron beam just misses one of the plates as it emerges.

Calculate the speed of the electrons as they enter the gap [Take the field between the plates to be uniform] [$\frac{e}{m}$=1.8x10^{11}Ckg^{-1}]

Solution

Since the beam enters symmetrically y=$\frac{d}{2}$, y=$\frac{0.02}{2}$, y=0.01m



d=0.02m, L=0.1m V=1000V

required to get is u=m/s

but specific charge $\frac{e}{m}$=1.8x10^{11}Ckg^{-1}

using y=$\frac{vet^2}{2md}$

when the beam just emerges t=$\frac{l}{u}$

0.01=$\frac{1000x(0.1)^2}{2x0.02xu^2}$ x$\left(\frac{e}{m}\right)$

0.01=$\frac{1000x(0.1)^2x1.8x10^{11}}{2x0.02xu^2}$

u^2 =4.5x10^{15}

u=6.71x10^{7}ms^{-1}

6. In an evacuated tube, electrons are accelerated through a p.d of 500V. Calculate their final speed and consider whether this depends on the accelerating field being uniform.
   After this acceleration, the electrons pass through a uniform electric field which is perpendicular to the direction of travel of the electrons as they enter the field. This electric field is produced by applying a potential difference of 10V to two parallel plates which are 0.06m long and 0.02m apart. Assume that the electric field is uniform and confined to the space between the two plates.
   Determine the angular deflection of the electron beam produced by the field.
   [e/m for the electron =1.76x10^{11}Ckg^{-1}]

Solution

For the first case

½ mu^2 =eVa

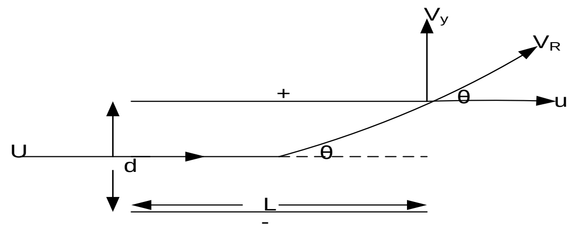
U^2= 2Va$\frac{e}{m}$

$U=\sqrt{2Va\frac{e}{m}}$

$U=\sqrt{2x500x1.76x10^{11}}$

$U=1.33x10^7 m/s$

For the second case



$v_y=3.97x10^5 m/s$

but

$\Theta=\tan^{-1}\frac{v_y}{u}$

$\Theta=\tan^{-1}\frac{3.97x10^5}{1.33x10^7}$

$\Theta=1.71^0$

The angular deflection $=1.71^0$

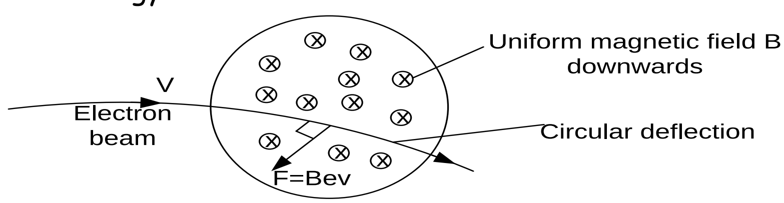
d=0.02m, L=0.06m ,V=10V, $\frac{e}{m}=1.76x10^{11}$

$v_y=\frac{vel}{mdu}$

$v_y=\frac{10x0.06x1.76x10^{11}}{0.02x1.33x10^7}$

2. Motion is a magnetic field

Consider an electron beam, moving with a speed V which enters a uniform magnetic field of magnitude B acting perpendicular to the direction of motion. The force F on an electron then $F = BeV$. The direction of the force is perpendicular to both B and V, the magnetic force cannot change the energy of the electron. It only deflects the electron but does not change its speed or kinetic energy.



The force BeV is always normal to the path of the beam. If the field is uniform the force is constant in magnitude and the beam then travels in a circle of radius r, since BeV is the centripetal force towards the centre.

$$Bev=\frac{mv^2}{r}$$

$$Be = mv$$

$$\boxed{r = \frac{mv}{Be}}$$

$$r=\frac{momentum}{Be}$$

If the velocity V of the electron decreases continuously due to may be collision, its momentum decreases, so from the relation for r above, the radius of its path decreases and the electron will then tend to spiral instead of moving in a circular path of constant radius.

Note

- When an electron beam having a common velocity enters a uniform magnetic field in a direction normal to the radius r, in accordance with Flemings left hand rule they experience a magnetic force $F = BeV$ at right angles to both B and V and so the field will provide the centripetal force which makes the electron beam to move in a circular path of constant radius.

Examples

1. An electron is moving in a circular path at $3.0x10^6 ms^{-1}$ in a uniform magnetic field of flux density $2.0x10^{-4}$T. Find the radius of the path [mass of electron=$9.1x10^{-31}$kg,charge on electron= $1.6x10^{-19}$C]

   Solution

   $V=3x10^6 ms^{-1}$
   $B=2x10^{-4} T$
   $m=9.1x10^{-31} kg$
   $e=1.6x10^{-19} c$

   $\therefore \frac{mv^2}{r} = Bev$

   $r = \frac{mv}{Be}$

   $r=\frac{9.1x10^{-31}x3x10^6}{2x10^{-4}x1.6x10^{-19}}$

   $r=8.53x10^{-2}m$

2. Electrons accelerated from rest through a potential difference of 3000V enters a region of uniform magnetic field, the direction of the field being at right angles to the motion of the electrons. If the flux density is 0.01T, calculate the radius of the electron orbit.

   [Assume that the specific charge e/m for the electron =$1.8x10^{11}Ckg^{-1}$]

   Solution

   Va= 3000V
   $\frac{e}{m}$= $1.8x10^{11}$
   $\frac{1}{2}mu^2=eVa$
   U=$\sqrt{2Vax}\,\frac{e}{m}$
   U=$\sqrt{2x3000x1.8x10^{11}}$
   U=$3.29x10^7 m/s$
   But
   $\frac{mv^2}{r}=Bev$

   $r = \frac{mv}{Be}$
   $v = 3.29x10^7 m/s$
   $\frac{e}{m}$= $1.8x10^{11}$
   $\frac{m}{e}=\frac{1}{1.8x10^{11}}$
   $r = \frac{v}{B}x\frac{m}{e}$
   r=$\frac{3.29x10^7}{0.01}$x$\frac{1}{1.8x10^{11}}$
   r=$1.83x10^{-2}$m

3. A beam of protons is accelerated from rest through a potential difference of 2000V and then enters a uniform magnetic field which is perpendicular to the direction of the proton beam. If the flux density is 0.2T, calculate the radius of the path which the beam describes

[proton mass=1.7×10^{-27}kg electronic charge =-1.6×10^{-19}C]

Solution

Va=2000V

$\frac{1}{2}mu^2 = eVa$

$U = \sqrt{2xVax}\,\frac{e}{m}$

$U = \sqrt{\frac{2x2000x1.6x10^{-19}}{1.7x10^{-27}}}$

$U = 6.14 \times 10^5$m/s

Bev $= \frac{mv^2}{r}$

Bev $= \frac{mv^2}{r}$

r $= \frac{mv}{Be}$

$r = \frac{v}{B}x\frac{m}{e}$

u=v=6.14×10^5m/s

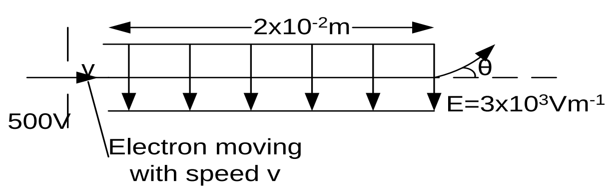
r=$\frac{6.14x10^5x1.7x10^{-27}}{0.2x1.6x10^{-19}}$

r=3.26×10^{-2}m

Exercise1

1. Calculate the speed of a proton which has been accelerated through a $p.d$ of 400V. [mass of proton =1.67×10^{-27}kg, charge on proton = 1.60×10^{-19}c] **Ans[2.77×10^5ms^{-1}]**

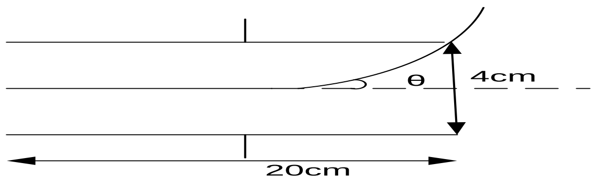
2.



In the figure above, a beam of electrons is accelerated through a $p.d$ of 500V and then enters a uniform electric field of strength E=3×10^3Vm^{-1}. Created by two parallel plates each of length 2×10^{-2}m. Calculate;

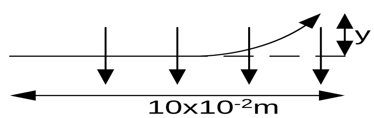
(a) The speed v of the electrons as they enter the field
(b) The time t that each electron spends in the field
(c) Angle Θ through which the electrons have been deflected by the time they emerge from the field [e/m of electron =1.76×10^{11}Ckg^{-1}**]** **An[1.33×10^7m/s, 1.51×10^{-9}s, 3.4^0]**

3. (a) In a cathode ray tube electrons are accelerated through a potential difference of 9000V and focused into a narrow beam. Calculate the velocity of electrons in the beam.
(b) The same beam along a line midway between the electrostatic deflecting plates 20cm long and 4.0cm apart.



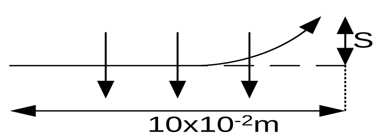
What is the value of the $p.d$ between the plates needed to deflect the beam through the greatest angle possible?[e=$1.6\times10^{-19}C$, m=9.1×10^{-31} kg] **An[$5.63\times10^{7}ms^{-1}$, 721V]**

4. (a) Electrons are accelerated to a $p.d$ of 3x10V and pass at right angles into a uniform magnetic field of strength 1.5×10^{-2}T. find the radius of their paths.
   (b) An identical beam is projected perpendicular into an electric field of $3\times10^{5}Vm^{-1}$. Calculate the deviation y of the electron path at a point 10×10^{-2}m perpendicularly into a field as measured from the point of entry of the beam.



[Specific charge of electron e/m =$1.76\times10^{11}Ckg^{-1}$] **An [$3.25\times10^{7}ms^{-1}$, 0.25m]**

5. (a) Electrons are accelerated by a $p.d$ of 2×10^{3}V and passes at right angles into a uniform magnetic field of flux density 1.0×10^{-2}T. find the radius of the path described.
   (b) An identical beam of electrons is projected perpendicular into an electric field of intensity $2.0\times10^{6}Vm^{-1.}$



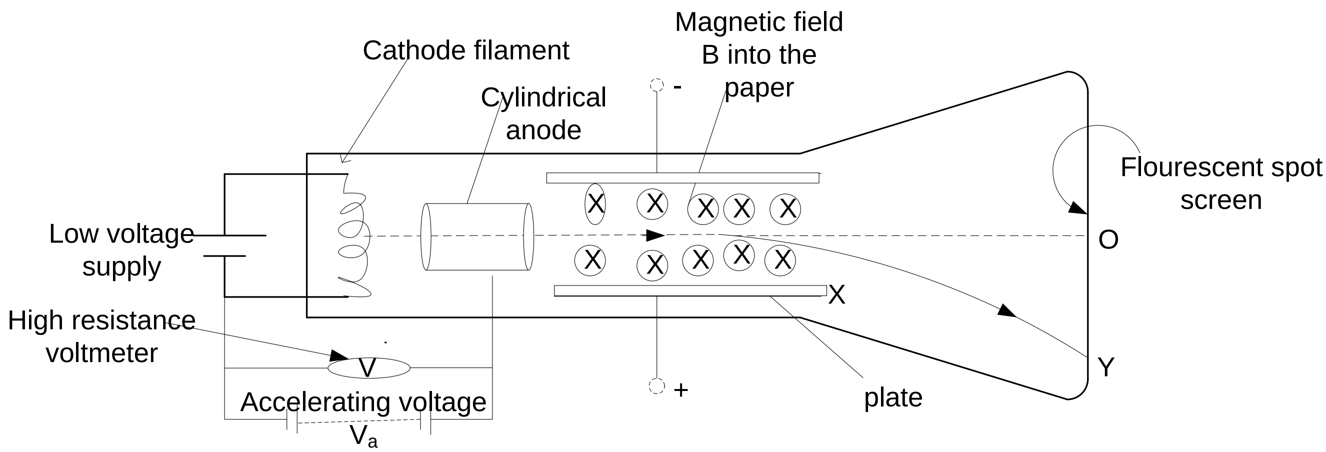
Calculate the deviation **S** of the electrons path at a point 10×10^{-2}m perpendicular to the field measured from the point of entry of the beam. **An[1.51×10^{-2}m, 2.5m]**

6. A narrow horizontal beam of electrons passes symmetrically between two vertical metal plates mounted one at each side of the beam. The velocity of the electrons is $3\times10^{7}ms^{-1}$, the plates are 0.03m long and 0.01m apart. It is found that when a battery of 568V is connected to the plates, the electron beam just strikes the end of one the plates. Calculate the value of the specific charge (e/m) of the electron. **An[$1.76\times10^{11}Ckg^{-1}$]**

7. In a cathode ray tube the electrons are accelerated through a $p.d$ of 500V and then pass between deflecting plates which are 0.05m long.
   (i) Find the time it takes an electron to pass between the plates
   (ii) If a $p.d$ across the plates is 10V $d.c$ and the plates and 1cm apart, calculate the angle through which electrons are deflected [e/m=$1.76\times10^{11}Ckg^{-1}$] **An[$3.8\times10^{-9}$s, 2.9^{0}]**

1.1.7: DETERMINATION OF SPECIFIC CHARGE OF AN ELECTRON [e/m] BY J.J THOMSON'S EXPERIMENT

The charge per unit mass or specific charge of an electron can be measured by the apparatus below.



- Electrons are emitted thermionic ally by the filament And are accelerated towards the cylindrical anode.
- With no electric and no magnetic fields applied at plate X, the electron beam strikes at the screen at point O which is noted
- A magnetic field of flux density, B is applied at X to deflect the electron beam to a point Y which is noted.
- An electric field of intensity, E is applied at X at right angles to the flux B at X and adjusted until the position of the beam on the screen is restored to point O.
- The p.d V, the plated separation, d and velocity, u of the movement of the electron beam are noted
- The electric force = magnetic force

  $Beu = eE$

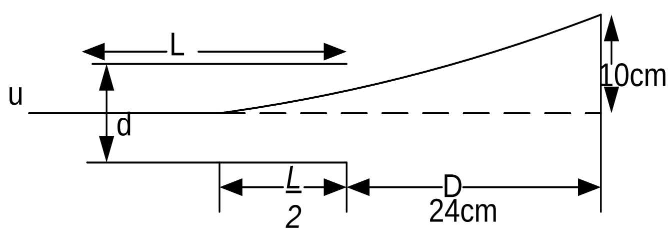
  u= $\frac{E}{B}$ Substituting for u into the equation $eVa = \frac{1}{2}$ mu^2 , where V_a is accelerating p.d

  $\frac{e}{m} = \frac{E^2}{2VaB^2}$, which gives the charge to mass ratio of an electron
- The value of E is found from E= $\frac{V}{d}$ whereV is $p.d$ between the deflecting plates d is their separation

Examples

1. Two plates are 2cm long and separated by a distance of 0.5cm in a uniform magnetic field of flux density $4.7x10^{-3}$T. An electron beam incident midway between the plates is deflected by magnetic field through a distance of 10cm on a screen placed 24cm from the end of the plate. When a $p.d$ of 1000V is applied to the plate , the electron is restored to the un deflected position. Calculate the specific charge of the electron

Solution



d=$0.5x10^{-2}$ m, L=$2x10^{-2}$m

B=$4.7x10^{-3}$T V=1000V

For **no** deflection

magnetic force =Electric force

$Beu = E.e$

But E=v/d

$Beu=\frac{Ve}{d}$

u=$\frac{V}{Bd}$

u=$\frac{1000}{4.7x10^{-3}x0.5x10x10^{-2}}$

u=$4.26x10^{7}ms^{-1}$

but tan Θ=$\frac{V_y}{u}$

tan Θ =$\frac{Vel}{mdu/u}$

tan Θ =$\frac{Vel}{mdu^2}$……………………[1]

also tan Θ=$\frac{10x10^{-2}}{\left(D+\frac{l}{2}\right)}$ ------------[2]

Equating 1 and 2

$\frac{10x10^{-2}}{\left(D+\frac{l}{2}\right)}=\frac{Vel}{mdu^2}$

$$\frac{e}{m} = \frac{10x10^{-2}du^2}{\left(D+\frac{l}{2}\right)lv}$$

$$\frac{e}{m} = \frac{10x10^{-2}x(4.26x10^{7})^2x0.5x10^{-2}}{(24x10^{-2}+1x10^{-2})x2x10^{-2}x1000}$$

$\frac{e}{m}$ =$1.8x10^{11}Ckg^{-1}$

2. An electron beam in which the electrons are $2x10^{7}ms^{-1}$ enters a magnetic field in a direction perpendicularly to the field direction. It is found that the beam can pass through without change of speed or direction. When an electric field of strength $2.2x10^{4}Vm^{-1}$ is applied in the same region at a suitable orientation.

(i) Calculate the strength of the magnetic field

(ii) If the electric field were switched off, what would be the radius of curvature of the electron path. [e=$1.6x10^{-19}$C]

Solution

v=$2x10^{7}ms^{-1}$ E=$2.2x10^{4}Vm^{-1}$

When the beam passes without change of speed or direction then the magnetic force is equal and opposite to the electric force

$Bev = Ee$

B=$\frac{E}{v}$

B =$\frac{2.2x10^4}{2x10^7}$

B=$1.1x10^{-3}$T

If the electric field were switched off, the magnetic force would provide the necessary centripetal force.

Bev $=\frac{mv^2}{r}$

r$=\frac{mv}{Be}$

r$=\frac{9.1x10^{-31}x2x10^{7}}{1.1x10^{-3}x1.6x10^{-19}}$

r=1.03×10^{-1}m

Exercise2

1. (a) A beam of singly ionized carbon atoms is directed into a region where a magnetic and an electric field are acting perpendicular both to each other and to the beam. The fields have intensities 0.1T and 10^4NC^{-1}respectively. If the beam is able to pass undeviated through this region. What is the velocity of the ions

   (b) The beam then enters a region where a magnetic field alone is acting. As a result the beam describes an arc of radius 0.75m. Calculate the flux density of this magnetic field. [Mass of carbon atom =2.0×10^{-26}kg, e=$1.6\times10^{-19}C$] **An[1×10^5ms^{-1}, 1.7×10^{-2}T]**

2. Radio waves from outer space are used to obtain information about interstellar magnetic fields. These waves are produced by electrons moving in circular orbits. The radio wave frequency is same as the electron orbital frequency. [The mass of an electron is 9.1×10^{-31}kg, and it's charge is $-1.6\times10^{-19}C$]. If waves of frequency 1.2MHZ are observed, calculate

(i) The orbital period of the electrons

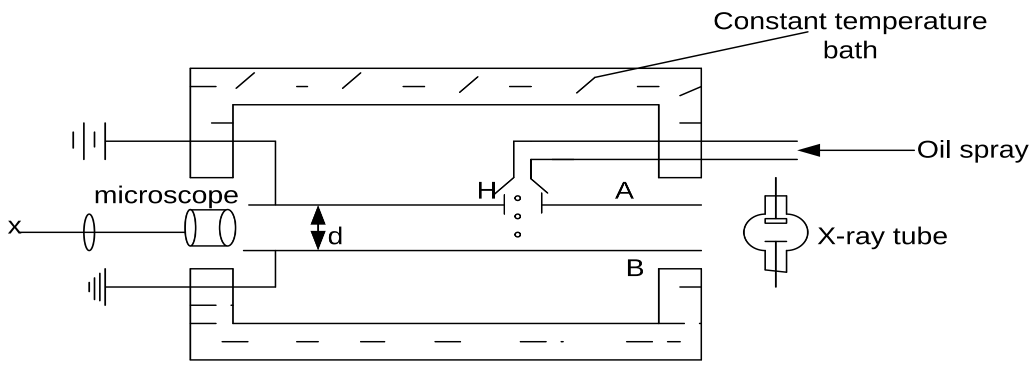
(ii) The flux density of the magnetic field **An[8.3×10^{-7}s, 4.3×10^{-5}T]**

3. A beam of cathode rays is directed mid way between two parallel metal plates of length 4cm and separation 1cm, the beam is deflected though 10cm on a fluorescent screen placed 20cm beyond the nearest edge of the plate when a $p.d$ of 200V is applied across the plate. If this deflected is annulled by a magnetic field of flux density 1.14×10^{-3}applied normal to electric field between the plates. Find the specific charge of the electrons. **An[1.75×10^{11}Ckg^{-1}**]

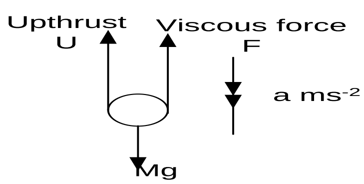
1.1.8: MILIKAN'S OIL DROP EXPERIMENT FOR MEASUREMENT OF e

When an object falls free in air it experiences a viscous drag which increases its velocity. As the object accelerates downwards, the upward force due to the viscous drag increases and eventually becomes equal to gravitational force. At this stage there is no further acceleration and the object is said to have reached its terminal velocity

Experimental procedure



- The apparatus is arranged as shown above, Millikan used two metal plates A and B with a small hole H in the center of the upper plate.
- A fine spray was used to atomize the oil and create tiny droplet, when one drop finds its way through the hole H, the drop is observed in a lower power travelling microscope by reflected light when the chamber is brightly illuminated.
- The drop acquires a charge by friction using x-rays and terminal velocity of selected drop is measured by timing its fall through a known distance by means of the scale on the eye piece of the microscope.



At terminal velocity $a=0m/s^2$

Mg=U+F

$\frac{4}{3}x\pi r^3\rho_o g=\frac{4}{3}\pi r^3\rho_a g+6\pi\eta r V_t$

$\frac{4}{3}\pi r^3 g(\rho_o-\rho_a)=6\pi\eta r V_t$ -----------------------[1]

Where V_t is terminal velocity

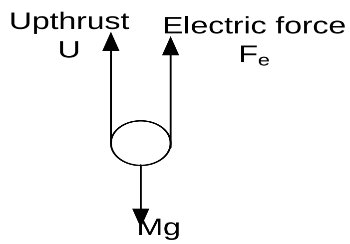
ρ_o is density of oil

ρ_a is density of air

from equation 1

$\mathbf{r}=\left[\frac{9\eta v_t}{2g(\rho_o-\rho_a)}\right]^{\frac{1}{2}}$

- A battery connected across plates A and B provides the field and its intensity $E=\frac{v}{d}$ is known where V is $p.d$ between the plates and d is the distance apart. The $p.d$ across the plates is now varied until the drop remain stationary.



$\boldsymbol{Mg = U + Fe}$

4/3 $\pi r^3 \rho_o g$=4/3 $\pi r^3 \rho_a g$+EQ

4/3 $\pi r^3 g(\rho_o - \rho_a)$= EQ ------------------ [2]

Equating equation 1 and equation 2

EQ=$6\pi\eta r V_t$

Q=$\frac{6\pi\eta r V_t}{E}$

But E=v/d

Q=$\frac{6\pi\eta r V_t d\ r}{v}$

r=$\left[\frac{9\eta V_t}{2g(\rho_0 - \rho_a)}\right]^{\frac{1}{2}}$

Therefore $\quad \mathbf{Q} = \frac{\mathbf{6\pi\eta} V_t \mathbf{d}}{v}\left[\frac{\mathbf{9}\eta V_t}{\mathbf{2}g(\rho_o - \rho_a)}\right]^{\frac{1}{2}}$

Note

- A constant temperature enclosure surrounded Millikan's apparatus in order to eliminate convection currents. It also served to shield the apparatus from draught hence drop under investigation does not drift side ways. This keeps viscosity and density of air and oil constant.
- Millikan used low- vapour pressure oil because it does not evaporate easily hence size of oil does not change appreciably.
- Stoke's law was assured in fall through a homogeneous medium

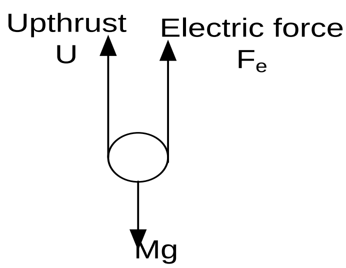
Results

- Millikan's measured the charges of several oil drops. The results showed that the charges were always integral multiple of 1.6×10^{-19}C and he concluded that electric charges can never exist in fractions of this amount and that the magnitude of the electronic charge e is 1.6×10^{-19} i.e Millikan established that charge is quantized.
- Therefore Q=ne where n is the number of electrons

Example

1. Oil droplets are introduced into the space between 2 flat horizontal plates set 5mm apart. The plate voltage is then adjusted exactly to 780V so that one of the droplets is held stationery. Then the plate voltage is switched off and the selected droplet is observed to fall a measured distance of 1.5mm in 11.2s. Given that the density of the oil used is 900kgm^{-3} and viscosity of air=1.8×10^{-5}Nm^{-2}. Calculate the charge on the droplet and determine the number of electronic charges

   Solution



At the terminal velocity a=0

$Mg = U + Fe$

4/3 $\pi r^3 \rho_o g$=4/3 $\pi r^3 \rho_a g$+EQ

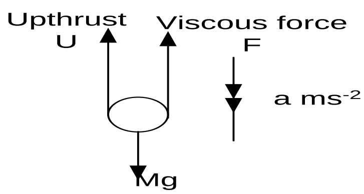
EQ=4/3 $\pi r^3 \rho_0 g$

Q=$\frac{\frac{4}{3}\pi r^3 \rho_0 g}{E}$

But E=v/d

Q=$\frac{4\pi r^3 \rho_0 gd}{3v}$ ------------------------------- [1]

When the plate voltage is switched off there is no electric force but only viscous drag acts



At terminal velocity a=0

$Mg = U + Fe$

4/3 $\pi r^3 \rho_0 g$=4/3 $\pi r^3 \rho_a g$+$6\pi\eta r V_t$

ρ_a=0 (negligible)

$6\pi\eta r V_t$=4/3 $\pi r^3 \rho_0 g$

r=$\left[\frac{9\eta v_t}{2g\rho_0}\right]^{\frac{1}{2}}$ ----------------------------------

[2]

but V_t=$\frac{1.5x10^{-3}}{11.2}$

V_t=1.34x10^{-4}s

But from equation 2

$$\therefore r = \left[\frac{9x1.8x10^{-5}x1.34x10^{-4}}{2x900x9.81}\right]^{\frac{1}{2}}$$

r =1.11x10^{-6}m

But from Equation 1 Q=$\frac{4\pi r^3 \rho_0 gd}{3v}$

Q=$\frac{4x\frac{22}{7}x(1.11x10^{-6})^3x900x9.81x5x10^{-3}}{3x700}$

Q=3.23x10^{-19}c

∴Electronic charge =3.23x10^{-19}c

Number of charges are obtained from

Q=ne

n=Q/e

n=$\frac{3.23x10^{-19}}{1.6x10^{-19}}$

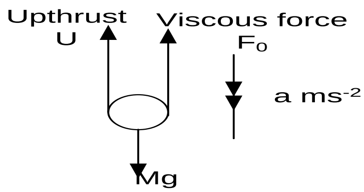
n=2.01875

n ≈ 2 charges

2. In measurement of the electron charge by Millikan's method, $p.d$ of 1.5kV can be applied between horizontal parallel metal plates 12mm apart.
   With the field switched off, a drop of oil of mass 1x10^{-14} kg is observed to fall with a constant velocity 400μms^{-1}. When the field is switched on the drop rises with constant velocity 80μms^{-1}. How many electron charges are there on the drop (Assume that air resistance is proportional to the velocity of the drop and that air buoyancy may be neglected) [electronic charge =1.6x10^{-19}C, g=10m/s^2]

Solution

when electric forceis swtiched off only the viscous drag acts.



At terminal velocity a =0m/s

$Mg=U+F_0$

$4/3\ \pi r^3\rho_0 g=4/3\ \pi r^3\rho_a g+F_0$

But $\rho_0=0$

$4/3\ \pi r^3\rho_0 g= F_0$

$Mg= F_0$

But from the assumption (viscous force ∝ velocity)

$F_0 \propto V_0$

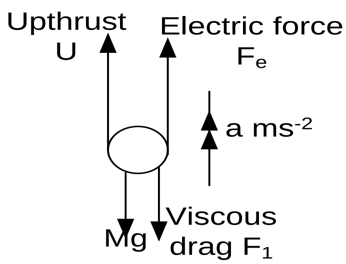
$F_0=kV_0$ where k is a constant of proportionality

$k=\frac{mg}{V_o}$

$k=\frac{1x10^{-14}x10}{400x10^{-6}}$

$k=2.5x10^{-10}$

When the field is switched on both the field and drag act but in opposite direction



At terminal velocity a=0

$U+F_e$=mg+F_1

U=0

$F_e=mg+F_1$

$F_1 \propto V_1$ and $F_e=EQ$

$F_1=kV_1$

$\therefore EQ=mg+kV_1$

Also $E=\frac{v}{d}$

$\frac{QV}{d}= mg+kV_1$

$Q=\frac{(mg+kV_1)}{v}d$

$Q=\frac{(1x10^{-14}x10+2.5x10^{-10}x80x10^{-6})x12x10^{-3}}{1.5x10^{3}}$

$Q=9.6x10^{-19}C$

Number of charges is obtained from

Q=ne

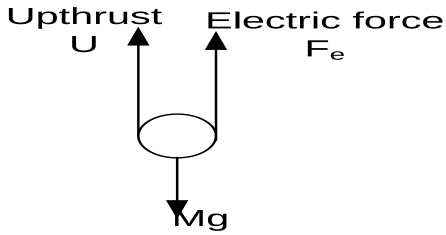
n=Q/e

$n=\frac{9.6x10^{-19}}{1.6x10^{-19}}$

n=6 charges

3. In Millikan's experiment an oil drop of mass $1.92x10^{-14}$kg is stationary in the space between the two horizontal plates which are $2x10^{-2}$m apart, the upper plate being earthed and the lower one at a potential of -6000V. Neglecting the buoyancy of the air. Calculate the magnitude of the charge.

Solution



At terminal velocity a=0

$Mg=U+F_e$ ------------------------[1]

But u=0 [neglecting air buoyancy]

EQ=mg

Q=mg/E

But E=v/d

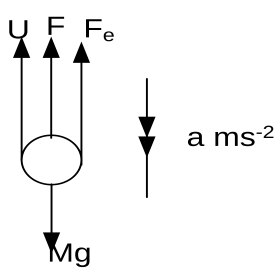
$Q=\frac{mgd}{v}$

$Q=\frac{1.92x10^{-14}x9.81x2x10^{-2}}{6000}$

$Q=6.28x10^{-19}C$

4. A small oil drop carrying negative electric charges is falling in air with a uniform speed of $8x10^{-5}ms^{-1}$ between the two horizontal parallel plates. The upper plate is maintained at a positive potential relative to the lower one.
   Draw a diagram showing all the forces acting on the drop, stating the cause of each force and use the following data to determine the charge on the oil drop and the number electronic charges.
   [Radius of drop=$1.60x10^{-6}$m, density of oil=$800kgm^{-3}$, density of air=$1.30kgm^{-3}$, viscosity of air =$1.80x10^{-5}Nm^{-2}$,Distance between the plates=$1x10^{-2}$m, $p.d$ between plates =$2x10^{3}$V, g=$10ms^{-2}$]

Solution



- U- upthrust due to air buoyancy (upwards)
- F_e-electric fields created between the plates due to the $p.d$
- F-viscous drag due to viscosity of air
- Mg-weight of the drop (downwards) due to gravitational pull

[r=$1.6x10^{-6}$, ρ_0=$800kgm^{-3}$, ρ_a=$1.3kgm^{-3}$, η=$1.8x10^{-5}Nm^{-2}$, d=$1x10^{-2}$m, v=$2x10^{3}$,V_t= $8x10^{5}ms^{-1}$]

At terminal velocity a=0

Mg=U+ F_e + F

$\frac{4}{3}\pi r^3\rho_0 g=\frac{4}{3}\pi r^3\rho_a g+EQ+6\pi\eta r v_t$

$EQ=\frac{4}{3}\pi r^3\rho_0 g\ (\rho_0-\rho_a)\ -6\pi\eta r v_t$

But $E=\frac{v}{d}$

$Q=\frac{4\,\pi r^3\rho_0 g\,(\rho 0-\rho a)-\,18\pi\eta r v_t}{3v}$

$Q=\frac{\left[4x\frac{22}{7}x\left(1.6x10^{-6}\right)^3x10(800-1.3)-18x\frac{22}{7}x1.8x10^{-5}x1.6x10^{-6}x8x10^{-5}\right]x10^{-2}}{3x2x10^{3}}$

$Q=\frac{10^{-2}x\left[4x\frac{22}{7}x\left(1.6x10^{-6}\right)^3x10(800-1.3)-18x\frac{22}{7}x1.8x10^{-5}x1.6x10^{-6}x8x10^{-5}\right]}{3x2x10^{3}}$

$Q=4.68x10^{-19}C$

But also Q=ne

n=$\frac{Q}{e}$

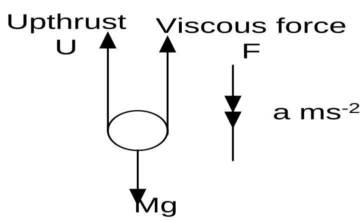
n=$\frac{4.68x10^{-19}}{1.6x10^{-19}}$

n=2.925

n $\approx$ 3 electrons

5. Calculate the radius of drop of oil of density 900kgm^{-3} which falls with a terminal velocity of 2.9x10^{-2}ms^{-1} through air of viscosity 1.8x10^{-5}Nsm^{-2}. Ignore the density of air if the charge on the drop is -3e. what p.d must be applied between two plates 5cm apart for the drop to be held stationary between them [e=1.6x10^{-19}C]

Solution



u=0 and at terminal velocity a=0

F=mg

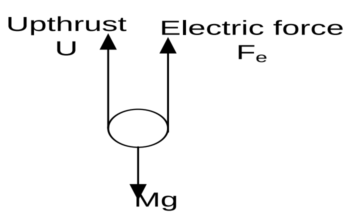
$6\pi\eta r v_t = \frac{4}{3}\pi r^3 \rho_0 g$

$r^2 = \frac{9\eta v_t}{2g\rho_0}$

r=$\left[\frac{9x1.8x10^{-5}x2.9x10^{-2}}{2x900x9.81}\right]^{\frac{1}{2}}$

r=1.63x10^{-5}m

For the drop to be held stationary then there is no viscous drag



u=0 and a=0ms^{-2}

mg = fe

mg= EQ

E = $^{V}/_{d}$ $Q = 3e$

V=$\frac{mgd}{Q}$

V=$\frac{\frac{4}{3}\pi r3\rho 0g}{3Q}$

V=$\frac{4x\frac{22}{7}x(1.63x10^{-5})^3x900x9.81x5x10^{-2}}{3x3x1.6x10^{-19}}$

V=1.67x10^{7}V

Exercise 3

1. A spherical oil drop of radius of 2x10^{-6}m is held stationery between two parallel metal plates to which a $p.d$ of 4500V is applied , the separation of the plates is 1.5cm, calculate the charge on the drop if the density of oil is 800kgm^{-3}. Assume no air resistance. **An[9.64x10^{-19} C]**

2. (a) A charged oil drop falls at a constant speed in the Millikan oil drop experiment when there is no $p.d$ between the plates explain this,

(b) Such an oil, of mass $4x10^{-15}$kg is held stationary when an electric field is applied between the two horizontal plates. If the drop carried six electric charges each of value $1.6x10^{-19}$C. Calculate the value of the electric field strength. **An[$4.2x10^{4}Vm^{-1}$]**

3. An oil drop carrying a charge of 3e falls under gravity with a constant velocity of $4.6x10^{-4}$ m/s between two metal plates 5mm apart. When a $p.d$ of 4600V is applied to the plates, the drop rises steadily. **Calculate;**

(i) Radius of oil drop

(ii) Velocity with which the oil drop rises [density of oil $900kgm^{-3}$,viscosity of air = $1.8x10^{-5}Nsm^{-1}$] assume the effect of air buoyancy is negligible **An[$2.05x10^{-6}$m, $6.35x10^{-4}ms^{-1}$]**

1.1.9: IONIC CHARGE AND IT'S RELATION TO FARADAY'S CONSTANT [F] AND ELECTRONIC CHARGE[e]

Faraday's constant [F]

- This is the charge required to liberate one mole of a monovalent element [ion] numerically equals to 96500C

Avogadro's constant [N_A]

- This is the number of ions [particles] in one mole of a monovalent ion.
  The charge carried by one mole or charge required to liberate one mole of monovalent ion is 96500C.

But in one mole we have $6.023x10^{23}$ions.

1 ion $=\frac{96500}{6.023x10^{23}}$ $=1.6x10^{-19}C$

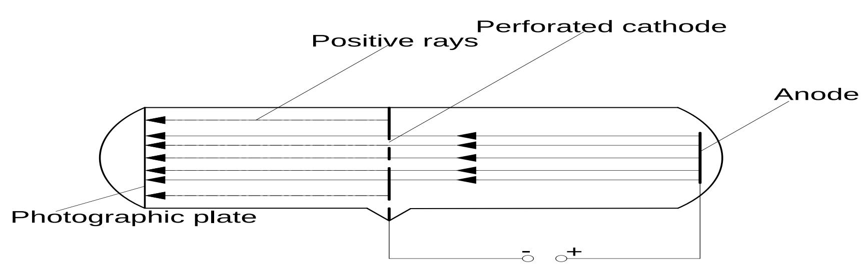
1 ionic charge(e) $=\frac{faraday'sconstant}{Avogadro'sconstant}$

$e=\frac{F}{N_A}$

$F=N_Ae$

1.1.10:POSITIVE RAYS

These are streams of positively charge particles that pass through a perforated cathode

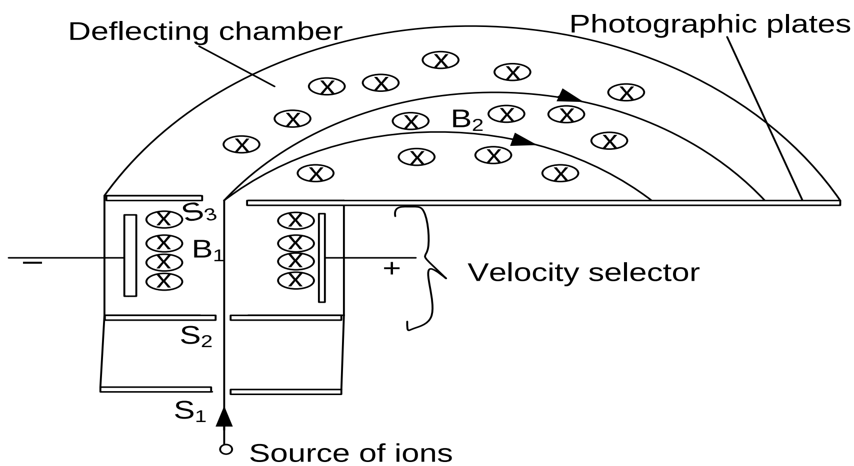


- Positive rays are produced when cathode rays in a discharge tube collide with gaseous atoms and strip off (knock out) some electrons from the atoms.
- The positive ions formed are accelerated to the cathode and these streams of positive ions constitute rays.

1.1.11:Properties of positive rays

- They are positively charged
- They are deflected in electric and magnetic field in a much small extend than cathode rays because they are more massive than cathode rays.
- They cause a fluorescence and affect a photographic plate
- They show a spectrum of different velocities
- They are dependent on the gas in the tube

1.1.12 DETERMINATION OF THE SPECIFIC CHARGE OF POSITIVE RAYS USING MASS SPECTROMETER



- Positive of ions from a source is directed though slits S_1 and S_2 into the velocity selector where there are crossed electric field of intensity, E and magnetic field of flux density, B_1
- Ions of charge, Q leave the velocity selector un deflected with velocity, u given by $B_1Qu = EQ$, that is $u = \frac{E}{B_1}$
- The selected ions pass through S_3 and enter a deflection chamber with a uniform magnetic field of flux density, B_2
- The ions move along a semi circular path and strike the photographic plate where they are detected. The radius, r of the path described is measured and recorded.
- In a circular path, $B_2Qu = \frac{mu^2}{r}$, that is $\frac{Q}{m} = \frac{E}{B_2r}$

- On substituting for u, the charge to mass ratio is got from $\boxed{\frac{Q}{m} = \frac{E}{B_1 B_2 r}}$

DIFFERENCE BETWEEN CATHODE RAYS AND POSITIVE RAYS

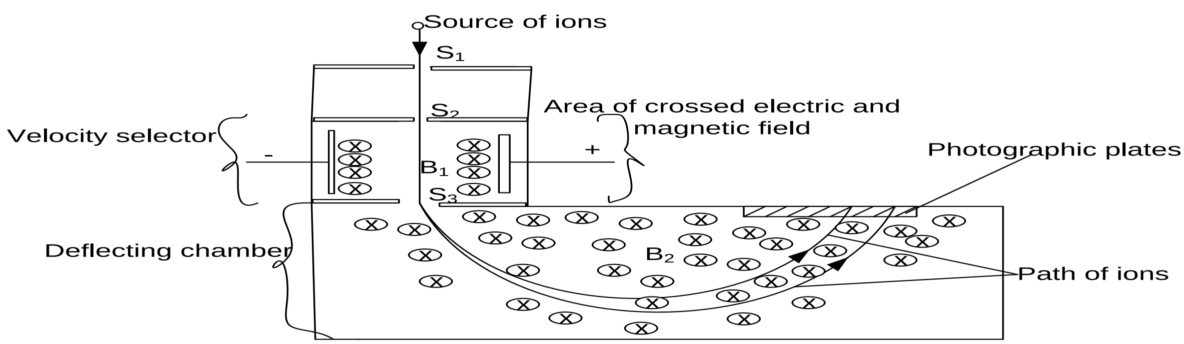
Cathode rays	Positive rays
They are light (less massive)	They are massive
They are negatively charged	They are positively charged
They travel with same velocity	Have a range of velocities
They produce x-rays when they bombard matter	They do not produce x-rays when they bombard matter

SPECIFIC CHARGE OF AN ION

This is the ratio of charge to mass of an ion

S.I unit is $C\ kg^{-1}$

1.1.12: DETERMINATION OF THE SPECIFIC CHARGE OF IONS USING A BAIN BRIDGE MASS SPECTROMETER



- Streams of ions from a source is directed though slits S_1 and S_2 into the velocity selector where there are crossed electric field of intensity, E and magnetic field of flux density, B_1
- Ions of charge, Q pass through the selector un deflected with velocity, u given by $B_1 Qu = EQ$, that is $u = \frac{E}{B_1}$
- The selected ions pass through S_3 and enter a deflection chamber with a uniform magnetic field of flux density, B_2
- The ions move along a semi circular path and strike the photographic plate where they are detected. The radius, r of the path described is measured and recorded.
- In a circular path, $B_2Qu = \frac{mu^2}{r}$, that is $\frac{Q}{m} = \frac{E}{B_2 r}$
- On substituting for u, the charge to mass ratio is got from $\boxed{\frac{Q}{m} = \frac{E}{B_1 B_2 r}}$

Note

r=$\frac{mE}{QB_1B_2}$

r∝ $^{e}/_{m}$

Since E, B_1 and B_2 are constant , r depends only on the charge to mass ratio. It follows that the position at which an ion strikes the photographic plate depends on its charge to mass ratio [ions with large Q/m fall on the near end of the photographic plate].

Bain bridge mass spectrometer is used to separate different isotopes of a single element.

Example

1. A beam of protons is accelerated through a $p.d$ of 10kV and is allowed to enter a uniform magnetic field B of 0.5T perpendicular to their path. Find the radius of the circle they travel. [mass of proton =$1.67x10^{-27}$kg, e=$1.6x10^{-19}C$]

Solution

B=0.5T

m=$1.67x10^{-27}$kg

Va= $10x10^3$V

$\frac{1}{2}$ mu^2=eVa

u=$\sqrt{\frac{2eVa}{m}}$

u=$\sqrt{\frac{2x1.6x10^{-19}x10x10^3}{1.67x10^{-27}}}$

u=$1.38x10^6ms^{-1}$

In the magnetic field

$\frac{mu^2}{r}$ =Beu

$r = \frac{mu}{Be}$

r=$\frac{1.6x10^{-27}x138x10^6}{0.5x1.6x10^{-19}}$

r=$2.9x10^{-2}$m

2. In a Bain bridge mass spectrometer singly ionized atoms of ^{35}Cl, ^{37}Cl pass into the deflection chamber with a velocity of 10^5ms^{-1}. If the flux density of the magnetic field in the deflecting chamber is 0.08T, calculate the difference in the radii of the path of the ion.

Solution

Let r_1 be radius for ^{35}Cl

r_2 be radius for ^{37}Cl

1u=$1.66x10^{-27}$kg

35u=($1.66x10^{-27}$x35)kg

37u=($1.66x10^{-27}$x37)kg

$\frac{mu^2}{r_1}$ =Beu

r_1=$\frac{m_1u}{Be}$

r_1=$\frac{35x1.66x10^{-27}x10^5}{0.08x1.6x10^{-19}}$

r_1=0.454m

Also

r_2=$\frac{m_2u}{Be}$

r_2=$\frac{35x1.66x10^{-27}x10^5}{0.08x1.6x10^{-19}}$

r_2=0.480m

Difference r_2-r_1=0.48-0.454

=0.026m

3. The mass of the singly charged neon isotope. ${}^{20}_{10}Ne^{+}$ is 3.3x10^{-26}kg. A beam of these ions enters a uniform transverse magnetic field of 0.3T and describes a circular orbit of radius 0.22m. What is?

a) The velocity of the ions
b) The potential difference which has been used to accelerate them to this velocity[e=1.6x10^{19}C]

Solution

Magnetic field provides the centripetal force

$\text{Beu} = \frac{mu^2}{r_1}$

$u = \frac{Ber}{m}$

$u = \frac{0.3x1.6x10^{-19}x0.22}{3.36x10^{-26}}$

u=3.2x10^{5}m/s

The accelerating $p.d$ provides the kinetic energy

$\frac{1}{2}$ mu^{2}=eVa

$Va = \frac{mu^2}{2e}$

$Va = \frac{3.3x10^{-26}x(3.2x10^{5})^2}{2x1.6x10^{-19}}$

Va=10560V

4. In a mass spectrograph consisting of doubly charged ions, it is required that the radius of the path of the ion with a mass number 72 be exactly 1m. If the electric field intensity across the velocity selector is 80Vm^{-1}. What will be the magnetic field intensity across the deflection chamber [1U=1.67x10^{-27}kg]

Solution

Since its doubly charge Q=2e

r=$\frac{mE}{QB_1B_2r}$

but $B_1=B_2=B$

r=$\frac{mE}{QB^2}$

r=$\frac{72x1.67x10^{-27}x80}{2x1.6x10^{-19}B^2}$

B=5.5x10^{-3}T

Exercise4

1. Singly charged ions having masses close 14U and 15U are accelerated by a $p.d$ of 800V and then passed perpendicular to the lines of force of a uniform magnetic field of flux density

0.2Wbm^{-2}. Calculate the radii of curvature for the path followed by the ions in the magnetic field. **An[7.64cm, 7.91cm]**

2. In a mass spectrometer, the magnetic flux density in both magnetic field is 0.4T and the electric field in the velocity selector is $2x10^4Vm^{-1.}$

(i) What is the velocity of an ion which goes un deviated through the slit system.

(ii) The source is set to produce singly-charged ions of magnesium isotopes. Mg-24 and Mg-26. Find the distance between the images formed by them on the photographic plate. [$1U=1.67x10^{-27}kg$ $e=1.6x10^{-19}C$] **An[$5x10^4m/s$ $5.22x10^{-3}m$]**

3. A velocity selector employs a magnet that produces a flux density of 0.004T and parallel plate capacitor with a plate separation of 1cm for the electric field. What $p.d$ must be applied to the capacitor in order to select charged particles having a speed of $4.0x10^6ms^{-1}$ **An[160V]**

4. The following measurement were made in a mass spectrograph for a beam of doubly ionized Neon atoms B=0.005T, r=0.053m, $V=2.5x10^4ms^{-1}$. Calculate the mass of Neon atom. **An[$3.4x10^{-26}kg$]**

UNEB 2013 Q.8

(a) Explain briefly how **positive rays** are produced (03marks)

(b) An electron of charge, e and mass, m, is emitted from a hot cathode and then accelerated by an electric field towards the anode. If the potential difference between the cathode and anode is V, show that the speed of the electron. U. is given by

$$u = \sqrt{\left(\frac{2eV}{m}\right)}$$

(03marks)

(c) An electron starts from rest and moves in an electric field intensity of $2.4x10^3Vm^{-1}$. Find the;

(i) Force on the electron. **An** $(3.84x10^{-16}N)$ (02marks)

(ii) Acceleration of the electron **An** $(4.22x10^{14}ms^{-2})$ (02marks)

(iii) Velocity acquired in moving through a $p.d$ of 90V **An** (5.6210^6ms^{-1}) (02marks)

(d) A beam of electron each of mass, m, and charge, e, is directed horizontal metal plates separated by a distance, d.

(i) If the $p.d$ between the plates is V, show that the deflection y of the beam is given by

$$y = \frac{1}{2m}\left(\frac{eV}{du^2}\right)x^2$$

Where, x, is the horizontal distance travelled

(06marks)

(ii) Explain the path of the electron beam as it emerges out of the electric field (02marks)

UNEB 2012 Q.8

(a) (i) What are cathode rays

(01mark)

(ii) With the aid of a diagram, describe an experiment to show that cathode rays travel in a straight lines

(04mraks)

(b) A beam of electrons is accelerated through a potential difference of 500V. The beam enters midway between two similar parallel plates of length 10cm and is 3cm apart. If the potential difference across the plates is 600V, find the velocity of an electron as it leaves the region between the plates. **An [2.96x10^{7}m/s $\theta = 63.4^o$]**

(08marks)

UNEB 2011 Q.8

(c) Explain why

(i) the apparatus in Millikan's experiment is surrounded with a constant temperature enclosure

(03marks)

(ii) low vapour pressure oil is used (02marks)

(d) In Millikan's experiment , the radius r of the drop is calculated from

$$r=\sqrt{\frac{9\eta v}{2pg}}$$

Where η is the viscosity of air and ρ is the density of oil. Identify the symbol v and describes briefly how it is measured. (02marks)

UNEB 2010 Q.8

(a) (i) With the aid of a labeled diagram, describe what is observed when a high tension voltage is applied across a gas tube in which pressure is gradually reduce to very low values

(05marks)

(ii) Give two applications of a discharge tubes

(01mark)

(b) Describe Thomson's experiment to determine the specific charge of an electron (06marks)

(c) In Millikan's oil drop experiment, a charged oil drop of radius 9.2x10^{-7}m and density 800 kgm^{-3} is held stationary in an electric field of intensity 4x10^{4}Vm^{-1}.

(i) How many electron charges are on the drop [04marks]

(ii) Find the electric field intensity that can be applied to move the drop with velocity $0.005ms^{-1}$ upwards (density of air $=1.29kgm^{-3}$, $\eta=1.8x10^{-5}Nsm^{-1}$) [04marks]

An[4, $2.48x10^6$ Vm^{-1}]

UNEB 2009 Q.8

(a) State four differences between cathode rays and positive rays [02marks]

(b) An electron having energy of $4.5x10^2eV$ moves at right angles to a uniform magnetic field of flux density $1.5x10^{-3}T$. Find the;

(i) Radius of the path followed by the electrons [04marks]

(ii) Period of motion [03marks]

(c) (i) Define the terms Avogadro's constant and Faraday's constant [02marks]

(ii) Use the Avogadro's constant and faraday constant to calculate the charge on an anion of a mono atomic element **An[$1.6x10^{-19}C$]** [03marks]

UNEB 2007 Q.9

(a) What are isotopes [01marks]

(b) With the aid of a diagram, describe the operation of brain bridge spectrometer in determining the specific charge of ions. [06marks]

UNEB 2006 Q.9

(a) (i) A beam of electrons, having a common velocity, enters a uniform magnetic field in a direction normal to the field. Describe and explain the subsequent path of the electrons

(ii) Explain whether a similar path would be followed if a uniform electric field were substitutes for the magnetic field (05marks)

(b) Describe an experiment to measure the ratio of the charge to mass of an electron [7mk]

(c) Electrodes are mounted at opposite ends of a low pressure discharge tube and a potential difference of 1.2kV applied between them. Assuming that the electrons are accelerated from rest, calculate the maximum velocity which they could acquire.

[specific electron charge $=-1.76x10^{11}Ckg^{-1}$] **An[$2.06x10^7ms^{-1}$]**

(05marks)

(d) (i) Give an account of the stages observed when an electric discharge passes through a gas at pressure varying from atmospheric to about 0.01mmHg as air is pumped out when the $p.d$ across the tube is maintained at extra high tension. [05marks]

(ii) State two disadvantages of discharge tubes when used to study cathode rays [01mk]

UNEB 2005 Q.8

(c) In the measurement of electron charges by Millikan's apparatus, a potential difference of 1.6kV is applied between two horizontal plates 14mm apart. With the $p.d$ switched off, an oil drop is observed to fall with constant velocity of $4x10^{-4}ms^{-1}$. When the potential difference is switched on, the drop rises with a constant velocity of $8x10^{-5}ms^{-1}$. If the mass of the oil drop is $1.0x10^{-14}kg$, find the number of electron charges on the drop. [Assume air resistance is proportional to the velocity of the oil drop and neglect the up thrust due to the air] [07marks] **An[4]**

UNEB 2004 Q.8

(b) A beam of electrons is accelerated through a potential difference of 2000V and is directed mid way between two horizontal plates of length 5.0cm and a separation of 2.0cm. The $p.d$ across the plates is 80V.

(i) Calculate the speed of the electrons as they enter the region between the plates [03marks]

(ii) Explain the motion of the electrons between the plates

(iii)Find the speed of the electrons as they emerge from the region between the plates.

An[$2.65x10^7ms^{-1}$, $2.653x10^7ms^{-1}$]

UNEB 2003 Q.8

(b) Explain how Millikan's experiment for measuring the charge of the electron proves that the charge is quantized.

(c) A beam of positive ions is accelerated through a $p.d$ of 1000V into a region of uniform magnetic field of flux density 0.2T. While in the magnetic field it moves in a circle of radius 2.3cm. Derive an expression for the charge to mass ratio of the ions and calculate its value.

An[$9.45x10^7Ckg^{-1}$]

UNEB 2002 Q.9

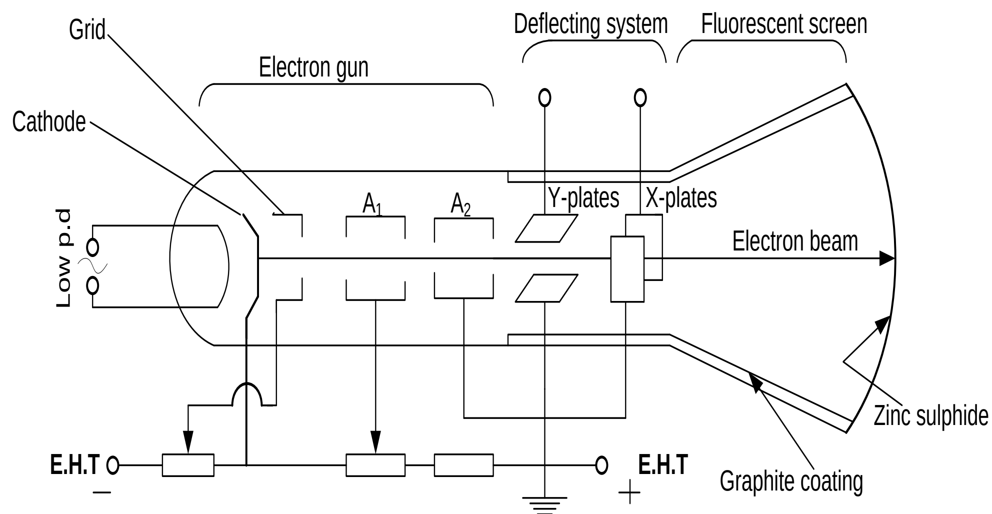
(a) (i) What are cathode rays? [01mark]

(ii) An electron gun operating at $3x10^3V$ is used to project electrons into the space between two oppositely charged parallel plates of length 10cm and separation 5cm, calculate the deflection of the electrons as they emerge from the region between the charged plates when the $p.d$ is 1000V. **An[$1.66x10^2m$]**

[04marks]

CHAPTER 2: ELECTRONIC DEVICES

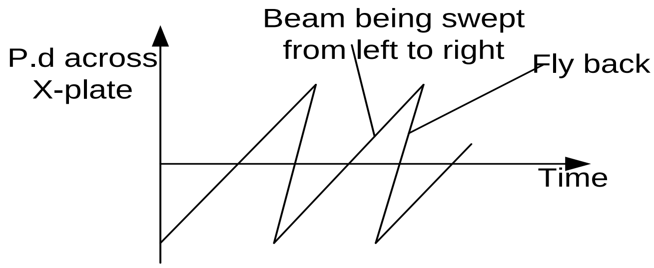
2.1.0: THE CATHODE RAY OSCILLOSCOPE (CRO)



- Cathode is heated and emits electrons thermionic ally. The electrons are focused and accelerated by the anodes to the screen. Grid controls number of electrons reaching the screen hence brightness of the spot
- Y-plates deflect electron beam vertically and X-plates deflect electron beam horizontally.
- The screen glows to form a spot when struck by electrons. Graphite coating shields electrons from external fields and conducts stray electrons to the earth.

Time base

- The time base is connected to the x-plates and provides a saw tooth $p.d$ that sweeps the electron beam from left to right of the screen at constant speed.



- The saw tooth then returns the beam to the initial position at the extreme left of screen almost instantaneously. The time taken for this right to left sweep is called **fly-back time.**

2.1.1:USES OF THE CRO

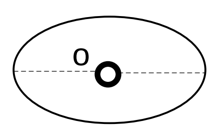
- It is used to display wave forms, the signal to be investigate is connected to the y-plate and the time base to the x-plate
- It measures voltage (AC or DC)
- Measures frequencies
- Used to measure phase differences
- Measures small time intervals

2.1.2:Advantages of CRO over a voltmeter

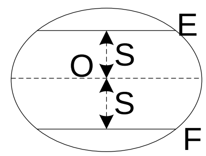
- It measures both AC and D.C voltage unlike a voltmeter measures only D.C voltage unless a rectifier is used
- It has an instantaneous response since the electron beam behaves as a pointer of negligible inertia.
- It draws very little current since it has nearly infinite resistance to DC and a very high impedance to AC
- It has no coil to burn out.

2.1.3: APPEARANCE OF ELECTRON SPOT ON THE SCREEN

- When a signal is **not** connected to the y-plate and time base switched **off**, a bright spot is formed on the screen.



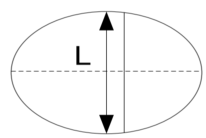
- When $the\ \boldsymbol{d.c}$ **voltage** is connected to the y-plate such that the top plate is positive the line is displaced to E. If the lower plate is positive the line is displaced to F. the displacement in either case is proportional to $the\ d.c$ voltage applied.



If in the CRO the gain control of the y-deflection amplifier is V_g Vcm^{-1} then
$V \propto S$

$$\boxed{\boldsymbol{V = VgS}}$$

- When A.C is connected to y-plate and time base switched off. The spot is a vertical line

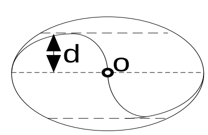


The length L represents peak to peak voltage
$2V_0 \propto L$
$2V_0 = VgL$ where V_0 is peak voltage

$$\boxed{V0\ = \frac{VgL}{2}}$$

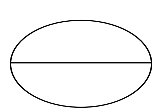
Also $\boxed{Vr.m.s\ = \frac{V_0}{\sqrt{2}}}$

- When the A.C is connected to Y-plate and time base also switched on the a stationery wave is obtained



$V_0 \propto d$
$V_0 = Vgd$
$Vr.m.s = \frac{V_0}{\sqrt{2}}$

- Y-plate off and time base on



Horizontal line formed at the centre of the screen

Examples

1. If the voltage gain is $20Vcm^{-1}$ and an A.C voltage connected to Y-plate products a vertical trace of 12cm long with time base off. Find the peak value of the voltage and its r.m.s value

Solution

$2V_0 = VgL$

$V_0 = \frac{20x12}{2}$

$V_0 = 120v$

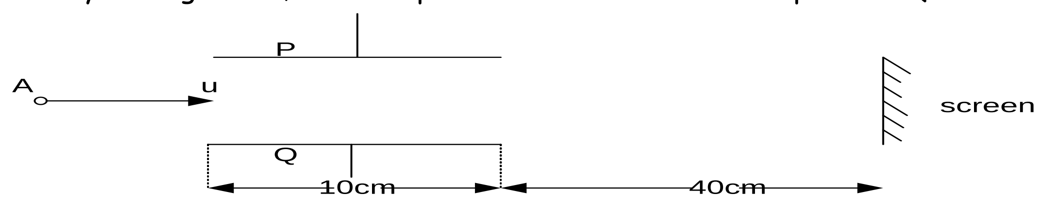
Peak value =120V

$V_{r.m.s} = \frac{v_0}{\sqrt{2}}$

$V_{r.m.s} = \frac{120}{\sqrt{2}}$

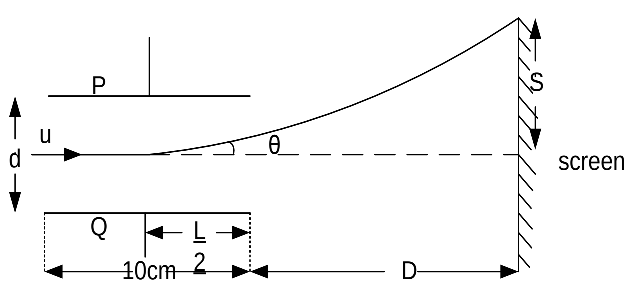
$V_{r.m.s} = 84.85V$

2. The sketch below shows part of the deflecting system of a cathode ray oscilloscope. At the point A, a beam of electrons has a velocity of $3x10^7ms^{-1}$ along the axis of the system. The plates which are 4cm apart provides a uniform electric field in the space between them. Edge effects may be neglected, P is at a potential of +200V with respect to Q



Find the position at which the electron beam strikes the screen ($e/m=1.76x10^{11}Ckg^{-1}$)

Solution



$L=10x10^{-2}m$, $d=4x10^{-2}m$, $D=40x10^{-2}m$,
$V=200$, $e/m = 1.76x10^{11}Ckg^{-1}$

$\text{Tan } \theta = \frac{S}{D+\frac{l}{2}}$ ------------------------- [1]

But also $\text{Tan } \theta = \frac{v_y}{u}$ ------------------- [2]

Equating 1 and 2

$\frac{S}{D+\frac{l}{2}} = \frac{v_y}{u}$

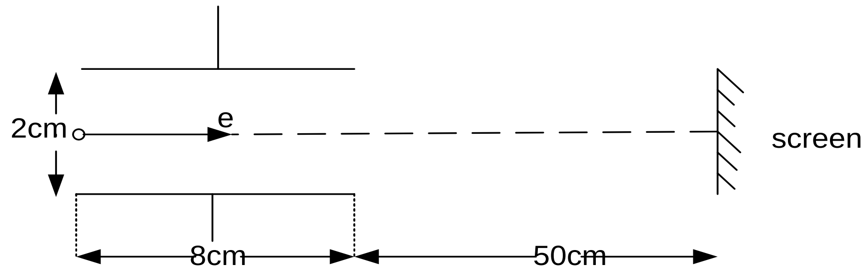
$S = \frac{v_y}{u}(D+\frac{l}{2})$

But $V_y = \frac{vel}{mdu^2}(D+\frac{l}{2})$

$S = \frac{200x1.76x10^{11}x10x10^{-2}x}{4x10^{-2}x(3x10^7)^2} x [4x10^{-2} + \frac{10x10^{-2}}{2}]$

$S=4.4x10^{-2}m$

3. The figure below shows two metal plates 8cm long and 2cm apart. A fluorescence screen is placed 50cm from the one end of the plates. An electron of kinetic energy $6.4x10^{-16}J$ is incident midway between the plates



Calculate the $p.d$ which must be applied across the plates to deflect the electron 4.2cm on the screen. Assume that the space through which to electron moves is evacuated.
[$e=1.6x10^{-19}C$, $m=9.1x10^{-31}kg$]

Solution

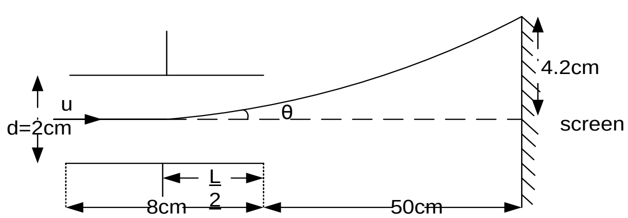
$Ke= \frac{1}{2} mu^2$

$6.4x10^{-16}=\frac{1}{2} mu^2$

$U=\sqrt{\frac{2x6.4x10^{-16}}{9.1x10^{-31}}}$

$U=3.75x10^{7}ms^{-1}$

$d=2x10^{-2}$, $D=50x10^{-2}$, $L=8x10^{-2}$,
$\frac{L}{2}= 4x10^{-2}m$, $S=4.2x10^{-2}m$



$Tan\ \theta =\frac{S}{D+\frac{l}{2}}$

$Tan\ \theta = \frac{4.2x10^{-2}}{(50x10^{-2}+4x10^{-2})}$ ----------- [1]

Also

$Tan\ \theta =\frac{v_y}{u}$

$Tan\ \theta =\frac{vel}{mdu^2}$ ----------------------------
[2]

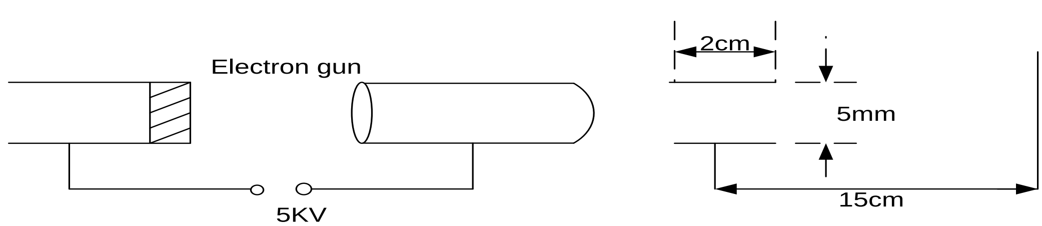
$\frac{vel}{mdu^2}=\frac{4.2x10^{-2}}{(50x10^{-2}+4x10^{-2})}$

$V=\frac{4.2x10^{-2}x9.1x10^{-31}x2x10^{-2}x(3.75x10^{7})^2}{(1.6x10^{-19}x8x10^{-2})}$

V=83.98V

Exercise 4

1.



Calculate the deflection sensitivity (deflection of spot in mm per volt potential difference) of the cathode ray tube from the following data.

Electrons are accelerated by a potential difference of 5kV between the cathode and anode. [length of deflection plates =2cm, separation of deflector plates =5mm, distance of mid point of deflector plates from screen =15cm] **An [6×10^{-2}mmV^{-1}]**

2. In one type of CRO, the electrostatics deflecting system consists of two parallel metal plates of length 2cm and 0.5cm apart the centre of the plates is situated 15cm from the screen and $p.d$ of 80V is applied between the plates to provide a uniform electric region between the plates at right angles to the field. Calculate.
(i) Speed with which electrons leave the plates
(ii) Deflection of electron beam on the screen. [$e=1.6\times10^{-19}C$, $m=9.1\times10^{-31}kg$]

An[$3.11\times10^{7}ms^{-1}$, $8.76\times10^{-3}m$]

UNEB 2011 Q.8

(a) (i) Describe with the aid of a well labeled diagram, the structure and mode of operation of CRO
[06marks]
(ii) State the advantages of CRO over a moving coil voltmeter [02marks]

UNEB 2004 Q.8

(a) (i) Describe with the aid of a labeled diagram the main features of a cathode ray oscilloscope (CRO)
[08marks]
(ii) State two uses of a CRO [01marks]
(iii) The gain control of a CRO is set on 0.5Vcm^{-1} and an alternating voltage produces a vertical trace of 2cm along with the time base off. Find the root mean square value of the applied voltage. **An[0.354V]** [02marks]

UNEB 2005 Q.9

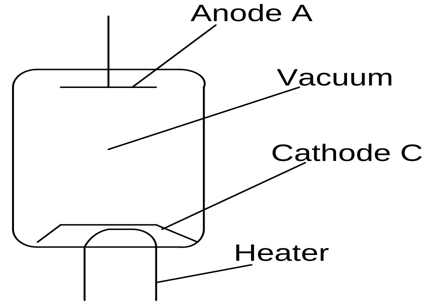
(b) Describe, with the aid of a diagram, the structure and mode of operation of a cathode ray oscilloscope (CRO) [06marks]
(c) A CRO has its y-sensitivity set to 20Vcm^{-1}, a sinusoidal input voltage is suitably applied to give a steady time base switched on so that the electron beam takes 0.01s to traverse the screen. If the trace seen has a peak –to-peak height of 4cm and contains two complete cycles. Find the
(i) r.m.s value of the input voltage [03marks]

(ii) frequency of the input signal **An[14.14V, 200Hz]**
[02marks]

2.2.0: THERMIONIC DIODE

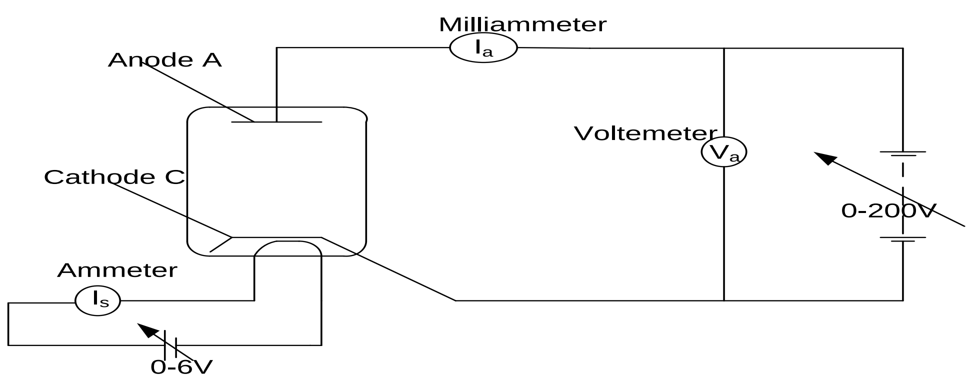
A thermionic diode is a device which is used to change alternating voltage to direct voltage. This process is called **rectification**

Circuit symbol



A diode consists of cathode (c) and a metal Anode (A). these two elements constitute the electrodes of the valve which are placed inside an evacuated glass envelope.

2.2.1:DIODE CHARACTERISTICS CIRCUIT



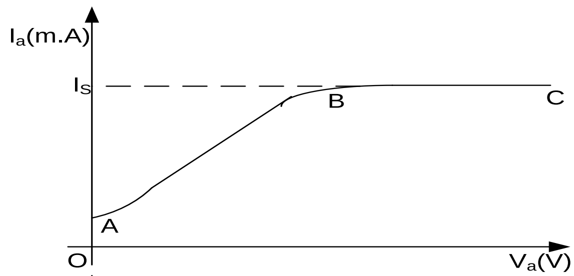
- When the cathode filament is heated with a low $p.d$ electrons are emitted thermionically.
- If the anode A is kept at **positive potential** (Va) with respect to the cathode c, some electrons move from cathode to anode and the diode conducts due to **attractive effect** on them.
- However if anode is at **negative potential** with respect to the cathode, no electrons reach the anode and the diode does not conduct due to the **repulsive effect on** them.

The diode therefore allows current to flow in only one direction.

Anode current (Ia) which flows is read from the Milliameter and the Voltmeter reading gives anode potential (Va)

Therefore anode resistance Ra = $\frac{\Delta Va}{\Delta Ia}$

2.2.2:A graph of I_a against V_a (diode characteristic graph)



- Along OA, electrons are released with some velocities that some reach the anode. Though the majority have low K.E and do not reach the anode **(space charge)** hence a small current is detected.
- AB, as voltage increases the number of electrons reaching the anode increases and therefore increase in current.
- BC, As anode potential is increased further, all the electrons leaving the cathode reach the anode. This is the **saturation region**.

Note:

Space charge:

This is the cloud of negative charges around the cathode at low anode p.d

Space charge limitation:

When the anode potential is not sufficient to attract all the electrons emitted from the cathode, emitted electrons tend to collect in the form of electron cloud above the cathode. This cloud of negative charge electrons constitutes space charge. Space charge exerts a repelling force on electrons being emitted from the cathode there by decreasing the anode current.

Saturation

This occurs when the anode potential is increased to a value such that the number of emitted electrons is equal to number of collected electrons

2.3.0:A DIODE AS A RECTIFIER

2..3.1:RECTIFICATION

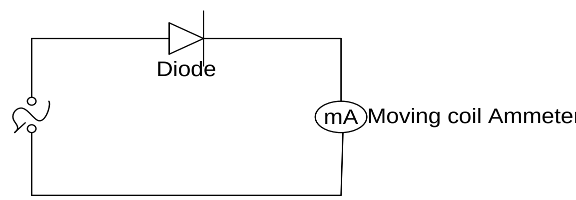
Rectification involves converting Alternating current to Direct current.

This can be done by use of

- Thermionic diodes.
- Semiconductor diode

When a rectifier is connected to a supply its supposed to conduct and when it does so its said to be **forward biased.** And when connected in a reverse way it fails to conduct therefore its said to be **reverse-biased.**

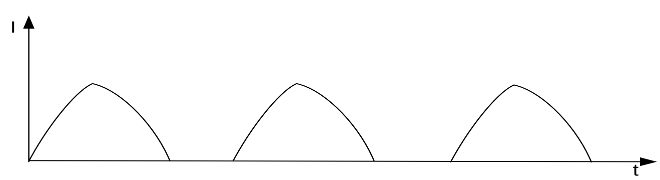
a) Half wave Rectification



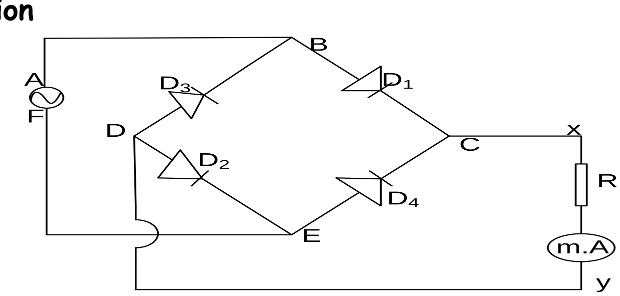
$A.c$ to be measured is first passed through the rectifier which converts it to $d.c$. The $d.c$ obtained is then measured using a moving coil ammeter.

N.B: The Arrow head in the rectifier symbol shows the direction of flow of current through the circuit.

A graph of I against t is drawn

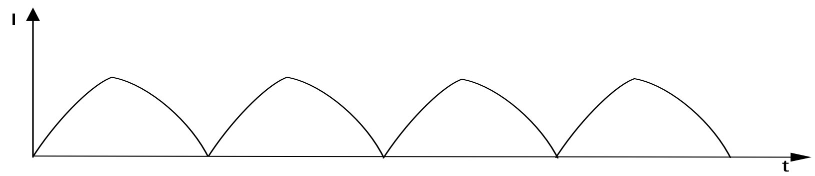


b) Full wave rectification



- Four diodes are arranged in a bridge network as shown above. If A is positive during the first half cycle, diodes 1 and 2 conduct and current takes the path ABCRDEF

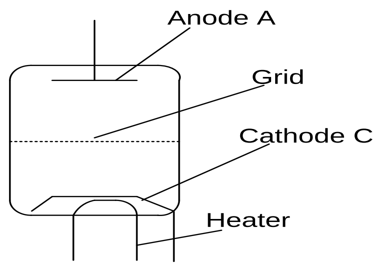
- During the next half cycle when F is positive and A is negative diodes D_3 and D_4 conduct while D_1 and D_2 do not conduct in this cycle and current (I) flows through path FECRDBA. The current through R is in the same direction throughout and it can be measured by moving coil ammeter.



UNEB NO 6(e) 1999

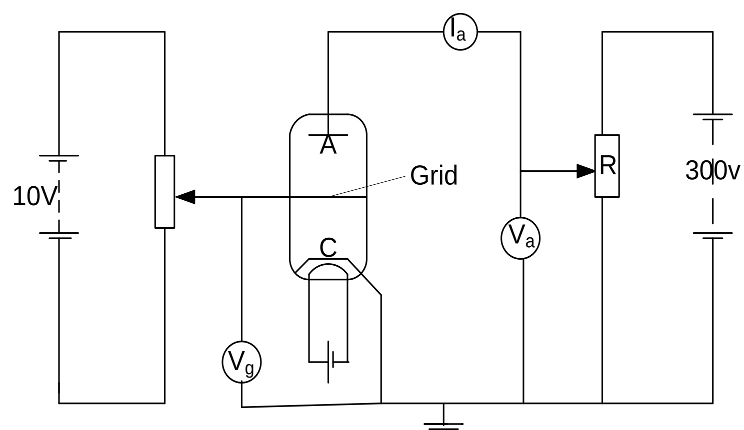
With the aid of a circuit diagram, describe the mode of action of a full wave rectifier.

2.4.0:THE TRIODE



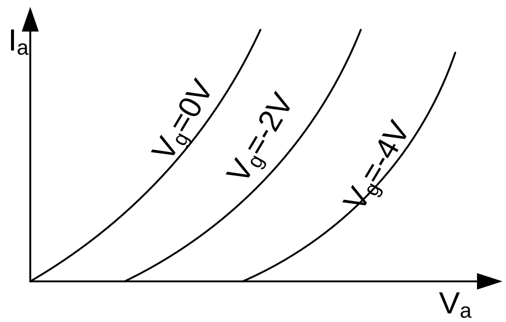
It consists of three electrodes, with grid placed between the cathode and anode.
The relationship between the grid potential (V_g), Anode potential (V_a) and anode current (I_a) for a given heating current gives the triode characteristics.

2.4.1:TRIODE CHARACTERISTICS CIRCUIT



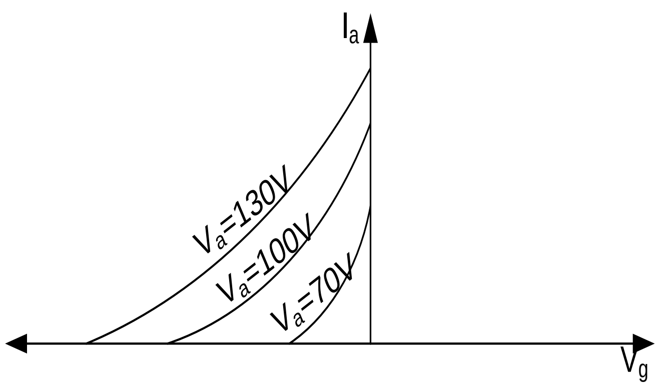
The circuit can be used to generate a set of readings to a give a triode characteristics.

2.4.2:TYPICAL ANODE CHARACTERISITCS



As the anode voltage increases, the anode current also increases

2.4.3:TYPICAL MUTUAL CHARACTERISITCS



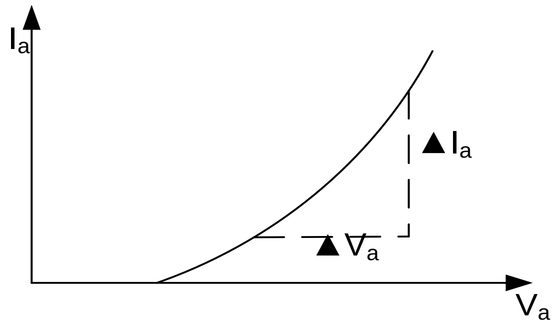
- When the anode is voltage is 70V (for example)the negative voltage on the grid creates the resultant electric field intensity at the cathode and hence no electrons move through the grid and hence the anode current is zero (I_a=0)
- As the negative voltage increases and reaches a certain value, the attraction effect of the positive anode overcomes repulsive effect of the grid and electrons now reach the anode.

2.4.4:TRIODE CONSTANTS

1:ANODE RESISTANCE (R_a)

It is defined as $R_a = \frac{\Delta V_a}{\Delta I_a}$ at constant V_g

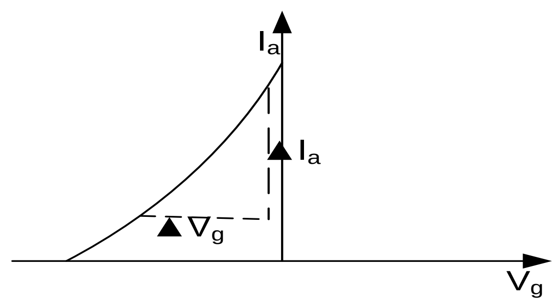
$V_a$is anode voltage and Ia is anode current which can be obtained from the straight part of the anode characteristics curve.



2: MUTUAL CONDUCTANCE (g_m)

It is defined as $\boldsymbol{g_m} = \frac{\Delta I_a}{\Delta V_g}$ for constant V_a

Vg is grid voltage



3:AMPLIFICATION FACTOR (μ)

It is defined as μ= $\frac{\Delta V_a}{\Delta V_g}$ for constant I_a

2.4.5:RELATION BETWEEN $\boldsymbol{R_a}$, $\boldsymbol{g_m}$ AND μ

$R_a=\frac{\Delta Va}{\Delta I_a}$

ΔVa=$R_a \Delta I_a$

Also $g_m=\frac{\Delta I_a}{\Delta Vg}$

$\Delta Vg=\frac{\Delta I_a}{g_m}$

But μ =$\frac{\Delta V_a}{\Delta V_g}$

μ =$\frac{R_a \Delta I_a}{\left(\frac{\Delta I_a}{g_m}\right)}$

μ =$R_a \Delta I_a$x$\frac{g_m}{\Delta I_a}$

$\boxed{\boldsymbol{\mu = R_a x g_m}}$

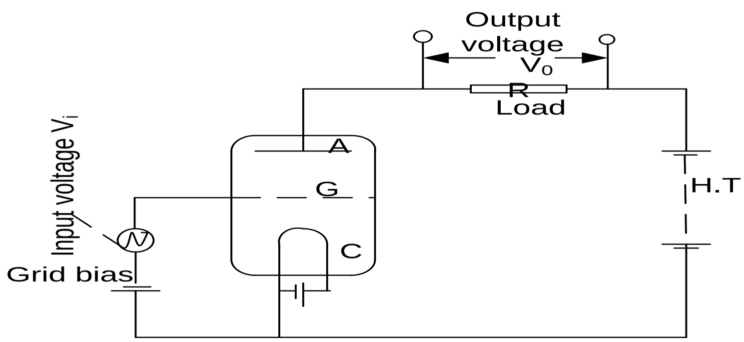
2.4.6:USES OF A TRIODE

- It is used as an amplifier in a radio receiver
- It is used an oscillator in a radio transmitter
- It is used as a detector in a radio receiver

2.4.7:TRIODE AS A SINGLE STAGE VOLTAGE AMPLIFIER

The amplifiers are used to boost the level of small voltage /current in radio receivers. Signals in form of alternating currents (voltages) are usually very weak and therefore need amplification. This can be achieved by means of a triode.

- Single stage amplification means signals to be amplified, pass through the amplifying circuit only once.



- The alternating input is supplied in the grid cathode circuit while the out put is taken across a high resistance in series with the anode.
- A triode should not only increase the value of alternating voltage but also give a wave for which is also a replication of the input without any distortion.
- A small negative voltage called a grid bias in the grid cathode is to prevent the distortion.

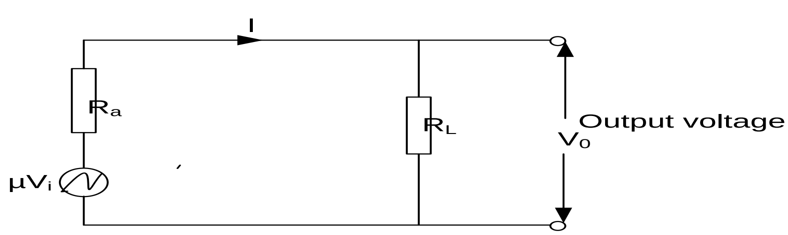
2.4.8:VOLTAGE GAIN

This is the ratio of output voltage V_0 to the input voltages (V_i)

$$\boxed{Voltage\ gain\ = \frac{V_0}{V_i}}$$

2.4.9:EQUIVALENT CIRCUIT OF TRIODE AS AN AMPLIFIER

To obtain the magnitude of voltage gain, the triode circuit is replaced with an equivalent circuit. The input voltage V_i in the grid cathode circuit is equivalent to (μV_i)



Total resistance in the circuit $= R_a + R_L$

$E.M.F$ of the source = μV_i

Therefore $\mu V_i = I(R_a + R_L)$

$I = \frac{\mu V_i}{R_a + R_L}$ ------------ [1]

Output voltage $V_0 = IR_L$

Therefore $V_0 = \frac{\mu V_i R_L}{(R_a + R_L)}$

$V_0 = \frac{\mu V_i R_L}{R_a + R_L}$ --------- [2]

Voltage gains = $\frac{V_0}{V_i}$

Voltage gains = $\frac{\mu V_i R_L}{R_a+R_L} \Big/ v_i$

Or $\frac{V_0}{V_i} = \frac{\mu R_L}{R_a+R_L}$

$$\boxed{Voltage\ gain\ = \frac{\mu R_L}{R_a+R_L}}$$

Example

1. A triode with mutual conductance of $3mAv^{-1}$, anode resistance $10^4\Omega$ and load resistance 20000 Ω is used as single stage voltage amplifier, calculate the voltage gain.

Solution

$g_m = 3mAV^{-1}$,

$Ra = 10^4\ \Omega$

$R_L = 20000\ \Omega$,

$g_m = 3x10^{-3}AV^{-1}$

$\mu = Ra \times g_m$

$\mu = 10^4 x3x10^{-3}$

$\mu = 30$

voltage gain $= \frac{\mu R_L}{R_a+R_L}$

$= \frac{30x20000}{10^4 x20000}$

Voltage gain =20

2. Calculate the voltage gain for triode whose amplification factor (μ) is 80 and whose anode slope resistance Ra is $10^4\ \Omega$ when used with an anode load of 20,000 Ω in a single stage voltage amplifier

Solution

$\mu = 80$,

$Ra = 10^4\ \Omega$

$R_L = 20000\ \Omega$

voltage gain $= \frac{\mu R_L}{R_a+R_L}$

$= \frac{80x20000}{10^4 x20000}$

Voltage gain = 53.3

3. A triode valve with an anode resistance of $3x10^3\ \Omega$ is used as an amplifier.
A sinusoidal alternating signal of amplitude 0.5V is applied to the grid of the valve. Find the $r.m.s$ value of the output voltage if the amplification factor is 15 and anode load is 50k Ω

Solution

$\frac{V_0}{V_i} = \frac{\mu R_L}{R_a+R_L}$

$V_0 = \left(\frac{\mu R_L}{R_a+R_L}\right) V_i$

$V_0 = \frac{15x0.5x50x10^3}{[50x10^3+3x10^3]}$

V_0=7.075v

$Vr.m.s = \frac{V_0}{\sqrt{2}}$

$Vr.m.s$=5.003v

Excercise5

1. A sinusoidal voltage of 0.2V is applied to the grid of the triode of an amplification factor 10. If the anode resistance of the triode is 15kΩ. Calculate the output voltage. **An[0.125V]**

2. A triode with mutual conductance of $4mAV^{-1}$ and anode resistance Ra=5kΩ is connected to a load resistance of 10kΩ. Estimate the out voltage obtained from an alternating output signal of 25mV. **An[0.333V]**

3. A triode with mutual conductance $4mAV^{-1}$ and the anode resistance 15kΩ and a load resistance 30kΩ is used as a single stage amplifier. Calculate the voltage gain. **An[40].**

2.5.0:THE TRANSISTOR

A transistor is made from three layers of p and n-semi conductor called the emitter (E), base (B) and collectors (C). the base is thinner. It can be pnp type or npn type transistor.

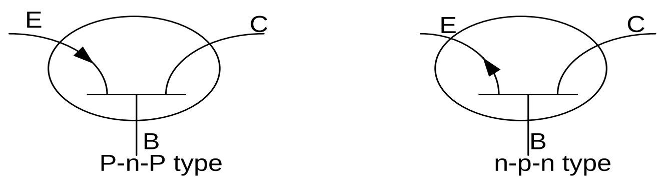
The junction transistor is called a bipolar transistor because its action is due to two charge carriers $i.e$ the electrons (-) and the holes (+).

There are two types of junction transistor

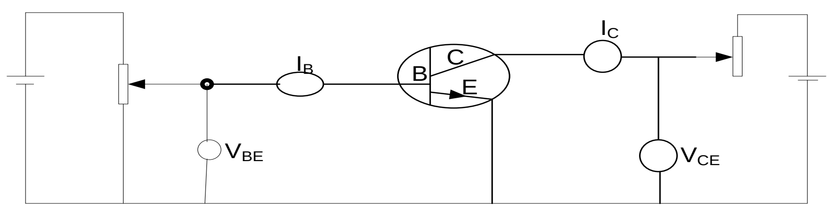
i) n-p-n transistor where the electrons, are the majority charge carriers

ii) p-n-p transistor where the holes are the majority charge carriers.

Symbols



2.5.1:Common – emitter mode (CE mode) for n-p-n transistor



The circuit can be used to obtain three types of characteristics

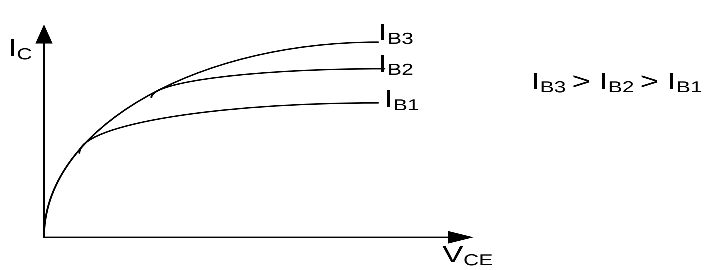
(1) Out put characteristics

(2) Input characteristics

(3) Transfer characteristic

2.5.2: Collector current (I_C) Against collector emitter voltage (V_{CE})

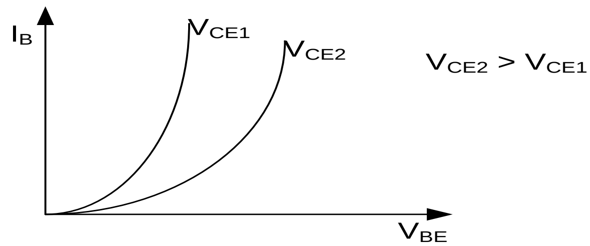
Output characteristics



For small V_{CE} the output current I_C increases slightly with V_{CE}.

At Higher V_{CE}, I_C varies linearly with V_{CE} for a given base current I_B. the linear part of the characteristics is used as amplifier circuit so that the output voltage variation is undistorted.

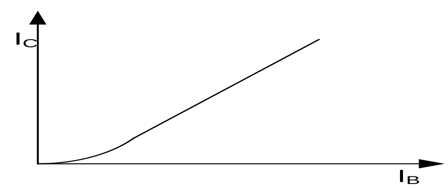
2.5.3: I_B Against V_{BE}(Input characteristics)



I_B varies exponentially with V_{BE} i.e its input characteristics is non linear for a given V_{CE}

Input resistance $R_B = \frac{\Delta\ V_B}{\Delta\ I_B}$

2.5.4: A graph of I_C Against I_B (Transfer characteristics)

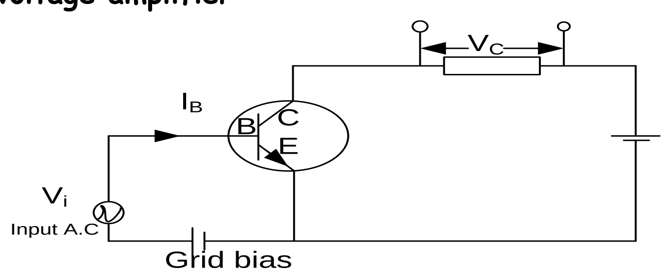


Output current I_C varies fairly linearly with the input current I_B.

Current transfer ratio B or (current gain)

$B = \frac{output\ current}{input\ current}$ $\qquad\qquad$ $B = \frac{\Delta I_c}{\Delta I_A}$

2.5.5: Transistor as a voltage amplifier



The small A.C voltage V_i is applied to the base emitter circuit and causes small changes of base current ΔI_B which produces large changes ΔI_C in the collector current flowing through the load R which converts these current changes into voltage charges which form the A.C output voltage $V_0 = \Delta I_C R$.

UNEB 2013 Q.10

(c) **(i)** Define **space charge** as applied to thermionic diode (01marks)

(ii) Draw anode current-anode voltage curve of thermionic diode for two different filament currents and explain their main feature (06marks)

UNEB 2012 Q 10

(a) Define the terms below as applied to a triode

(i) Space charge [01marks]

(ii) Amplification factor [01marks]

(iii)Mutual conductance [01marks]

(b) With the aid of a labeled diagram explain full wave rectification [07marks]

(c) Derive an expression for the amplification factor μ in terms of anode resistance Ra and mutual conductance g_m for a triode valve. [03marks]

(d) A triode with mutual conductance $3mAV^{-1}$ and anode resistance of $10k\Omega$ is connected to a load resistance of $20k\Omega$. Calculate the amplitude of the output signal, if the amplitude of the input signal is 25mV **An[0.5V]** [04marks]

(e) i) Sketch the output characteristics of a transistor [02marks]

(ii) Identify on the sketch in e(i) the region over which the transistor can be used as an amplifier.
[01mark]

UNEB 2008 Q.10

(a) Describe the mechanism of thermionic emission [03marks]

(b) Explain the following terms as applied to a vacuum diode

(i) space charge limitation [03marks]

(ii) saturation [01mark]

(iii)rectification [02marks]

(c) Sketch the current- potential difference characteristics of thermionic diode for two different operating temperatures and explain their mean features
[05marks]

(d) (i) A triode valve with anode resistance of $3x10^3\Omega$ is used as an amplifier. A sinusoidal alternating signal of amplitude 0.5v is applied to the grid of the valve. Find the r.m.s valve of the output voltage if the amplification factor is 15 and anode load is 50k Ω. **An[5.003V]**

[05marks]

(ii) Draw an equivalent circuit of a triode as a single stage-amplifier [01marks]

UNEB 2007 Q.8

(a) Describe briefly the mechanism of thermionic emission [02marks]

(b) (i) Draw a labeled circuit to show a triode being used as single-stage voltage amplifier

[03marks]

(ii) With the aid of an equivalent circuit of the triode as an amplifier, obtain an expression for the voltage gain [04marks]

(iii) A triode with mutual conductance $3x10^{-3}AV^{-1}$ and anode resistance of $1x10^{4}\Omega$ is used as a single-stage amplifier. If the load resistance is $3x10^{4}\ \Omega$ calculate the voltage gain of the amplifier. **An[22.5]** [05marks]

(c) (i) Describe the structure of a junction transistor [02marks]

(ii) Sketch and describe the collector-current against the collector-emitter voltage characteristics of a junction transistor [03marks]

UNEB 2004 Q.10

(a) (i) Explain briefly the mechanism of thermionic emission [02marks]

(ii) Draw labeled diagram of the circuit used to determine the anode current and anode voltage characteristics of thermionic diode [02marks]

(iii) Sketch the characteristics expected in a)i) at constant filament current and account for its special features [04marks]

UNEB 2003 Q.9

(a) (i) What is meant by thermionic emission [04marks]

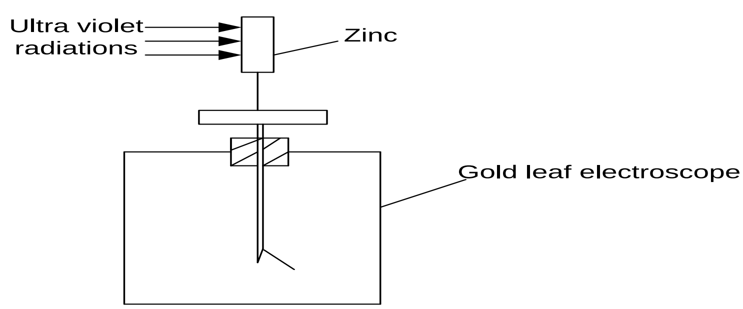
(ii) Sketch the current-potential difference characteristics of thermionic diode for two different operating temperatures and explain their main features. [05marks]

(iii) Describe one application of a diode [02marks]

CHAPTER 3: PHOTOELECTRIC EMISSION

It's defined as a process by which electrons are released from a clean metal surface when irradiated by electromagnetic radiations (light) of high enough frequency (energy).
The electrons emitted this way are called **photo electrons.**
The radiation falling on the metal surface is absorbed by the electrons and becomes internal energy which is sufficient to enable them overcome the inward attraction for the electrons to get loose and fly off the metal surface.

3.1.0:EXPERIMENT TO DEMONSTRATE PHOTO ELECTRIC EFFECT



- A cleaned zinc plate is placed on a cap of a negatively charged gold leaf electroscope.
- When ultraviolet radiations are directed on to the plate, the leaf is seen to collapse gradually.

- This is because the the plate and the cap lost charges (electrons). So the magnitude of the negative charge at the leaf and gold plate decreases thereby decreasing the divergence of the leaf gradually.

Note:

(1) If the intensity of UV radiation is increased for the positively charged electroscope there is no change on the divergence of the leaf. But for a negatively charged electroscope, the leaf collapses fast since the number of electrons emitted per unit time (photo current) from the zinc plate increases with intensity.

(2) If infrared radiations are used instead of UV **no effect** is observed on the leaf divergence because the frequency of the infrared is below threshold frequency for zinc. Hence it cannot eject electrons from the zinc plate no matter how intense it's radiation is.

(3) When ultraviolet radiations fall on a cleaned zinc plate placed on a cap of a positively charged gold leaf electroscope, there is no change in the divergence of the leaf. This is because the electrons that are emitted photo electrically are attracted back by the positively charged zinc plate. Hence there is no change in the magnitude or sign of charge on the electroscope.

3.1.1:EINSTEINS PHOTOELECTRIC EQUATION

Einstein proposed that when a photon collides with an electron, it must either be reflected with no reduction in energy or it must give up **all** its energy to the electron.

The energy of a single photon cannot be shared amongst the electrons i.e no more than one electron can absorb the energy of one photon.

Therefore in any given time the number of electron emitted by a surface is proportional to the number of incident photons i.e to the intensity of the radiation.

Further more , an electron can be emitted as soon as a photon reaches the surface, explaining why photoemission begins instantaneously.

Einstein also reasoned that some energy imparted by a photon is actually used to **release** an electron from the surface (overcome the binding force) and the rest appears as the kinetic energy of the emitted electron. This is summarized by Einstein's photoelectric equation

$$\boxed{\boldsymbol{hf = W_O + ½\, mv_{max}^2}}$$

where h = is plank's constant

hf=the energy of each incident photon of frequency

W_0 = the work function of the surface

$\frac{1}{2}$ m$v_{\max}^2$=maximum kinetic energy of the emitted electrons

$\boxed{W_0 = hf_0}$ where f_0 is threshold frequency

$\boxed{½\, mv_{\max}^2 = eV_S}$ V_s is stopping potential

Definition

Work function of metal (W_0)

It is the minimum energy that is needed to just remove an electron from the metal surface

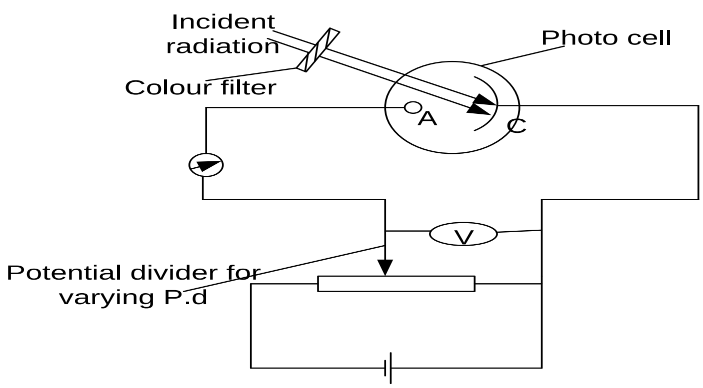
Threshold frequency (f_0)

It is the minimum frequency of the incident radiation below which no electron emission takes place from a metal surface

Stopping potential (V_s)

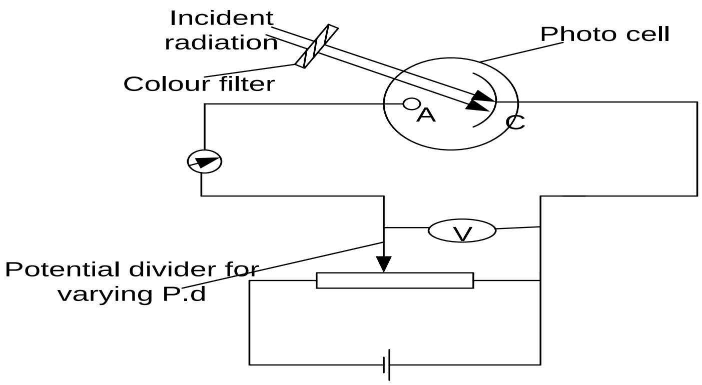
It is the minimum potential which reduces the photo current to zero.

3.1.2:AN EXPERIMENT TO MEASURE 0F STOPPING POTENTIAL



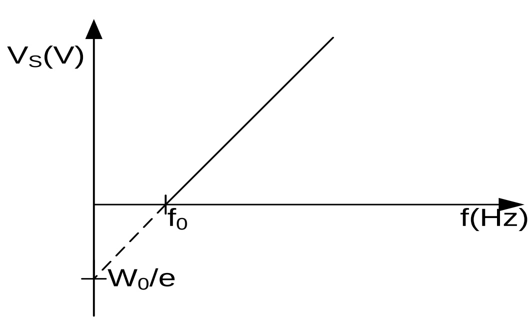
- The cathode C is made positive with respect to the anode by the potential divider.
- The beam of radiation is passed through a colour filter on to the cathode.
- The ammeter gives the photocurrent due to emitted electrons
- The applied p.d is increased negatively until the ammeter register zero reading.
- The $p.d$ (V_s) for which the photocurrent is zero is recorded from the voltmeter
- This p.d V_s is known as the stopping potential

AN EXPERIMENT TO VERIFY EINSTEIN'S EQUATION OR DETERMINE PLANCK'S CONSTANT



- The anode A is made negative with respect to the cathode
- The cathode C is also made positive with respect to the anode.
- A beam of radiation of known of frequency, f is passed through a colour filter on to the photo cathode.
- The ammeter gives the photocurrent due to emitted electrons
- The applied p.d V is increased negatively until the ammeter register zero reading.
- The $p.d$ (V_s) for which the photocurrent is zero is recorded from the voltmeter
- The procedure is repeated with other frequencies, f of radiation.
- A graph of V_s against f is plotted.
- A straight line graph is obtained and the slope s is found from it.
- The plancks constant h is got from $h = eS$ where e is the electronic charge

NOTE



$\boldsymbol{hf} = W_0 + \frac{1}{2} m\boldsymbol{v}^2_{\mathbf{max}}$

$\boldsymbol{h}f = W_0 + eV_s$

$\boldsymbol{eV_S} = hf + W_0$

$\boldsymbol{V_S} = \frac{\boldsymbol{h}}{\boldsymbol{e}}\boldsymbol{f} - \frac{\boldsymbol{w_0}}{\boldsymbol{e}}$

Slope $= \frac{\boldsymbol{h}}{\boldsymbol{e}}$

∴h=e x slope

Where e is electronic charge

- Photo electric emission is an instantaneous process.

3.1.3:LAWS/RESULTS/OBSERVATIONS OF PHOTO ELECTRIC EMISSION

Law 1- For any given metal surface there is a certain minimum frequency of radiation called threshold frequency below which no photo electrons are emitted.

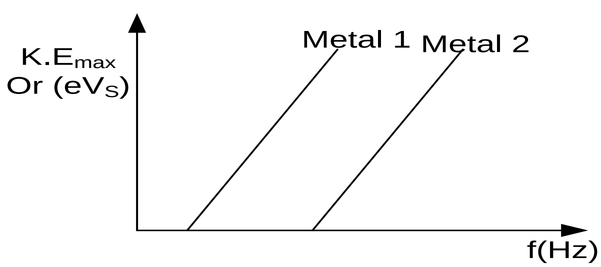
Law 2- Photo electrons are emitted with a range of kinetic energies, ranging from zero up-to some maximum value and K.E is proportional to the frequency of the incident radiation.

Law 3-The number of photo electrons emitted per second (photo current) is directly proportional to the intensity of incident radiation for a given frequency.

Law 4-There is no detectable time lag between irradiation of a metal surface and emission of electrons by the surface.

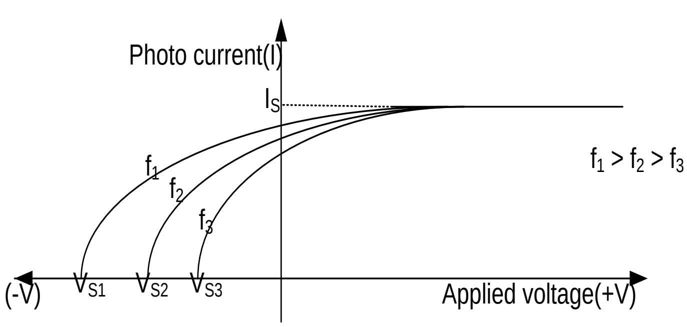
3.1.4:Results/observation from the experiment

1:Variation of $K.E_{max}$ with frequency [keeping applied $p.d$ and intensity constant]



- $K.E_{max}$ is directly proportional to frequency
- For any given surface, there is minimum frequency called threshold frequency to below which no electrons are emitted
- The metals have the same slope and h (plank's constant)

2:Variation of photo current I with applied voltage for different frequencies but keeping the intensity constant

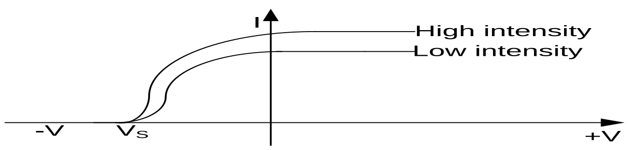


- $K.E_{max}$ (eV_S) increases with frequency
- I_s (saturation current) is independent of frequency

$$\boxed{Is = ne}$$

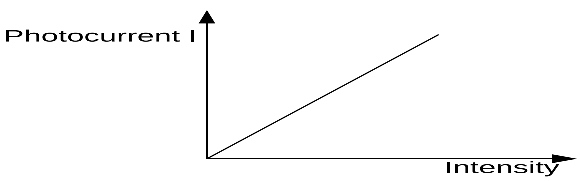
Where n is the number of photo electrons

3:Variation of photocurrent I with applied $p.d$ for different intensities keeping frequency constant



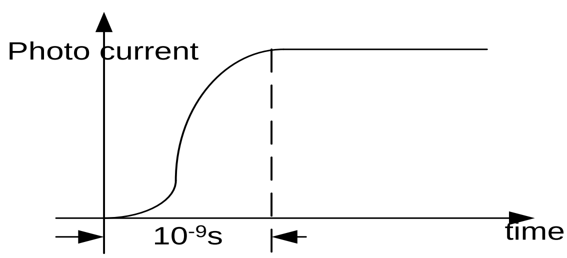
- Photo current increases with intensity
- Max $k.e$ (eV_S) is independent of intensity

4:Variation of photocurrent with intensity when both frequency and applied $p.d$ are constant



- Photocurrent is directly proportional to intensities
- **Max** $K.e$ (eV_S) is independent of intensity

5:Variation of photocurrent with time



3.1.5:THEORIES OF LIGHT

There are two theories of light

a) **Classical wave theory** which is considered as wave like propagation of light

It states that radiation is emitted with continuous energy

b) **Quantum theory**

It states that radiation is emitted in Quanta (packets of light energy).

A photon This is a packet of light energy

3.1.6:FAILURES OF CLASSICAL WAVE THEORY TO EXPLAIN PHOTO ELECTRIC EMISSION

- **Existence of threshold frequency**

  The theory allows continuous absorption and accumulation of energy; any radiation should eventually be able to provide electrons even if the it's below the threshold frequency provided it is intense enough. It therefore predicts no threshold frequency contrary to what was experimentally observed.

- **Variation of kinetic energy**

By the wave theory, an increase in intensity means more energy and hence greater value of maximum kinetic energy of electrons. However, maximum K.E depends on frequency of radiation and not.

- **Instantaneous emission**

  Since there is continuous absorption and accumulation of energy by an electron, the theory predicts a time lag between irradiation and emission of electrons, however such time lag is not experimentally observed

Note: classical theory only accounts for the increase in the number of electrons emitted per second with increasing intensity of the incident radiation.

3.1.7: QUANTUM THOERY EXPLANATION OF PHOTOELCRIC EMISSION

- Quantum theory considers radiations to be emitted and absorbed in discrete packets or quanta's called photons each of energy $\boldsymbol{E = hf}$.
- Quantum theory considers photoelectric emission to be one electron-one photon affair. Each electron can only absorb one photon. If the photon energy is less than the work function the photon is reflected back, but if the photon energy is greater or equal to the work function, then the photon is absorbed and an electron is released.
- Increasing intensity increases the number of photons in the beam since each electrons is only allowed to absorb only one photon, the number of electrons emitted per second (photo current) is proportional to the intensity.
- Quantum theory does not allow continuous absorption and accumulation of energy. Energy transfer process requires very small time [of order 10^{-9}S]. This predicts photoelectric emission to be an instantaneous process.

Examples

1. Work function of potassium is 2.25eV. Light having wavelength of 360nm falls on the metal. Calculate;
   (i) Stopping potential
   (ii) The speed of the most energetic electron emitted
   [h=6.60×10^{-34}Js, C=3×10^{8}ms^{-1}, e=1.6×10^{-19}C]

Solution

Work function W_0=2.25eV

$W_0=2.25\times1.6\times10^{-19}$J

$\lambda=360\times10^{-9}$m

Using Einstein's equation

$hf = W_0+\frac{1}{2}mv_{\max}^2$

but also $\frac{1}{2}mv_{\max}^2=eV_s$

$hf = W_0+eV_s$

$h\frac{C}{\lambda}= W_0+eV_s$

$V_s =\frac{h\frac{C}{\lambda}-W_0}{e}$

$V_S = \frac{\frac{6.6x10^{-34}x3x10^{8}}{360x10^{-9}} - 2.25x1.6x10^{-19}}{1.6x10^{-19}}$

V_S=1.188V

$\therefore \frac{1}{2} mv_{max}^2 = eVs$

$V_{max} = \sqrt{\frac{2eVs}{m}}$

$= \sqrt{\frac{2x1.6x10^{-19}x1.88}{9.1x10^{-31}}}$

V_{max}= 6.46x10^5m/s

2. If a surface has a work function of 3.0eV
(a) Find the longest wave length of light which will cause the emission of photo electrons on it.
(b) What is the maximum velocity of the photo electrons liberated from the surface having a work function of 4.0eV by ultraviolet radiations of wave length 0.2μm.

Solution

a) W_0 =3x1.6x10^{-19}J

$W_0=hf_0$

hf_0=3x1.6x10^{-19}

$h\frac{C}{\lambda_0}$=3x1.6x10^{-19}

$\lambda_0 = \frac{6.6x10^{-34}x3x10^{8}}{3x1.6x10^{-19}}$

λ_0=4.125x10^{-7}m

Longest wave length of light =4.125x10^{-7}m

b) Using Einstein's equation

$hf = W_0 + \frac{1}{2} mv_{max}^2$

W_0=4x1.6x10^{-19}J

λ =0.2x10^{-6}m

$h\frac{C}{\lambda} = W_0 + \frac{1}{2} mv_{max}^2$

$V_{max} = \sqrt{\frac{2x\left(\frac{6.6x10^{-34}x3x10^{8}}{0.2x10^{-6}} - 4x1.6x10^{-19}\right)}{9.1x10^{-31}}}$

V_{max} =8.77x10 ^{5}ms^{-1}

3. Calcium has a work function of 2.7eV
   (a) What is the threshold frequency for calcium
   (b) What is the maximum wavelength that will cause emission from calcium.

[e=1.6x10^{-19}C, h=6.6x10^{-34}Js,C=3x10^8ms^{-1}]

Solution

a) W_0=2.7x1.6x10^{-19}

hf_0=2.7x1.6x10^{-19}

$f_0 = \frac{2.7x1.6x10^{-19}}{6.6x10^{-34}}$

f_0=6.55x10^{14}Hz

b) Max wavelength is λ_0

$f_0 = \frac{C}{\lambda_0}$

$\lambda_0 = \frac{3x10^8}{6.55x10^{14}}$

λ_0=4.58x10^{-7}m

EXCERCISE6

1. Calculate the energy of;
   (a) A photon of frequency 7.0x10^{14}Hz.
   (b) A photon of wavelength 3x10^{-7}m

[h=6.6x10^{-34}Js, C=3x10^{8}ms^{-1}J]   **An[4.6x10^{-19}J, 6.6X10^{-19}J]**

2. Sodium has a work function of 2.3eV. Calculate;
   (a) Its threshold frequency
   (b) Maximum velocity of the photoelectrons produced when the sodium is illuminated by light of wavelength 5x10^{-7}m
   (c) The stopping potential with light of this wavelength [h=6.6x10^{-34}Js, C=3x10^{8}ms^{-1}, 1eV=1.6x10^{-19}J, mass of electron m=9.1x10^{-31}kg] **An[5.6x10^{14}Hz, 2.5x10^{5}ms^{-1}, 0.18V]**

3. Calculate the stopping potential for a platinum surface irradiated with ultraviolet light of wavelength 1.2x10^{-7}m. The work function of platinum is 6.3eV. [h=6.6x10^{-34}Js, C=3x10^{8}ms^{-1}, e= 1.6x10^{-19}C]
   An[4.0V]

4. Gold has a work function of 4.9eV
(a) Calculate the maximum kinetic energy in joules, of the electrons emitted when gold is illuminated with ultraviolet radiations of frequency 1.7x10^{15}Hz.
(b) What is the energy in eV
(c) What is the stopping potential for these electrons. [h=6.6x10^{-34}Js, e=1.6x10^{-19}C]

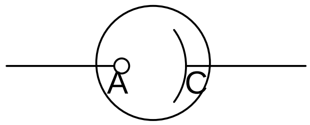
An [3.4x10^{19}J, 2.1eV, 2.1V]

5. Light of frequency 6x10^{14}Hz, incident on a metal surface ejects photoelectrons having a kinetic energy 2x10^{-19}J. Calculate the energy needed to remove an electron from the metal (work function). [h=6.6x10^{-34}Js]   **An[1.96x10^{19}J]**
6. Light of wave length 0.5μm incident on a metal surface ejects electrons with kinetic energies up to a maximum value of 2x10^{-19}J. What is the energy required to remove an electron from the metal? If a beam of light causes no electrons to be emitted, however great its intensity what condition must be satisfied by its wavelength? [h=6.6x10^{-34}Js, C=3x10^{8}ms^{-1}]
   An[1.96x10^{-19}, 1.01x10^{-6}m]
7. The maximum kinetic energy of photo electrons ejected from a tungsten surface by mono chromatic light of wavelength 248nm was found to be 8.6x10^{-20}J. Find the work function of tungsten. [h=6.6x10^{-34}Js, e=1.6x10^{-19}C, C=3x10^{8}ms^{-1}]   **An[4.45eV]**

3.1.8: PHOTO CELLS

These are devices that change radiations into current

Symbol



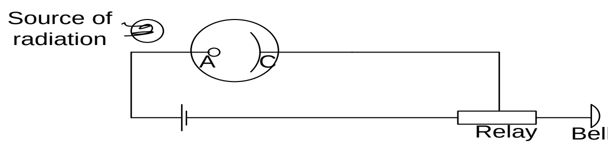
A- Anode
C- Cathode

When radiation falls on the cathode, elections are emitted and are collected by the anode if it is positive with respect to the cathode.

3.1.9: USES OF PHOTOCELLS

(i) They are used in Burglar alarms

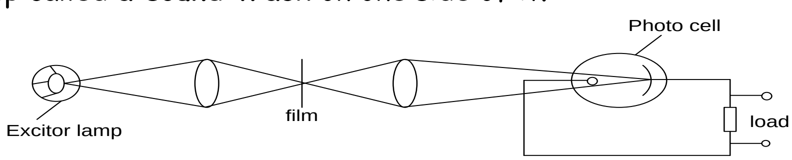
A burglar alarm consists of a photo cell forming a closed circuit and a source of radiation



Light from the source falls on the photocell and maintains a current in the circuit. When someone intersects the radiations, the current is automatically switched off. This automatically operates the relay switch and another circuit containing a bell gets closed, the bell starts ringing and can only stop when it has been switched off.

(ii) **Reproduction of sound track on a film**

A photo cell is used in the reproduction of sound which is recorded on a film in form of a thin transparent strip called **a sound track** on one side of it.



When the film runs, light from lamp goes through the film and falls on a photocell. A variable current is produced which is amplified and fed to a loud speaks to reproduce a sound.

Example

1. A 100mW beam of light of wave length $4x10^{-7}$m falls on caesium surface of a photocell

(i) How many photons strike the cesium surface per second.

(ii) If 65% of the photons emit photo electrons, find the resulting photo current

(iii) Calculate the kinetic energy of each photon if the work function of caesium is 2.20eV

Solution

(i) Photon energy E=hf or $E=h\frac{C}{\lambda}$

$E=\frac{6.6x10^{-34}x3x10^{8}}{4x10^{-7}}$

$E=4.95x10^{-19}$

Power of beam =photon energy x number of photons per second

$100x10^{-3}$=$4.95x10^{-19}$xnumber of photons

Number of photons per second =$2.02x10^{17}$ photons

(ii) Number of electrons emitted per second n=65% of photons

n=$\frac{65}{100}x2.02x10^{17}$

n=$1.31x10^{17}$ electrons

I=ne

I=$1.31x10^{17}x1.6x10^{-19}$

I=$2x10^{-2}$A

(iii) From Einstein's equation

$hf = W_0 + \frac{1}{2} mv_{\max}^2$

$K.E_{max}= hf - W_0$

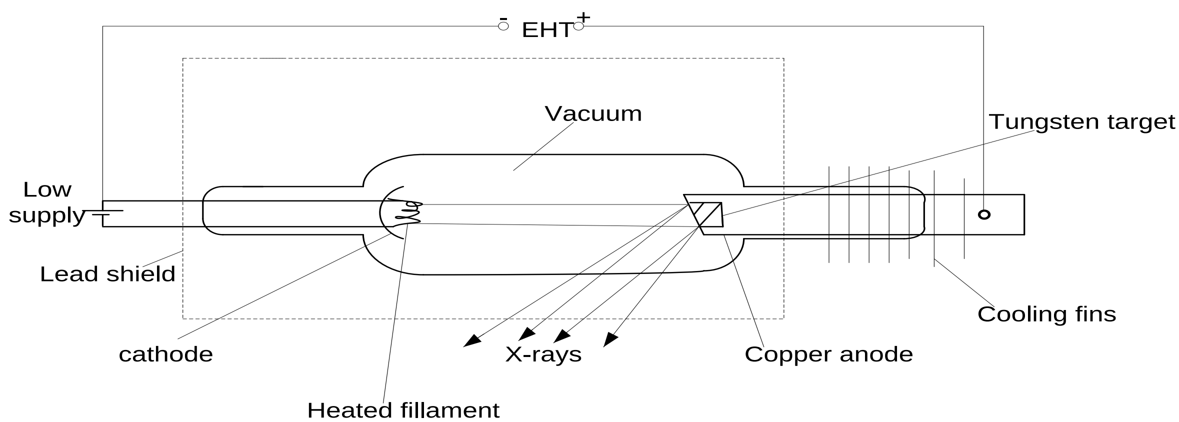
$K.E_{max}= 4.95x10^{-19}-2.2x1.6x10^{-19}$

$K.E_{max}= 1.43x10^{-19}J$

3.2.0:X-RAYS

These are electromagnetic radiations of short wavelength ($\sim 10^{-10}m$) which travel at a speed of light and produced when fast moving electrons (cathode rays) strike a metal target.

3.2.1: X-RAY TUBE [PRODUCITON OF X-RAYS]



Operation

- ❖ The cathode is heated with low voltage and electrons are emitted thermionically.
- ❖ Electrons are accelerated by a high p.d towards the anode.
- ❖ On striking the target, a small percentage of the electrons is converted to X-rays
- ❖ The anode is cooled by the cooling fins.

Note

(1) The energy changes in an x-rays tube are; electrical energy from low voltage source to heat energy used for heating the filament to kinetic energy of electrons and then to heat and x-rays.

(2) The intensity of x-ray beam increases with the number of electrons hitting the target, therefore intensity is controlled by filament current /heating current or supply voltage.
(3) The penetrating power (quality) of an x-ray beam is controlled by the accelerating $p.d$ between the cathode and the anode
(4) X-rays with high penetrating power are called hard x-rays while those with low penetrating power are called soft x-rays.
(5) The x-ray tube is totally evacuated to prevent collision of electrons with gas molecules.

3.2.2: PROPERTIES OF X-RAYS

(1) They travel in straight lines at the velocity of light.
(2) They cannot be deflected by electric or magnetic field(This is an evidence that they are not charged particles)
(3) They readily penetrate matter, penetration is least with materials of high density
(4) They can be reflected but not at very large angles of incidence
(5) Refractive indices of all materials are very close to unity (one) for x-rays so that very little bending occurs when they pass from one material to another
(6) They can be diffracted

The following properties 7 to 10 are used to detect x-rays

(7) They ionize gases through which they pass
(8) They affect photographic film
(9) They can produce fluorescence
(10) They can produce photoelectric emission

3.2.3: USES OF X-RAYS

Medical uses

- Used to detect fractures in bones
- Used to destroy cancer cell
- Used in detection of lung T.B
- Used for sterilization of medical equipments

Industrial use

- They are used to locate internal imperfection in welded joints and costing

Agric uses

- Tracing phosphate fertilizers using phosphorus
- Sterilization of insecticides for pest control
- X-ray crystallography

- Used to study crystal structures and determine structure of complex organic molecules

Health hazard of x-rays

- They have harmful effects on human cells which become eminent after sometime

Precaution

- Lead aprons should be worn while dealing with x-rays
- The brain and other delicate parts of the body should not be exposed to x-rays
- Unnecessary long time exposure to x-rays should be avoided.

3.2.4:X-RAY EMISSION SPECTRA

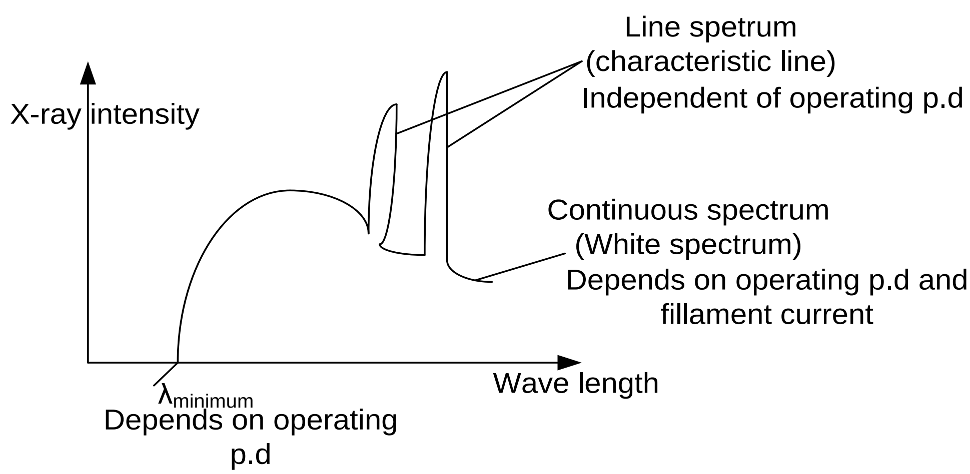
X-rays spectra have two distinct components

(1) A background of continuous radiation, the minimum wavelength of which depends on the operating voltage of the tube $i.e$ the energy of the incident electrons.

- At cut off wavelength, λ_{min} . Electrons from the cathode strike the target and lose all their kinetic energy in a single encounter with the target atoms. This results in the production of the most energetic x-ray photons of maximum frequency and corresponding, λ_{min} called cut off wavelength.

From $\boxed{\boldsymbol{E = hf}}$ $\boxed{\boldsymbol{hfmax = ev}}$ $\boxed{h\frac{C}{\lambda_{min}} = ev}$

(2) Very intense emission at a few discrete wavelength (an x-ray line spectrum). These wavelength are characteristic of the target material and are independent of the operating voltage.



a) Continuous spectrum

It is formed as a result of multiple collisions of energy electrons with a target atom and these electrons are decelerated. Different amounts of energy are lost, x-rays given off have wavelengths varying from a certain minimum value (λ_{min}) to infinity.

b) Line spectrum

When highly energetic electrons penetrate the atom, knock electrons from inner most shells and displace them to higher shells. This puts the atom in an excited state and therefore becomes unstable. The subsequent electron transition from higher energy levels into a vacancy in the lower energy levels causes a high energy x-ray photon of definite wavelength to be emitted whose energy is equal to the difference between the energy levels. This leads to x-ray line spectrum.

Note

- The frequency of the x-ray is given by $\boldsymbol{E = hf}$. Where E is the difference in the energy levels involved and h is plank's constant.
- Continuous spectrum is produced due to multiple collisions of electrons with target atoms while
- Line spectrum is produced by electronic transitions within the atoms as the electrons in them fall back to the lower energy levels.

Example

1. An x-ray tube operates at 30kV and current through it is 2mA. Calculate
   (i) The electrical power input
   (ii) Number of electrons striking the target per second
   (iii) The speed of electrons when they hit the target
   (iv) The lower wavelength limit of x-rays emitted
   [$h=6.6\times10^{-34}$Js, $e=1.6\times10^{-19}$C, $C=3\times10^{8}ms^{-1}$, $m=9.1\times10^{-31}$kg]

Solution

(i) Power input =IV

Power input $=2\times10^{-3}\times30\times10^{3}$

Power input $=60Js^{-1}$

(ii) I=ne

n=I/e

$n=\frac{2x10^{-3}}{1.6x10^{-19}}$

$n=1.25\times10^{16}$ electrons per second

(iii) $\frac{1}{2}mu^2 = eV$

$u=\sqrt{\frac{2ev}{m}}$

$u=\sqrt{\frac{2x1.6x10^{-19}x30x10^{3}}{9.1x10^{-31}}}$

$u=1.03\times10^{8}ms^{-1}$

(iv) $hf_{max}=eV$

$h\frac{C}{\lambda_{min}}=eV$

$\lambda_{min}=\frac{hC}{ev}$

$\lambda_{min}=\frac{6.6x10^{-34}x3x10^{8}}{1.6x10^{-19}30x10^{3}}$

λ_{min}=4.13x10^{-11}m

2. The $p.d$ between the target and cathode of an x-ray tube is 50kV and current in the tube is 20mA. If only 1% of the total energy is emitted as x-rays.

(i) What is the maximum frequency of the emitted radiations

(ii) At what rate must heat be removed from the target in order to keep it a steady temperature.

Solution

i) $hf_{max}=eV$

$f_{max}=\frac{1.6x10^{-19}x50x10^{8}}{6.6x10^{-34}}$

f_{max}=1.21x10^{19}Hz

ii) 1% of power produces x-ray, therefore 99% of power produces heat

For a steady temp the rate at which heat is supplied equals to rate at which heat is removed

Rate at which heat is supplied to the target 99% of power

=$\frac{99}{100}$ of IV

=$\frac{99}{100}$x20x10^{-3}x50x10^{3}

=990Js^{-1}

3. An x-ray tube operated at 1.8x10^{5}V with target made of a material of S.H.C of 250Jkg^{-1}K^{-1} and has a mass of 0.25kg. 1% of the electrical power supplied is converted into x-ray and the rest is dissipated as heat in the target. If the temp of the target rises by 8^{0}C per second. Find

(i) The number of electrons which strike the target per second

(ii) The shortest wavelength of x-rays produced

Solution

V=1.8x10^{5}V, C=250Jkg^{-1}K^{-1} m=0.25kg

$\frac{\Delta\theta}{t}$=8°Cs^{-1}

i) IVt =mc $\Delta\theta$

IV= mc$\frac{\Delta\theta}{t}$

I=$\frac{mc}{v}\frac{\Delta\theta}{t}$

I=$\frac{0.25x250x8}{1.8x10^{5}}$

I=2.78x10^{-3}A

Using I=ne

n=I/e

n=$\frac{2.78x10^{-3}}{1.6x10^{-19}}$

n=1.74x10^{6} electrons per second

ii) $hf_{max}=eV$

h$\frac{C}{\lambda_{min}}$=ev

$\lambda_{min}=\frac{hC}{ev}$

$\lambda_{min}=\frac{6.6x10^{-34}x3x10^{8}}{1.6x10^{-19}x1.8x10^{5}}$

λ_{min}=6.88x10^{-12}m

Exercise 7

1. Calculate the wavelength of the most energetic x-rays produced by the tube operating at 1x10^{5}V. [h=6.6x10^{-34}Js, e=1.6x10^{-19}C, C=3x10^{8}ms^{-1}] **An[1.24x10^{-11}m]**

2. The current in a water-cooled x-ray tube operating at 60kV is 30mA. 99% of the energy supplied to the tube is converted into heat at the target and removed by water flowing at a rate of $0.06kgs^{-1}$ calculate;
(a) the rate at which energy is being supplied to the tube.
(b) The increase in temperature of the cooling water [S.H.C =$4.2x10^3Jkg^{-1}℃^{-1}$]

An[$1.8x10^3Js^{-1}$, 7.1℃]

3. The most energetic x-rays produced by a particle x-ray tube have a wavelength of $2.1x10^{-11}$m. What is the operating $p.d$ of the tube. [h=$6.6x10^{-34}$Js, e=$1.6x10^{-19}C$, C=$3x10^8ms^{-1}$]

An[59kV]

4. An x-ray tube which is 1% efficient produces x-rays energy at a rate of $20Js^{-1}$. Calculate the current in the tube if the operating $p.d$ is 50kV **An[40mA]**

5. Explain how the radiation from an evacuated x-ray tube is affected by changing
(a) the filament current
(b) the filament target $p.d$
(c) the target material

6. State briefly how you would control electrically;
(a) the intensity
(b) the penetrating power of the emitted x-rays.

7. Electrons are accelerated from rest through a potential difference of 10kV in an x-ray tube calculate.
(i) the resultant energy of the electrons in eV
(ii) the wavelength of the associated electron waves
(iii) The maximum energy and the minimum wavelength of the x-ray radiation generated
[h=$6.6x10^{-34}$Js, e=$1.6x10^{-19}C$, C=$3x10^8ms^{-1}$, m=$9.11x10^{-31}$kg]

An[10000eV, $1.223x10^{-11}$m, $1.6x10^{-18}$J , $1.24x10^{-10}$m]

8. The $p.d$ between the target and cathode of an x-ray tube is 50kV and the current in the tube is 20mA only 1% of the total energy supplied is emitted as x-radiation.
(a) What is the minimum frequency of the emitted radiation

(b) At what rate must heat be removed from the target in order to keep it at a steady temperature. [h=6.6x10^{-34}Js, e=1.6x$10^{-19}C$] **An[1.2x10^{19}Hz, 9.9x10^{2}W]**

9. An x-ray tube works at $a\ d.c\ p.d$ of 50kV. Only 0.4% of the energy of the cathode rays is converted into x-radiation and heat is generated in the target at a rate of 600W. estimate;
(i) Current passed through the tube
(ii) velocity of the electrons striking the target [h=9x10^{-31}kg, e= -1.6x$10^{-19}C$]

An[12mA, 1.33x10^{8}ms^{-1}]

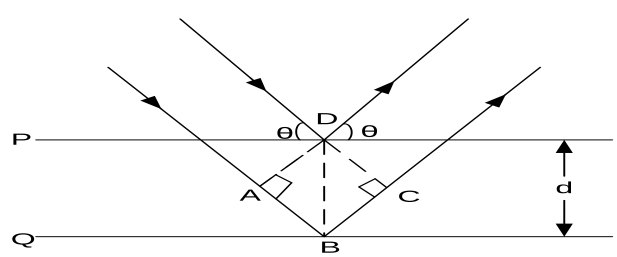
10. (a) A 900W x-ray tube operates at a $d.c\ p.d$ of 30kV. Calculate the minimum wavelength of the x-rays produced
(b) Calculate the current through the tube
(c) If 99% of the power is dissipated as heat estimate the number of x-ray photons produced per second. [h=6.6x10^{-34}Js, e=1.6x10^{-} 19c, C=3x10^{8}ms^{-1}J] **An[4.1x10^{-11}m, 30mA, 1.9x10^{15}s^{-1}]**

3.3.5: BRAGG'S LAW FOR X-RAY DIFFRACTION

It states that $2dsin\theta = n\lambda$ where d is interplanal separation, θ is glancing angle, λ is x-ray wavelength and n is an interger

DERIVATION OF BRAGG'S LAW FOR X-RAY DIFFRACTION

When x-rays are directed to a crystal each atomic plane of a crystal behaves like a reflecting surface.



- Constructive interference occurs when the path difference is **nλ**
  Where n is an integer and λ is wavelength the x-rays.

$$\therefore AB+BC=n\lambda$$

$$AB = BC = dsin\theta$$

$$dsin\theta + dsin\theta = n\lambda$$

$$2dsin\theta = n\lambda$$

generally

$$\boxed{2d\ sin\theta_n = n\lambda} \text{ Bragg's law}$$

where θ_n is glancing angle for the nth order maximum

Note

The small angle (θ_{min}) is given when n=1 and it's the first order maxima

n_{max} occurs when $Sin\theta$ =1

Example

1. A beam of x-rays of wavelength 0.15nm incident on the crystal. The smallest angle at which there is strongly reflected beam is 15^0. Calculate the distance between the successive layers between the crystal lattice.

   Solution

   $\lambda=0.15x10^{-9}$m, d=?

   for smallest angle n=1, θ_{min}=15

   from Bragg's law $2dsin\theta_n=n\ \lambda$

   $d=\frac{n\ \lambda}{2\sin\theta_n}$

   $d=\frac{1x0.15x10^{-9}}{2sin15}$

   $d=2.898x10^{-10}$m

2. A beam of x-rays of wavelength $8.42x10^{-11}$m is incident on a sodium chloride crystal of inter planal separation $2.82x10^{-10}$m. Calculate the first order diffraction angle.

   Solution

   $\lambda=8.42x10^{-11}$m, $d=2.82x10^{-10}$m, θ=? For first order diffraction n=1

   using Bragg's law $2dsin\theta_n= n\ \lambda$

   $\theta_1=\sin^{-1}\left(\frac{1x8.42x10^{-11}}{2x2.82x10^{-11}}\right)$

   $\theta_1=8.59^0$

3. A monochromatic x-ray beam of wavelength $1x10^{-10}$m is incident on a set of planes it a crystal of spacing $2.8x10^{-10}$m. What is the maximum order possible in these x-rays.

   Solution

   n_{max} occurs when $sin\theta$=1

   $2dsin\theta_n=n_{max}\ \lambda$

   $n_{max} = \frac{2x2.8x10^{-10}}{1x10^{-10}} = 5.6$

   $n_{max} \approx 6$ sixth order diffraction

4. A monochromatic beam of x-rays of wavelength $2x10^{-10}$m is incident on a set of cubic plane in a potassium chloride crystal. First order diffraction maxima are observed at a glancing angle of 18.5^0. find the density of potassium chloride. If its molecular weight is 74.55.

Solution

$\lambda=2x10^{-10}$m, n=1, $\theta=18.5^0$, d=?

∴m=74.55g, $m=74.55x10^{-3}$kg

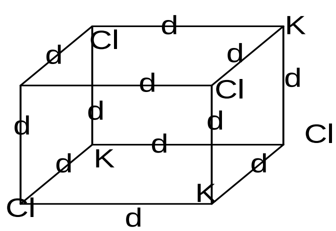
note that molecular weight is measured in grams unless given in kg

Using Bragg's law

$2d\sin\theta_n = n\lambda$

$d=\frac{1x2x10^{-10}}{2xsin18.5}$

$d=3.15x10^{-10}$m



its set of cubic planes

volume of one atoms $=d^3$

$=(3.15x10^{10})^3$

volume of one atoms $=3.13x10\text{-}^{29}m^3$

But one molecule of KCl has two atoms

∴ volume of the one molecule of KCl

$=2x3.13x10^{-29}$

$= 6.26x10^{-29}m^3$

Mass of one molecule of KCl =

$= \frac{molecular\ weight}{N_A}$

$=\frac{74.55x10^{-3}}{6.02x10^{23}}$

$=1.24x10^{-25}$kg

Density of one molecule of

$KCl=\frac{mass\ of\ one\ molecule}{volume\ of\ 1\ molecule}$

$\rho=\frac{1.24x10^{-25}}{6.26x10^{-24}}$

$\rho=1.984x10^3$ kgm^{-3}

5. Calculate the atomic spacing of sodium chloride if the relative atomic mass of sodium is 23 and that of chlorine is 35.5 [density of sodium chloride $=2.18x10^3kgm^{-3}$]

Solution

Mass of one mole = 23+35.5

=58.5g

Mass of one molecule of $NaCl$ = $\frac{58.5x10^{-3}}{N_A}$

$=\frac{58.5x10^{-5}}{6.02x10^{-23}}$

$= 9.718x10^{-26}$kg

Density of one molecule of $NaCl$

$=\frac{mass\ of\ one\ molecule\ of\ NaCl}{volume\ of\ 1\ molecule}$

$2.18x10^3= \frac{9.718x10^{-26}}{volume\ of\ 1\ molecule\ of\ NaCl}$

$volume\ of\ 1\ molecule\ of\ NaCl =\frac{9.718x10^{-26}}{2.18x10^3}$

$volume\ of\ 1\ molecule\ of\ NaCl$$=4.458x10^{-29}m^3$

$volume\ of\ 1\ molecule\ of\ NaCl$ has two atoms of Na and Cl

$volume\ of\ 1\ atom\ of\ either\ Na\ or\ Cl$

$=\frac{4.458x10^{-29}}{2}$

$=2.229x10^{-29}m^3$

∴ $d^3 = 2.229x10^{-29}m^3$

$d = (2.229x10^{-29})^{\frac{1}{3}}$

$d = 2.81x10^{-10}$m

EXCERCISE8

1. Calculate the smallest glancing angle at which x-rays of wave length $0.7x10^{-10}$m will be diffracted from a certain crystal which has inter-atomic separation of $1.5x10^{-10}$m. what is the highest diffraction order that can be observed from this radiation.
   An [13.5^0, $\approx$4, fourth order diffraction]

2. A beam of x-rays of frequency $3.56x10^{18}$ Hz is incident on potassium chloride (KCl) crystal and the first order Bragg reflection occurs at 7.68^0. The density of KCl is $1.98x10^3kgm^{-3}$ and its molecular mass is 74.5. Calculate the value of Avogadro's number. **An[$6.02x10^{23}mol^{-1}$]**

3.3.6:CONDITION FOR X-RAY DIFRACTION TO OCCUR

- Wave length of x-rays must be of the same order as the interplanar spacing.
- Parallel beam of x-rays must be incident on planes

3.3.7: DIFFERENCES BETWEEN CATHOD RAYS AND X-RAYS

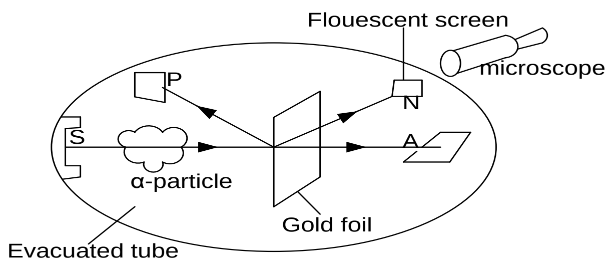
Cathode rays	X-rays
Are fast moving electrons	Are electromagnetic waves
They are negatively charged	They have no charge
Can be deflected by electric and magnetic fields	Can not be deflected by electric and magnetic fields
Have a low penetrating power	Have a high penetrating power
They produce x-rays on striking matter	They eject electrons from matter
They are relatively slower compared to x-rays	Move very fast at a speed of light

3.4.0: RUTHERFORD'S MODEL OF THE ATOM

An atom consists of a very small central core called the **nucleus** with strong electric charge surrounded by electrons of the opposite charge which fill the rest of the atom.

Rutherford's model states: that the positive charge of the atom and nearly all its mass is concentrated in a very small volume at the centre with electrons in motion in a circular orbit around the nucleus.

3.4.1: RUTHERFORD'S ALPHA PARTICLE SCATTERING EXPERIMENT



- Alpha particles from a radioactive source were allowed to strike a thin gold foil placed in the centre of an evacuated vessel and the scattering of alpha particles when they collide with the gold foil was observed from a fluorescent screen mounted on a focal plane of a microscope.
- Alpha particles produce tiny, but a visible flash of light when they strike a fluorescent screen.
- Surprisingly, alpha particles not only struck the screen at A but also at N and some were even found to be back scattered to P.
- The greatest flash was observed at position A.

Conclusion

From the experiment, it was observed that;

- **Most** of the alpha particle went through the gold foil **un deflected** . This is because the atom of the foil contains very tiny nuclei, most of the space of an atom is an empty space.
- **Few** alpha particles were scattered through angles **greater than** $90°$. This is because nucleus occupies a very small volume of the atom. Therefore very few alpha particles are incident close to the nucleus and alpha particles incident close to the nucleus are strongly repelled almost backwards.

Question: Explain what is observed when a beam of $\propto$-particles is incident on a gold foil.

Note

The experiment was done in a vacuum in order to avoid

- Deflection of ∝-particles by wind
- Absorption of ∝-particles by air which would lead to ionization of the air atoms

Structure of the atom

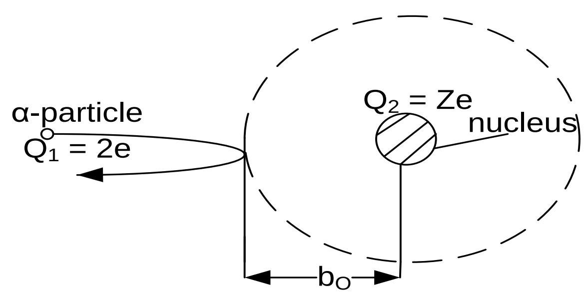
- It is electrically neutral
- The nucleus is at the centre of the atom and is positively charged
- The nucleus contains protons and neutrons
- Orbiting around the nucleus are electrons and are negatively charged
- The atom is largely an empty space
- The nuclear occupies only small volumes of the available space

3.4.2: Failure of Rutherford's model of the atom

(1) An orbiting electron is constantly changing it's direction and therefore has an acceleration. In classical physics charges undergoing acceleration emit electromagnetic radiation continuously and therefore they would loose energy. This implies that the electron would spiral towards the nucleus and the atom would collapse and cease to exist within a short time, yet the atom is a stable structure. Therefore Rutherford's model can not explain the stability of the atom.

(2) Since electrons are continuously accelerating around the nucleus, continuous emission spectra should be emitted by the atom. However experimental observations reveal that it is atomic like spectra which occur.

3.4.3: RUTHERFORD'S α –PARTICLE SCATTERING FORMULA



b_0 is distance of closest approach

Kinetic energy of alpha particle= $\frac{1}{2}mv^2$ where v is speed before collision

Electrostatic potential energy= $\frac{(2e)(Ze)}{4\pi\varepsilon_0 b_O}$

At closest distance of approach $\frac{1}{2}mv^2 = \frac{(2e)(Ze)}{4\pi\varepsilon_0 b_O}$

$$\frac{1}{2}mv^2 = \frac{Ze^2}{2\pi\varepsilon_0 b_O}$$

$$\boxed{\boldsymbol{b_0} = \frac{\boldsymbol{Ze^2}}{\pi\varepsilon_0 mv^2}}$$

OR $$\boxed{\mathbf{K.e} = \frac{\boldsymbol{Ze^2}}{2\pi\varepsilon_0 \mathbf{bo}}}$$

Example

1. A beam of 4.7MeV alpha particle is incident normally on a thin gold foil. What is the closest distance of approach of the alpha particle to the gold nucleus.
   (Atomic number of gold = 79). What is the significance of this result.

   Solution

   $K.e$ = 4.7MeV

   $K.e$ = $4.7 \times 10^{6} \times 1.6 \times 10^{-19}$J

   $K.e$ = 7.52×10^{-13}J

   $\boldsymbol{K.e} = \frac{\boldsymbol{Ze^2}}{2\pi\varepsilon_0 \mathbf{bo}}$

   $\boldsymbol{b_0} = \frac{\boldsymbol{Ze^2}}{2\pi\varepsilon_0 \boldsymbol{K.e}}$

   $\boldsymbol{b_0} = \frac{79x(1.6x10^{-19})^2}{2x\frac{22}{7}x8.85x10^{-12}x7.52x10^{-13}}$

   $\boldsymbol{b_0}$ = 4.84×10^{-14}m

   The distance of closest approach is an estimate of the radius of the nucleus.

2. In a head on collision between an alpha particle and a gold nucleus, the minimum distance of approach is 5×10^{-14}m. Calculate the energy of the alpha particle in (MeV)
   (Atomic number of gold = 79)

   Solution

   Z = 79

   $bo = 5 \times 10^{-14}$

   £o = 8.85×10^{-12}

   $\boldsymbol{K.e} = \frac{\boldsymbol{Ze^2}}{2\pi\varepsilon_0 \mathbf{bo}}$

   $$K.e = \frac{79x(1.6x10^{-19})^2}{2x\frac{22}{7}x8.85x10^{-12}x5x10^{-14}}$$

   K.e = 7.274×10^{-13}J

   $K.e = \frac{7.274x10^{-14}}{10^6 x1.6x10^{-19}}$

   $K.e$ = 4.55MeV

Exercise9

1. An alpha particle with kinetic energy of 5MeV is in a head on collision with an atom of a gold foil (it is deflected through 180°). If the atomic number of gold is 79. Calculate the distance of closest approach of alpha particles to the nuclear centre of the atom.

   An (4.55×10^{-14}m)

3.5.0:BOHR'S THEORY OF HYDROGEN ATOM

Definition

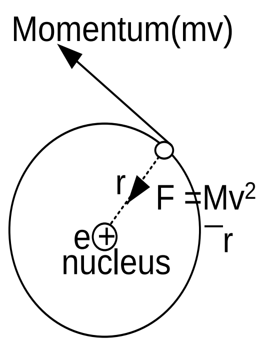
Abohr atom is one with a small central positive nucleus with electrons revolving around it in only certain allowed circular orbits and while in these orbits they do not emit radiations.

3.5.1:POSTULATES OF BOHR

Bohr made the following assumption

(1) In allowed circular orbits, the angular momentum is a multiple of $\frac{h}{2\pi}$ where h is Planck's constant.
(2) When electrons are orbiting in this allowed orbits they do not emit radiations
(3) Electromagnetic radiation is emitted when the electron makes a transition between orbits
(4) In those orbits where the angular momentum is a multiple of $\frac{h}{2\pi}$ the energy is constant

3.5.2: EXPRESSION FOR TOTAL ENERGY



From Bohr's assumption $mvr = \frac{nh}{2\pi}$

$$V = \frac{nh}{2\pi mr} \text{-------------- } [1]$$

From circular motion

Force on electron $\frac{mV^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$

$mV^2 = \frac{e^2}{4\pi\varepsilon_0 r}$......................2

Put equation(1) into equation(2)

$m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{e^2}{4\pi\varepsilon_0 r}$

$m\frac{n^2h^2}{m^2r^2 4\pi^2} = \frac{e^2}{4\pi\varepsilon_0 r}$

$r = \frac{n^2h^2\varepsilon o}{\pi m e^2}$.....................3

Also from equation (2) $mV^2 = \frac{e^2}{4\pi\varepsilon_0 r}$

Multiplying both sides by ½

$\frac{1}{2}mV^2 = \frac{e^2}{8\pi\varepsilon_0 r}$

$k.e = \frac{e^2}{8\pi\varepsilon_0 r}$

Also $p.e = \frac{e}{4\pi\varepsilon_0 r} x - e$

Total energy $E = K.e + P.e$

$= \frac{e^2}{8\pi\varepsilon_0 r} + \frac{e}{4\pi\varepsilon_0 r} x - e$

$$E = \frac{-e^2}{8\pi\varepsilon_0 r}$$

Putting value of r in equation (3)

$$E = \frac{-e^2}{8\pi\varepsilon o\left(\frac{n^2h^2\varepsilon_0}{\pi m e^2}\right)}$$

$$E = \frac{-e^4 m}{8n^2h^2\varepsilon_0{}^2}$$

Where n is quantum number

h is Planck constant

ε_0 permittivity of free space

M is mass of the electron

e is charge of electron

Note

- Total energy of electron is negative because electrons are bound to the nucleus of the atom and work must be done to remove the electrons from the atom. This work is done against nuclear attraction bending electrons to the atoms.
- Increasing values of r are associated with increasing values of n and therefore with increasing values of E (less negative).

3.5.3: EMMISSION LINE SPECTRA

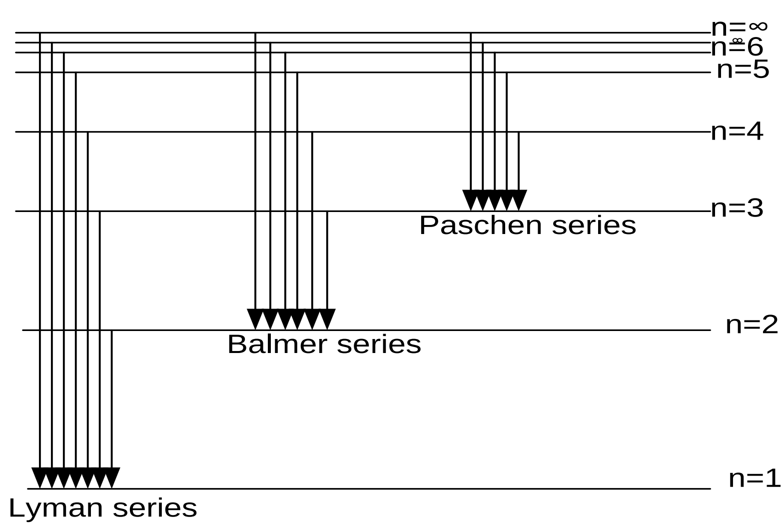
- When a gas at low pressure is heated to a very high temperature atoms are excited and jump to high energy levels. When electrons fall back to lower energy levels and emit radiations of definite wavelength.
- Since the frequency is definite for a particular element, then it implies that the energy levels are discrete (quantized). The frequency of the line can also be used to uniquely identify the element. The line formed in the spectrum appears bright against a dark background.

Question

1. Explain how line spectra can be used to account for the existence of discrete energy levels in an atom.

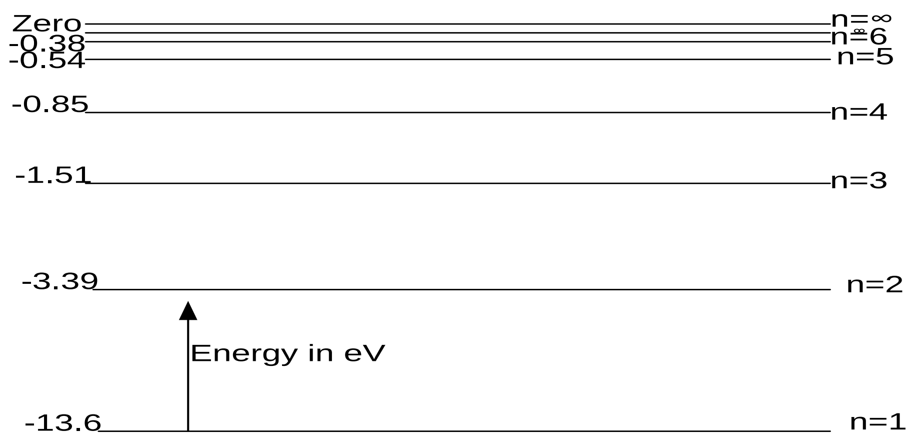
Answer is all the sentences above

3.5.4: MAIN SPECTRAL TRANSITION OF ATOMIC HYDROGEN



The spectrum of atomic hydrogen contains distinct groups of lines. The three most obvious groups are the $\boldsymbol{lyman}$ series, the $\boldsymbol{Balmer}$ series and the $\boldsymbol{paschen}$ series. The wavelength of the lines in the lyman series are in the ultraviolet and each is associated with a transition involving the level with n = 1. The Balmer series involves transitions to the level with n= 2 and as a result smaller energy differences are involved and the wavelength are in the visible. The lines of the $\boldsymbol{paschen}$ series are in the infrared.

3.5.5: ENERGY LEVELS IN HYDROGEN ATOM



- The lowest level with n = 1 is called the **ground state**. The electron will always occupy this lowest level unless it absorbs energy. Ground state is also the lowest energy state for the atom.
- When the atom absorbs energy is some way, the electron may be promoted into one of the higher energy levels, the atom becomes unstable and it is said to be in **Excited state**.
- The top level with n = ∞ is the ionization state. An electron raised to this level will be removed from the atom.

Note

All levels have negative energy values because the energy of an electron at rest outside the atom is taken as zero (eV) and work has to be done to remove the electron to infinity.

Definition

An electron volt **($\boldsymbol{eV}$)** is the kinetic energy gained by electron in being accelerated through a potential difference of one volt.

3.5.6: IONISATION AND EXCITATION POTENTIAL

(1) **Ionization energy** of an atom is the minimum amount of energy required to remove it's most loosely bound electron when the atom is in it's ground state.

Ionization energy of hydrogen = $E_\infty - E_0$

= $0 - -13.6$

= 13.6eV

It follows from definition of an eV that 13.6eV is the kinetic energy gained by an electron in being accelerated through a $p.d$ of 13.6V thus the ionization potential of hydrogen is 13.6V.

(2) **Excitation energy** of an atom is the energy required to an electron from an atom which is in it's ground state to higher energy level.

For example the first and second excitation energies of hydrogen are 10.2eV and 12.1eV respectively. The corresponding excitation potentials are 10.2V and 12.1V.

Note

If the energy absorbed is more than that for ionization then the rest appears as kinetic energy of the electrons from which it's velocity can be calculated.

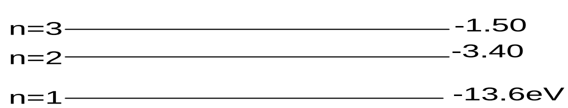
Examples

1. If heat energy absorbed by a hydrogen atom is 1.5eV. Calculate the energy of the excited electron given that ionization energy of hydrogen is 13.6eV.

$K.e$ of electron = 15 – 13.6

$\frac{1}{2}mv^2$ = 1.4eV

2.



Calculate; i) first ionization energy

ii) second ionization energy

iii) state the corresponding excitation potentials

Solution

1^{st} ionization energy $= -3.40 - -13.6$

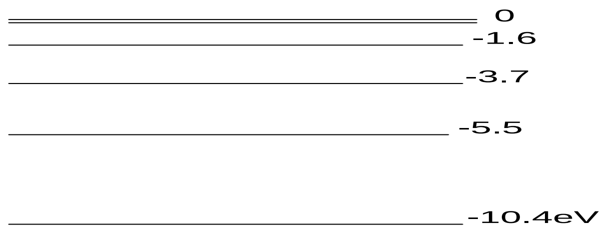
= 10.2eV

2^{nd} ionization energy $= -1.50 - -13.6$

= 12.1eV

1^{st} and 2^{nd} excitation potentials 10.2V and 10.4V

3.



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The diagram shows energy levels of mercury

(i) What is the ionization energy and the corresponding ionization potential, if mercury vapour atom is in the ground state.

(ii) If mercury vapour atom in aground state has collision with an electron of energy 5eV. How much energy might be retained by electrons in this case.

Solution

i) Ionization energy $= 0 - -10.4$

= 10.4eV

ii) first ionization energy = -5.5 - -10.4

= 4.9eV

Since 5eV is more than 4.9eV the electron retains = 5 – 4.9

= 0.1eV

4. The energy levels in a mercury atom are -10.4eV, -5.5eV, -3.7eV and -1.6eV.

(i) Find the ionization energy in Joules

(ii) What is likely to happen if a mercury atom in an unexcited state is bombarded with an electron of energy 4.0eV, 6.7eV or 11.0e.V

Solution

ionization energy $= 0 - -10.4$

= 10.4eV

= 10.4 x 1.6 x 10^{-19}J

Ionization energy = 1.664 x 10^{-18}J

1^{st} ionization energy $= -5.5 - -10.4$

= 4.9eV

- ❖ Since 4.0eV is less than 4.9eV, the atom remain unexcited.

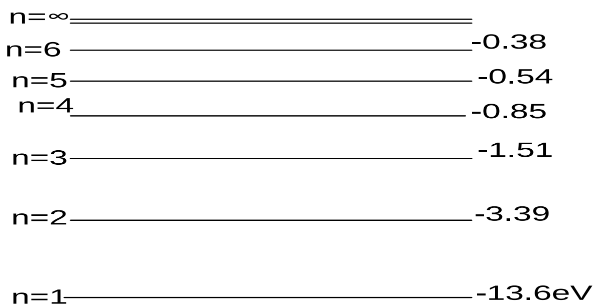
2^{nd} ionization energy = -3.7 - -10.4

= 6.7eV

- ❖ So an electron of 6.7eV excites the atom such that an electron jumps from the ground state to energy level -3.7eV.

- For a electron of 11eV, it will cause ionization because its value is greater than that at ground state $i.e$ 10eV.

5.



Calculate the frequency and wavelength of radiations resulting from the following transitions
a) n = 4 to n = 2 b) n = 2 to n = 1 [$h = 6.6 \times 10^{-34}$J, $c = 3 \times 10^{8}ms^{-1}$]

Solution

(a)

$hf = E_4 - E_2$

$hf = -0.85 - -3.39$

hf = 2.54eV

$f = \frac{2.54x\ 1.6 \text{ x } 10^{-19}}{6.6x10^{-34}}$

$f = 6.16 \times 10^{14}$Hz

$\lambda = \frac{C}{f}$

$\lambda = \frac{3x10^8}{6.16x10^{14}}$

$\lambda = 4.87 \times 10^{-7}$m

(b)

$hf = E_2 - E_1$

$hf = -3.39 - -13.6$

hf = $10.21eV$

$f = \frac{10.21x\ 1.6 \text{ x } 10^{-19}}{6.6x10^{-34}}$

$f = 2.48 \times 10^{15}$Hz

$\lambda = \frac{C}{f}$

$\lambda = \frac{3x10^8}{2.48x10^{15}}$

$\lambda = 1.21 \times 10^{-7}$m

6. An electron of energy 20eV comes into collision with the hydrogen atom in it's ground state, the atom is excited into a state of higher state of internal energy and electrons is scattered with a reduced velocity, the atom subsequently returns to it's ground state with emission of a photon of wavelength 1.216×10^{-7}m. Determine the velocity of the scattered electron ($e = 1.6 \times 10^{-19}$C, $h = 6.6 \times 10^{-34}$J, $m = 9.1 \times 10^{-31}$kg, $c = 3 \times 10^{8}ms^{-1}$)

Solution

E = hf

$E = h\frac{C}{\lambda}$

$E = \frac{6.6x10^{-34}x3x10^8}{1.216x10^{-7}}$

$E = 1.63 \times 10^{-18}$J

K.E of scattered electron $= 20eV - 1.63 \times 10^{-18}$

$= 20 \times 1.6 \times 10^{-19} - 1.63 \times 10^{-18}$

$= 1.57 \times 10^{-18}$J

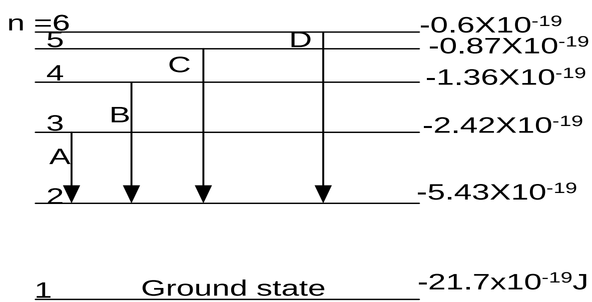
$\frac{1}{2}mV^2 = 1.57 \times 10^{-18}$J

$V = \sqrt{\frac{2x1.57x10^{-18}}{9.1x10^{-31}}}$ | $V = 1.86 \times 10^{6} ms^{-1}$

Exercise9

1. The ionization potential of the hydrogen atom is 13.6V. Use the data below to calculate
   (a) The speed of an electron which could just ionize the hydrogen atom.
   (b) The minimum wavelength which the hydrogen atom can emit
   (charge on an electron = 1.6×10^{-19}, $M = 9.11 \times 10^{-31}$kg, $h = 6.63 \times 10^{-34}$Js $C = 3 \times 10^{8}ms^{-1}$)
   An (2.19×10^{6}ms-1, 9.14×10^{-8}m)

2. The figure below representing the lowest energy level of the electron in the hydrogen atom, gives the principle quantum number n associated with each and the corresponding value of the energy measured in Joules.



i) Calculate the wavelength of the lines arising from the transition marked A, B, C, D on the figure.

ii) The level n = 1 is the ground state of the un excited hydrogen atom. Explain why hydrogen in it's ground state is quite transparent to light emitted by the transitions A, B, C, D and also what happens when 21.7×10^{-19}J of energy is supplied to a hydrogen atom in it's ground state.

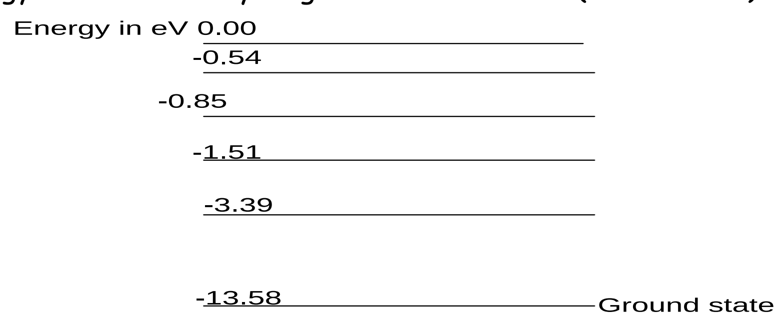
(take the value of the speed of light in vacuum C to be $3 \times 10^{8}ms^{-1}$ and that of the Planck constant h to be 6.63×10^{-34}Js). **An (661nm, 489nm, 436nm, 412nm)**

3. The ionization energy for a hydrogen atom is 13.6eV, if the atom is in it's ground state. It is 3.4eV if the atom is in the first excited state. Explain he terms ionization energy and excited state.
   Calculate the wavelength of the photon emitted when a hydrogen atom returns to the ground state from the first excited state. Name the part of the electromagnetic spectrum to which this wavelength belong.($e = -1.6 \times 10^{-19}$C, $h = 6.63 \times 10^{-34}$Js, $C = 3 \times 10^{8}ms^{-1}$)

4. The energy levels of the hydrogen atom are given by the expression
   En = $\frac{-2.16x10^{-18}}{n^2}$
   Where n is an integer.
   (a) What is the ionization energy of the atom
   (b) What is the wavelength of the Hα line which arises from transition between n = 3 and n = 2 level. (h = 6.6 x 10^{-34}Js, c = 3 x 10^{9}ms^{-1}) **An (2.16 x 10^{-18}J, 6.6 x 10^{-7}m)**

5. The lowest energy level in a helium atom (the ground state) is -24.6eV. There are a number of other energy levels, one of which is at -21.4eV.
   (a) -Define an eV
   (b) – (i) Explain the significance of the negative signs in the values quoted.
   (ii) What is the energy, in J, of a photon emitted when an electron return to the ground state from the energy level at -21.4eV?
   (iii)Calculate the wavelength of the radiation emitted in this transition.
   The electronic charge e = 1.6 x 10^{-19}C. The speed of electromagnetic radiation C = 3 x 10^{8}ms^{-1}. The Planck's constant h = 6.6 x 10^{-34}Js. **An (5.1 x 10^{-19}J 3.9 x 10^{-7}m)**

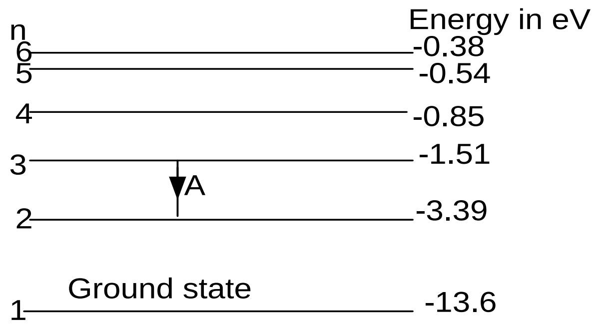
6. Some of the energy levels of the hydrogen atom are shown (not to scale) in the diagram.



   (a) Why are the energy levels labeled with negative energies
   (b) State which transition will result in the emission of radiation of wavelength 487nm. Justify your answer by calculation.
   (c) What is likely to happen to a beam of photons of energy (i) 12.07eV (ii) 5.25eV when passed through a vapour of atomic hydrogen

7. The diagram below represents the lowest energy levels of the electron in the hydrogen atom, giving the principal quantum number n associated with each level and the corresponding values of the energy.

(i) Why are the energies quoted with negative values

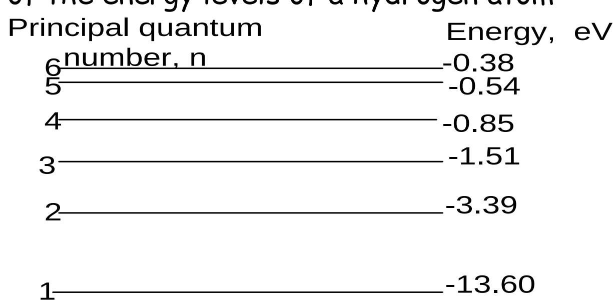


(ii) Calculate the wavelength of the line arising from the transition A, indicating in which region of the electromagnetic spectrum this occurs.

(iii) What happens when 13.6eV of energy is absorbed by a hydrogen atom in it's ground state **An (6.6×10^{-7}m)**

UNEB 2013 Q.9

(a) Figure shows some of the energy levels of a hydrogen atom



(i) Why are the energies for the different levels negative (01marks)

(ii) Calculate the wavelength of the line arising from a transition from the third to the second energy level **An (6.6×10^{-7}m)** (03marks)

(iii) Calculate the ionization energy in joules of hydrogen **An ($2.176x10^{-18}J$)** (02marks)

(b) Explain the physical process in an X-ray tube that accounts for

(i) Cut off wavelength (03marks)

(ii) Characteristic line (04marks)

(c) Calculate the maximum frequency of radiation emitted by an X-ray tube using an accelerating voltage of 33.0kV **An ($8x10^{18}Hz$)** (03marks)

(d) Derive Bragg's law of X-ray diffraction in crystal (04marks)

UNEB 2013 Q.10

(a) A beam of $\alpha - particles$ directed normally to a thin metal foil. Explain why

(i) Most of the $\alpha - particles$ passed straight through the foil (02marks)

(ii) Few $\alpha - particles$ are deflected through angles more than $90°$ (02marks)

(b) Calculate the least distance of approach of **a** $\mathbf{3.5 MeV}$ $\alpha - part\square cles$ to the nucleus of a gold atom (atomic number of gold = 79) **An($\mathbf{6.495x10^{-14}m}$)** (04marks)

UNEB 2012 Q.8

(c) State the laws of photo electric emission (04marks)

(d) Explain how line emission spectra are produced (03marks)

UNEB 2011 Q.9

a) (i) Explain how X-rays are produced in an X-ray tube

(ii) Explain the emission of X-ray characteristic spectra (03marks)

(iii) Derive the Bragg X-ray diffraction equation (04marks)

(iv) Under what conditions does X-ray diffraction occur (02marks)

UNEB 2010 Q.10

c) (i) show that when an alpha particle collides head on with an atom of atomic number. The closest distance of approach to the nucleus, $\mathbf{Z_0}$ is given by $\mathbf{Z_0} = \frac{Ze^2}{\pi\varepsilon_0 mv^2}$

Where e is the electronic charge $\boldsymbol{\varepsilon_0}$ is the permittivity of free space, m is the mass of the alpha particle and V is the initial speed of the alpha particle (04marks)

UNEB 2010 Q.9

(c) An X-ray of wavelength 10^{-10}m is required for the study of it's diffraction in a crystal. Find the least accelerating voltage to be applied to an X-ray tube in order to produce these X-rays.

(04marks)

(d) Sodium has a work function of 2.0eV and is illuminated by radiation of wavelength 150nm. Calculate the maximum speed of the emitted electrons **An(1.24 x $\mathbf{10^4}$V)**

(04marks)

(e) With aid of a labeled diagram describe how stopping potential of metal can be measured.

UNEB 2009 Q.9d

(d) Distinguish between continuous and line spectra in an X-ray tube.

UNEB 2009 Q.10

(a) (i) Explain the observations made in the Rutherford's alpha particle scattering experiment

(06marks)

(ii) Why is a vacuum necessary in this experiment (01mark)

(b) Distinguish between excitation and ionization energies of an atom (02marks)

(c) Draw a labeled diagram showing the main components of an X- ray tube. (03marks)

(d) An X-ray tube is operated at 50kV and 20mA. If 1% of the total energy supplied is emitted as X-radiation, calculate the;

(i) Maximum frequency of the emitted radiation (3mk)

(ii) Rate at which heat must be removed from the target in order to keep it at a steady temperature

(03marks)

(e) A beam of X-rays of wavelength 0.20nm is incident on a crystal at a glancing angle of 30°. If the inter planar separation is 0.20nm, find the order of diffraction.

(An (1.21 x 10^{19}Hz, 990W, n = 1 (first order diffraction))

UNEB 2008 Q.8

(a) What is meant by a line spectrum (02marks)

(b) Explain how line spectra accounts for the existence of discrete energy level in atoms (4mk)

(d) Describe with aid of a labeled diagram, the action of an X-ray tube

(e) An X-ray tube is operated at 20kV with an electron current of 16mA in the tube estimate the;

(i) Number of electrons hitting the target per second (02marks)

(ii) Rate of production of heat, assuming 99.5% of the kinetic energy of electrons is converted to heat (e = 1.6 x 10^{-19}C) **An (1.0 x 10^{17} electron per second, 318.4W**) 02mks)

UNEB 2007 Q.10

(c) Explain X-ray diffraction by crystals and derive Bragg's law (06marks)

(d) The $p.d$ between the cathode and the anode of an X-ray tube is 5 x 10^{-4}V. If only 0.4% of the kinetic energy of the electrons is converted into X-rays and the rest is dissipated as heat in the target at a rate of 600W. Find the;

(i) Current that flows

(03marks)

(ii) Speed of the electrons striking the target (03marks)

An(1.21×10^6A, $1.33 \times 10^4 ms^{-1}$)

UNEB 2006 Q.8

(a) (i) What is photon (01marks)

(ii) Explain, using quantum theory, the experimental observations on the photoelectric effect (06marks)

(iii) When light of wavelength 450nm falls on a certain metal, electrons of maximum kinetic energy 0.76eV are emitted. Find the threshold frequency for the metal. (04marks)

An (4.83×10^{14}Hz)

(b) Explain, using suitable sketch graphs, how X-ray spectra in an X-ray tube are formed (6mks)

(c) A beam of x-rays of wavelength 8.42×10^{-11}m is incident on a sodium chloride crystal of inter planal separation 2.82×10^{-10}m. Calculate the first order diffraction angle (03marks)

An ($\theta = 8.6^o$)

UNEB 2005 Q.8

(a) (i) Draw a labeled diagram of an X-ray tube (02marks)

(ii) Use the diagram in (a) (i) to describe how X-rays are produced. (03marks)

(iii) State one industrial and one biological use of X-rays. (01marks)

(b) (i) Sketch a graph of intensity versus wavelength of X-rays from an X-ray tube and describe it's main features. (04marks)

(ii) Calculate the maximum frequency of X-rays emitted by an X-ray tube operating a voltage of 34kV **An (8.34×10^{18} Hz)** (03marks)

UNEB 2005 Q.9

(a) (i)State the laws of photo electric emission (04marks)

(ii) Write down Einstein's equation for photoelectric emission (02marks)

(iii) Ultra-violet light of wavelength 3.3×10^{-8}m is incident on a metal. Given that the work function of the metal is 3.5eV, calculate the maximum velocity of the liberated electron

An ($3.46 \times 10^6 ms^{-1}$)

(03marks)

UNEB 2004 Q.9

(a) Explain the term stopping potential are applied to photoelectric effect.

(b) Explain how intensity and penetrating power of X-rays from an X-ray tube would be affected by changing

(i) the filament current (02marks)

(ii) the high tension potential difference across the tube (02marks)

(c) When a $p.d$ of 60kV is applied across an X-ray tube a current of 30mA flows. The anode is cooled by water flowing at a rate of 0.06kgs^{-1}. If 99% of the power supplied is converted into heat at the anode, calculate the rate at which the temperature of the water rises.

(S.H.C = 4.2 x 10^{3}Jkg^{-1}k^{-1}) **An (7.07K)**

(05marks)

(d) (i) Derive Bragg's law of X-ray diffraction (05marks)

UNEB 2003 Q.9

(b) (i) What features of an X-ray tube make it suitable for continuous production of X-rays

(03marks)

(ii) Sketch a graph of intensity versus frequency of a radiation produced in an X-ray tube and explains it's features (05marks)

(c) A mono chromatic X-ray beam of wavelength 1 x 10^{-10}m is incident on a set of planes in a crystal of spacing 2.8 x 10^{-10}m. What is the maximum order possible with these X-rays

An (6)

(04marks)

UNEB 2003 Q.8

(a) (i) State Rutherford's model of the atom (02marks)

(ii) Explain two main failures of Rutherford's model of the atom (03marks)

UNEB 2002 Q.8

(a) What is meant by

(i) Bohr atom (01marks)

(ii) Binding energy of a nucleus (02marks)

(b) The total energy E of an electron in an atom may be expressed as

E = $\frac{-mq^4}{8\mathrm{n}^2\mathrm{h}^2\varepsilon o^2}$

(i) Identify the quantities m, q, n and h in this expression (02marks)

(ii) Explain the physical implication of the fact that E is always negative. (02marks)

(iii) Draw an energy level diagram for hydrogen to indicate emission of ultraviolet visible and infra-red spectral lines (03marks)

(d) The atomic nucleus may be considered to be a sphere of positive charge with a diameter very much less than of the atom. Discuss the experiment evidence which supports this view.

(03marks)

UNEB 2002 Q.9

(b) (i) Describe a simple experiment to demonstrate photo electric emission (04marks)

(ii) Explain why the wave theory of light falls to account for the photoelectric effect (6mk)

(iii) Describe an experiment to verify Einstein's equation for the photoelectric effect and explain how plank's constant may be obtained from the experiment (06marks)

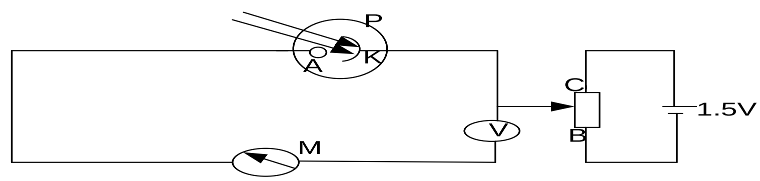
UNEB 2001 Q.8

(a) (i) Write down the Einstein photoelectric equation

(ii) Explain how the equation in (i) above accounts for the emission of electrons from metal surface illuminated by radiation

(04marks)

(b)



P is a vacuum photo cell with anode A and cathode K, made from the same metal of work function 2eV. The cathode is illuminated by monochromatic light of constant intensity and of wavelength 4.4×10^{-7}m.

(i) Describe and explain how the current shown by the micro ammeter M will vary as the slider of the potential divider is moved from B to C. (03marks).

(ii) What will the reading of the high resistance voltmeter V be when photo-electric emission just ceases?

(03marks)

(c) With the slider set mid-way between B and C, describe and explain how the reading of M would change if;

(i) The intensity of the light was increased

(03marks)

(ii) the wavelength of the light was changed to 5.5×10^{-7}m (06marks)

Solution

b)(i) As the slider moves from B to C, the cathode will become more positive. Hence more of the photo electrons that are emitted by the cathode are attracted by it. This causes a reduction in the number of the photo electrons reading the anode and hence the photo electric current that is measured by the micro-ammeter M reduces as the slider moves towards C.

ii) When photo electric emission ceases, it gives stopping potential (Vs)

$hf = W + \frac{1}{2}mv^2$

$hf = Wo + eVs$

Vs = $\frac{hf - w_0}{e}$

Vs = $\frac{\frac{6.6x10^{-34}x3x10^{8}}{4.4x10^{-7}} - 2x1.6x10^{-19}}{1.6x10^{-19}}$

Vs = 0.8125V

c)(i) with the slider mid way between B and C the $p.d$ V = $\frac{1.5}{2}$ = 0.75V which is less than the stopping potential of 0.8125V. Since increasing the intensity leads to an increase in the number of photo electrons being emitted per second then it implies that the micro-ammeter reading would increase with increasing intensity.

ii)

$W_0 = hf_0$

$W_0 = h\frac{C}{\lambda_{max}}$

λ_{max} x = $\frac{hc}{w_0}$

λ_{max} = $\frac{6.6x10^{-34}x3x10^{8}}{2x1.6x10^{-19}}$

λ_{max} = 6.19x10^{7}m

Since the wavelength of 5.5 x 10^{-7}m is less than λ_{max} which is the maximum wavelength of the incident radiation that would cause photoelectric emission then it means that there will be photo electric emission. But since the new wavelength less than that of the previous radiation then the kinetic energy of the photo electrons will be less than before. However if the intensity is maintained, the rate of emission of the photo electrons is the same and consequently the reading of M is unaltered.

UNEB 2000 Q.8

(a) State the laws of photo electric emission (04marks)

(b) (i) Describe an experiment to determine Planck's constant (05marks)

(ii) Violet light of wavelength 0.4μm is incident on a metal surface of threshold wavelength 0.65μm. Find the maximum speed of the emitted electrons (04marks)

(iii) Explain why light whose frequency is less than the threshold frequency can't cause photo emission (02marks)

(c) (i) What are X-rays (01marks)

(ii) Explain how intensity and penetrating power of X-rays produced by an X-ray tube can be varied.

CHAPTER4: NUCLEAR STRUCTURE

The nucleus is the central positively charged part of an atom.
Nuclei contain protons and neutrons which are collectively referred to as **nucleons** (nuclear number).

4.1.0: ATOMIC NUMBER Z, MASS NUMBER A AND ISOTOPES

Atomic number Z of an element is the number of protons in the nucleus of an atom of the element.

Mass number A of an atom is the number of nucleons in its nucleus.

Isotopes are atoms of the same element which have the same number of protons but different number of neutrons and therefore different mass numbers.

Isotopy: it is the existence of atoms of the same element with the same atomic number but different mass numbers.
Isotopes of an element whose chemical symbol is represented by X can be distinguished by using the symbol $^{A}_{Z}X$
Where **A** is mass number and **Z** is atomic number

Example of isotopes

Isotopes of Lithium $^{7}_{3}\boldsymbol{Li}$ and $^{6}_{3}\boldsymbol{Li}$

Isotopes of uranium $^{235}_{92}\boldsymbol{U}$ and $^{238}_{92}\boldsymbol{U}$

Isotones are nuclei with the same number of neutrons

Isobars are nuclei with the same number of nucleons.

4.1.1: EINSTEIN'S MASS – ENERGY RELATION

Einstein showed from his theory of relativity that mass (m) and energy (E) can be changed from one form to another.

The energy Δ**E** produced by a change of mass Δ**M** is given by the relation.

$$\boxed{\Delta \mathbf{E} = \Delta \boldsymbol{MC^2}}$$

Where *C* is the speed of light (*C*= $3 \times 10^{8}ms^{-1}$)

Energy produced in change of mass of 1kg

$\Delta E = 1 \times (3 \times 10^{8})^{2}$

$\Delta E = 9 \times 10^{16}J$

4.1.2: UNIFIED ATOMIC MASS UNIT [U]

It is defined as $^{1}/_{12}$ of the mass of carbon–12 atom.

The number of molecules in 1mole of carbon 12 is $6.02 \times 10^{23}mol^{-1}$

6.02×10^{23} atoms has a mass of 12g of carbon -12

6.02×10^{23} atoms = $12 \times 10^{-3}kg$

1atom = $\frac{12 \times 10^{-3}}{6.02 X 10^{23}}$

1atom = $1.993355482 \times 10^{-26}kg$

1unified atomic mass = $^{1}/_{12} \times 1.993355482 \times 10^{-26}$

=$1.661129568 \times 10^{-27}kg$

$$\boxed{1\text{U} = 1.66 \text{ x } 10^{-27}\text{kg}}$$

From Einstein's mass – energy relation

ΔE = MC^2

C = 2.998×10^{8}mls

1U = $1.661129568 \times 10^{-27} \times (2.998 \times 10^{8})^{2}$

1U = $1.49302392 \times 10^{-10}J$

1eV = $1.602 \times 10^{-19}J$

1U = $\frac{1.49302392 \times 10^{-10}}{1.602 x 10^{-19}} eV$

1U = 931.97×10^{6}eV

$\boxed{1U = 931MeV}$

4.1.3: MASS DEFECT AND BINDING ENERGY

a) MASS DEFECT

- It is defined as the mass equivalence of the energy required to split the nucleus into its constituent particles.

  The mass of a nucleus is always less than the total mass of its constituent nucleus.

  The difference in mass is called the mass defect of the nucleus $i.e$

$$\boxed{\boldsymbol{Mass\ defect = (mass\ of\ nucleons\ and\ elctrons) - (mass\ of\ atom)}}$$

Note

The reduction in mass arises because the act of combining the nucleons to form the nucleus causes some of their mass to be released as energy (in form of γ-rays).

Any attempt to separate the nucleons would involve them being given this same amount of energy; it is therefore called the **binding energy** of the nucleus.

b) BINDING ENERGY (B.E)

- Binding energy of the **nucleus** is the energy required to break up the nucleus into it's constituent nucleons
- Bind energy per nucleon is the ratio of the energy needed to split a nucleus into it's constituent nucleons to the mass number.

$$\boxed{\mathbf{B.E\ per\ nucleon} = \frac{\boldsymbol{B\ E}}{\mathbf{Mass\ number}}}$$

Binding energy per nucleon is very useful in measure of the stability of the nucleus. The higher the binding energy per nucleon the more stable the nucleus is.

Binding energy (J) = mass defect (kg) x C^2 (ms^{-1})^2

Where 1U = 1.66 X 10^{-27}kg

OR

Binding energy (MeV) = mass defect (U) x 9.31 (MeV)

Where 1U = 931MeV

Example

1. Given atomic mass of $^{238}_{92}U$ = 238.05076U

$mass\ of\ neutron\ =\ 1.00867U$

$mass\ of\ proton\ =\ 1.00728U$

$mass\ of\ electron\ =\ 0.00055U$

1U = 931MeV

Find; a) mass defect b) B.E per nucleon for $^{238}_{92}\boldsymbol{U}$

Solution

a) Mass defect = (mass of nucleons and electrons) – (mass of nucleus)

number of protons = 92

number of electrons = 92

number of neutrons = (238 – 92)

=146

$$\text{Mass defect} = \begin{pmatrix} 146x1.00867 \\ + \\ 92x1.00728 \\ + \\ 92x0.00055 \end{pmatrix} - (238.05076)$$

= 239.98618 - 38.05076

Mass defect = 1.93542U

b) **B.E per nucleon** = $\frac{\boldsymbol{B\ E}}{\textbf{Mass number}}$

B.E = mass defect x 931

= 1.93542 x 931

= 1801.87602mev

B.E per nucleon = $\frac{1801.87602}{238}$

B.E per nucleon= 7.571MeV

2. Given mass of proton = 1.0080U

   Mass of neutron = 1.0087U

   Mass of alpha particle = 4.0026U

   1U = 931MeV

   Find:

a) mass defect in (i) U (ii) kg
b) Binding energy in (i) MeV (ii) J
c) Binding energy per nucleon in (i) MeV (ii) J

Solution

An alpha particle is a helium nuclei $^{4}_{2}\boldsymbol{He}$

a) Mass defect = (mass of nucleons) - (mass of a tom)

number of protons = 2

number of neutrons = 2

i) mass defect = (2 x 1.0080 + 2 x 1.0087) – 4.006
= 0.0308U

ii) mass defect in kg

1U = 1.66×10^{-27}kg

Mass defect = $0.0308 \times 1.66 \times 10^{-27}$kg
5.1128×10^{-29}kg

b)(i) Binding energy (MeV) = mass defect x 931MeV
= 0.00308 x 931
=28.6748MeV

ii) Binding energy (J) = $28.6748 \times 10^{6} \times 1.6 \times 10^{-19}$J
= 4.59×10^{-12}J

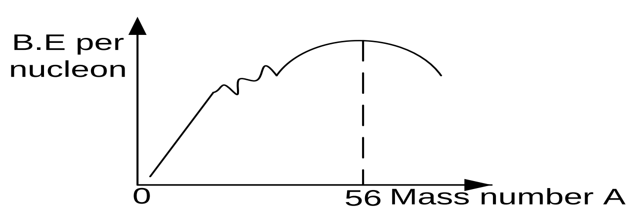
Or Binding energy (J)= mass defect (kg) xC^2(ms^{-1})^2
= $5.1128 \times 10^{-29} \times (3x10^8)^2$
= 4.60×10^{-12}J

c)(i) Binding energy per nucleon = $\frac{B\,}{\text{Mass number}}$

= $\frac{28.6748}{4}MeV$
= 7.17MeV

ii) Binding energy per nucleon = $7.17 \times 10^{6} \times 1.6 \times 10^{-19}$
= 1.15×10^{-12}J

4.1.4: VARIATION OF B.E PER NUCLEON WITH MASS NUMBER



Main features of the graph

- Binding energy per nucleon for very small and large nuclides is small.
- A few peaks for low mass numbers are for lighter nuclei that are comparatively stable.
- The binding energy per nucleon increases sharply to a maximum at mass number 56
- For A> 56 binding energy per nucleon gradually decreases

NOTE

The low binding energy per nucleon value for small and high mass number nuclide implies that they are potential sources of nuclear energy because they easily undergo fusion and fission respectively.

4.1.5: Explanation of fusion and fission using the graph

- **During nuclear fusion** two light nuclei unite to form a heavier nucleus that has a higher binding energy per nucleon. However the total mass of the heavier nucleus is less the sum of the two light nuclei and the mass difference is accounted for by the energy released.
- **During Nuclear fission**, a heavy nucleus splits to form two lighter nuclei having greater binding energy per nucleon. However the total mass of the two daughter nuclei is less than the mass of the heavyt nucleus and the difference in mass is accounted for by the release of energy.

4.2.0: RADIO-ACTIVITY (RADIOACTIVE DECAY)

This is the spontaneous decay of a heavy nucleus to daughter nuclei with emission of $\propto$ particles, β-particle or γ-rays.

Heavy nuclides are generally unstable hence this decay is in attempt to reach a stable state.

Radio-activity is said to be a random process because no particular pattern is followed.

4.2.1: TYPES OF IONISING RADIATIONS

a) **Alpha particles ($\propto$)**

They have a mass of 4times that of hydrogen atom and a charge of +2e where e is the numerical charge on an electron hence they are Helium nuclei

$[{}^{4}_{2}\mathrm{He}\ ]$

Properties

- They have the least penetrating power among the ionizing radiations.
- They are positively charged hence can be deflected by electric and magnetic field
- They are the best ionizers of gases
- They have the shortest range in air among the ionizing radiations
- When emitted, they are emitted with the same speed

Note

When a nucleus undergoes $\propto -$ decay it loses four nucleons, two of which are protons, therefore atomic number Z decreases by two.

Thus if a nucleus X becomes a nucleus Y as a result of $\propto -$decay then.

$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$

(Parent) (Daughter) (∝ Particle)

$E.g$ Uranium – 238 decays by $\propto -$emission to thorium 234 according to

$$^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}^{4}_{2}\propto$$

b) Beta particle (β)

It is an electron which is moving at a high speed. It is represented as [$_{-1}^{0}e$]

Properties

- It has a higher penetrating power than ∝particle
- It is negatively charged hence deflected by electric and magnetic field.
- It is a moderate ionizer of gases
- It has a moderate range in air
- β particles are emitted by nuclei with various speeds
- It is lighter than $\propto -$particle

Note

β-particles are emitted by nuclei which have too many neutrons to be stable. To gain a stable state one of its neutrons should change into a proton and an electron, when this happens the electron is immediately emitted as a β-particle.

Thus when a nucleus undergoes β-decay, it's mass number A does not change and it's atomic number Z increases by one

$$^{A}_{Z}X \rightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e$$

(Parent) (Daughter) (β - Particle)

$E.g$ Carbon-14 decays by β-emission to nitrogen- 14 according to

$$^{14}_{6}C \rightarrow {}^{14}_{7}N + {}_{-1}^{0}e$$

c) Gamma rays (γ)

They are electromagnetic waves of very short wave length and they travel with a velocity of tight.

Properties

- They have the highest penetrating power
- They are electrically neutral hence they can't be deflected by electric or magnetic field
- They are the poorest ionizers of gases
- They can be diffracted and refracted

Note

Gamma ray decay involves the release of only energy without the change in atomic mass and atomic number $e.g$

$${}_{Z}^{A}X \rightarrow {}_{Z}^{A}Y \quad + \gamma$$

4.2.2: ENERGY OF DISINTEGRATION (Q-value)

It is necessary to investigate whether a given disintegration takes in energy (**endothermic**) or gives out energy **(Exothermic**) by considering the total atomic mass of reactant and products when the total mass of reactant is greater than the total mass of products then the reaction is **exothermic**.

Example

1: $\quad {}_{84}^{210}Po \rightarrow {}_{2}^{4}He + {}_{82}^{206}Pb$

Atomic mass of ${}_{82}^{206}Pb$ =205.969U

Atomic mass of ${}_{2}^{4}He$ = 4.003U

Atomic mass of ${}_{84}^{210}Po$ = 209.983U

i) State whether the disintegration is endothermic or exothermic and calculate the energy of disintegration.

ii) Calculate energy of the α −particle

Solution

Mass of reactant = 209.983U

Mass of product = 205.909U + 4.003U

= 209.972U

Since mass of reactant is greater than the total mass of products then its exothermic.

Therefore $\quad {}_{84}^{210}Po \rightarrow {}_{2}^{4}He + {}_{82}^{206}Pb + Q$

Energy of disintegration = mass defect x 931mev

= (209.983 – 209.972) x 931mev

= 0.011 x 931MeV

= 10.24MeV

Note Q-value appears as the kinetic energy of the products

$K.e_{\alpha} = \frac{M}{M+m_{\alpha}}Q$ where

m_{α} is the atomic mass of the α -pa rticle

M is atomic mass of daughter atom

$K.e_{\alpha} = \frac{206}{206+4}10.24$

$K.e_{\alpha}$ =10.05MeV

2. Consider the equation $^{206}_{82}Pb + Q \rightarrow {}^{4}_{2}He + {}^{202}_{80}Hg$

Atomic mass of Hg = 201.971U

Atomic mass of He = 4.003U

Atomic mass of Pb = 205.969

Calculate i) Q –value

ii) kinetic energy of the α-particle

Solution

i) Q = mass x 931MeV

Q = ((201.971 + 4.003) – 205.969) x 931mev

0.005 x 931MeV

Q-value = 4.66MeV

ii) $K.e_{\alpha}$= $\frac{M}{M+m_{\alpha}}Q$

$K.e_{\alpha}$ = $\frac{202}{202+4}4.66$

$K.e_{\alpha}$ = 4.57MeV

Generally, A nucleus would tend to be unstable and emit an α-particle, if the sum of the atomic masses of the products are together less than that of the nucleus and it would be stable if the sum of the atomic masses of the possible reaction products are together greater than the atomic mass of the nucleus.

EXERCISE 10

1. $^{210}_{84}Po$ decays to Pb-206 by emission of alpha – particle of single energy

(i) Write down the symbolic equation for the reaction

(ii) Calculate the energy in MeV released in each disintegration

(iii) Explain why this energy does not all appear as kinetic energy of the alpha particle.

(iv) Calculate the kinetic energy of the alpha particle

210P$_O$	=	209.93673U
206Pb	=	205.929421U
4He	=	4.001504U
1U	=	931MeV

An (5.40MeV, 5.3MeV)

2. Consider the decay process represented by

$^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$

Calculate the kinetic energy of the alpha particle

$^{226}_{88}Ra$	=	226.0254U
$^{222}_{86}Rn$	=	222.0175U

$^{4}_{2}He \quad = \quad$ 4.0026U

An (4.93MeV)

4.2.3: ARTIFICIAL DISINTEGRATION (Nuclear reaction)

This is achieved by bombarding the nuclei with an energetic particle.

The bombarding particle acquires enough energy by being accelerated in a reasonable speed by use of electric fields except the neutron.

$$Eg \quad ^{14}_{7}N + ^{4}_{2}He \rightarrow \ ^{17}_{2}O + ^{1}_{1}H$$

Examples of nuclear reactions

1. Photo disintegration

This is a nuclear reaction in which absorption of γ −ray results into decay of the absorbing nuclei.

$e.g \quad ^{25}_{12}Na + \gamma \rightarrow \ ^{24}_{11}Na \quad + ^{1}_{1}H$

2. Neutron radioactive capture

In this reaction the neutron is captured by the parent atom and a γ-ray is emitted

$$^{27}_{13}Al + ^{1}_{0}n \rightarrow \ ^{28}_{13}Al \quad + \gamma$$

This method is used for production of isotopes in nuclear reactions.

Beta particle as a bombarding particle

Advantage

- It can be accelerated at a high speed using electric field.

Disadvantages

- It experiences electrostatic repulsion with shell electrons
- It is light

Alpha particle as bombarding particle

Advantages

- It can be accelerated to high speed using electric field
- It is fairly heavy

Disadvantage

- It experiences electrostatic repulsion with positive nucleus

Neutron as a bombarding particle

- This is the best particle for study of nuclear reactions. Being electrically neutral it nether experiences electrostatic repulsion in the shell electrons nor the nucleus.
- However, it cannot be accelerated to high speeds using electric fields.

Energetic neutrons for nuclear reactions are obtained from nuclear reactants by the process of fusion.

4.2.4: NUCLEAR FISSION

Nuclear fission is the disintegration of a heavy nucleus into two lighter nuclei accompanied by release of energy..

Energy is released by the process because the average binding energy per nucleon of the fission products is greater than that of the parent.

$e.g \quad {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{148}_{57}La + {}^{85}_{35}Br + 3\,{}^{1}_{0}n + Q$

Example

Calculate the energy released by 1kg of ${}^{235}_{92}U$ under going fission according to

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{148}_{57}La + {}^{85}_{35}Br + 3\,{}^{1}_{0}n + Q$$

Mass 235U = 235.1U

Mass of 148La = 148.0U

Mass of ${}^{1}_{0}n$ = 1.009U

Mass of 85Br = 84.9U

Solution

Mass of reactants = 235.1 + 1.009

= 236.109U

Mass of products = (148.0 + 84.9 + (3 x 1.009))

= 235.927U

Energy released = mass defect x 931MeV

= (236.109 – 235.927) x 931MeV

= 169.442MeV

Energy released = $169.442 \times 10^{6} \times 1.6 \times 10^{-19}$J

= 2.71×10^{-11}J

This is the energy released by one atom of uranium – 235

235×10^{-3}kg contains = 6.02×10^{23} atom

1 kg contains = $\frac{6.02 \times 10^{23}}{235 \times 10^{-3}}$

= 2.562×10^{24} atom

One atom released = 2.71×10^{-11}J

2.562 x 1024 atoms = $2.71 \times 10^{11} \times 2.562 \times 10^{24}$J

= 6.943×10^{13}J

Energy released by 1kg of uranium = 6.943×10^{13}J

Exercise

1. Calculate the energy released when 10kg of ${}^{235}_{92}U$ undergoes fission according to;

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n$$

(mass of 235U = 235.04U, of 41Ba = 140.91U, of 92Kr = 91.91U of 1n = 1.01U and 1U = 931MeV, N_A = 6.02×10^{23}mol^{-1}) **An (7.36×10^{14}J or 4.77×10^{27}MeV)**

2. Atypical fission reaction is as below;

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{95}_{42}Mo \quad + {}^{139}_{35}La + 2\,{}^{1}_{0}n \quad + 7\,{}^{0}_{-1}e$$

Calculate the total energy released by one gram of uranium – 235 undergoing fission, neglect the masses of the electron

(mass of 1n = 1.009U, of 95Mo = 94.906U of 139La = 138.906U of 235U = 235.044U, 1U = 931MeV). **An (8.51×10^{10}J)**

Note

The neutrons released in fission can be slowed to a reasonable speed and made to bombard more uranium targets, emission of more neutrons which can still bombard more target thus leading to chain reaction. This is the basis of atomic bombs and nuclear reactants to produce energetic neutrons.

Application of fission

- In the production of neutrons
- In production of atomic bombs

Condition for fission

- It requires an energetic particle like a neutron

4.2.5: NUCLEAR FUSION

Nuclear fusion is the union of two light nuclei to form a heavier nucleus accompanied by release of energy.

Energy is released in the process.

An example is the fusion of two deuterium nuclei to produce helium -

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He \quad + {}^{1}_{0}n$$

Examples

1. ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He \quad + {}^{1}_{0}n$

Calculate the amount of energy released by 2kg of Deuterium given
(2H = 2.015U, 1n = 1.009U, 3He = 3.017U)

Solution

Mass of reactant = 2.015 + 2.015
= 4.03U

Mass of products = 3.017 + 1.009
4.026U

Mass defect = 4.03 – 4.026
= 0.004U

Energy released = Mc^2
$= 0.004 \times 1.66 \times 10^{-27} \times (3 \times 10^8)^2$
$= 5.976 \times 10^{-13}$J

Energy released by 2 atoms of ^2_1H $= 5.976 \times 10^{-13}$J

Energy released by 1 atom of ^2_1H $= \frac{5.976 \text{ x } 10^{-13}}{2}$

Energy released by 1 atom ^2_1H $= 2.988 \times 10^{-13}$J

2g contains = 6.02×10^{23} atoms

2×10^{-3}kg contains = 6.02×10^{23} atoms

2kg contains = $\frac{= 6.02 \text{ x } 10^{23}}{2x10^{-3}} x2$
$= 6.02 \times 10^{26}$ atoms

1 atom of ^2_1H $= 2.988 \times 10^{-13}$J

6.02×10^{26} atoms $= 2.988 \times 10^{-13} \times 6.02 \times 10^{26}$
$= 1.799 \times 10^{14}$J

Energy released by 2kg = 1.799×10^{14}J

EXERCISE

$^2_1H + ^3_1H \rightarrow ^4_2He \quad +^1_0n$

How much Energy in Joule is released
(mass of 2H = 3.345×10^{-27}kg, of 3H = 5.008×10^{-27}kg, of 4He = 6.647×10^{-27}kg of 1n = 1.675×10^{-27}kg
c = $3 \times 10^8 ms^{-1}$) **An (2.79×10^{-12}J)**

Condition for fusion

- High temperatures (in excess of 10^8K) are required to provide the nuclei which are to fuse with the energy needed to overcome their mutual electrostatic repulsion.

Note

- Fusion is the basis of hydrogen bond
- Solar energy is produced by the process of fusion.

4.2.6: DETECTION OF IONISING RADIATIONS

In a detector energy transferred from the radiation to the atoms of the detector may cause

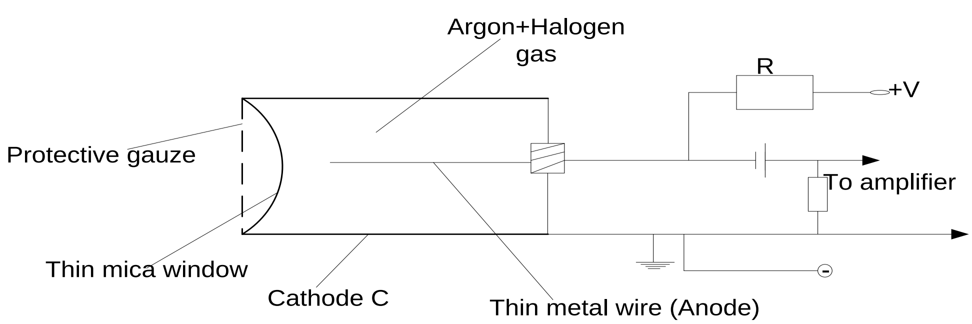
- Ionization of a gas (as in ionization chamber, cloud chamber, G.M tube)
- Fluorescence of a phosphor
- Exposure of a photographic emulsion

The following instruments are used for detecting radiation from a radioactive source.

1. Ionization chambers
2. Scintillation counter
3. Bubble chamber
4. Cloud chamber (Wilson and Diffusion)
5. Solid state detector
6. Photographic plates
7. Electroscope

1. THE GEIGER – MULLER TUBE / (GM) TUBE

Gm tube is a very sensitive type of ionization chamber which can detect single ionizing events



Operation

- Ionising radiations enter the G.M tube through the thin mica window and ionise the argon gas atoms, ion pairs are formed
- The electrons move very fast to the anode due to a high p.d between cathode and anode while the positive ions drift to the cathode.
- Fast moving electrons collide with other argon molecules producing more ion pairs, an avalanche of electrons is obtained.

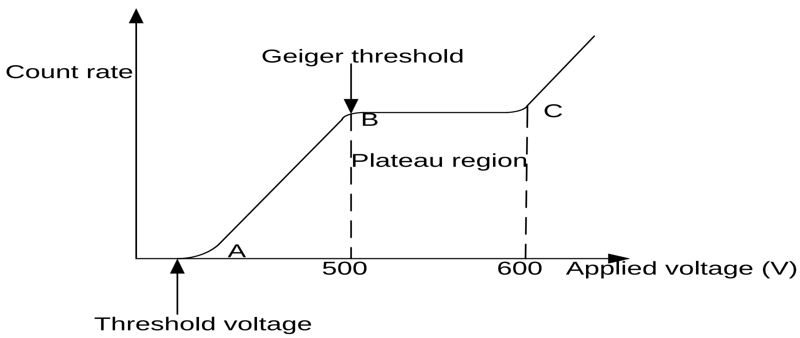
- ❖ When electrons reach anode, pulse of current is obtained.
- ❖ Voltage pulse is amplified and measure by scaler
- ❖ Positive ions move to cathode.
- ❖ Quenching agent is used to avoid secondary emission of electrons by absorbing kinetic energy of positive ions

Note

- ➢ The comparatively heavy positive ions move relatively slowly towards the cathode and reaching after the avalanche has occurred. Positive ions have appreciable energy and would cause emission of electrons from the cathode by bombardment causing a second avalanche. This would cause the discharge to persist for some time upsetting the recording of other ionizing particles following the first.
- ➢ To prevent a second avalanche due to positive ions, a halogen gas ($e.g$ Bromine) is mixed with the argon gas to form **a quenching agent**.
- ➢ Bromine water acts as a quenching agent so as to prevent secondary electrons to be emitted from the cathode by the positive ions bombarding it.
- ➢ An avalanche is a large number of moving ionised particles created as a result of secondary ionisation due to collisions between ions and the gas atoms, when the ions are accelerated by a high enough p.d where each ionisation leads to the formation of more ions pairs which themselves cause further ionisation.
- ➢ Time taken by the positive ions to travel towards the cathode is called **dead time**.

The GM tube may be fitted to a variety of detectors for investigating activity of radioactive sources.
1 - Scalar – simply records the total number of pulses
2- Rate meter – records actual count rate $\left(\frac{dN}{dt}\right)$and the out put may be fed into a meter
3- Speaker –Amplifier – gives an audible single each time a particle is detected. The signal becomes a continuous crackle when the activity is high.

G.M tube characteristic curve

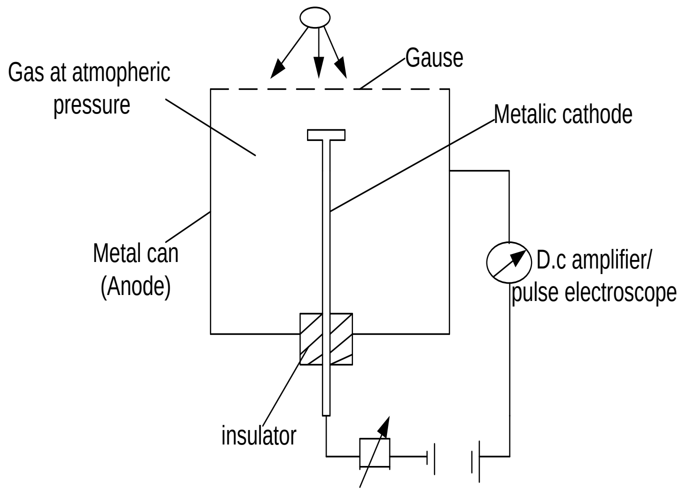


- Up to the threshold voltage no counts are recorded at all since the amount of electron amplification is not enough to give pulses of sufficient magnitude to be detected.
- Between A and B, the magnitude of pulse developed in the tube depends on the initial ionization which in turn depends on the energy of the incident ionizing particle. Only some of the freed electrons give pulses of sufficient magnitude to be recorded but their number increases with applied voltage.
- Between B and C (plateau region), the count rate is almost constant. A full avalanche is obtained along the entire length of the anode and all particles whatever their energy produce detectable pulses.
- Beyond C, the count rate increases rapidly with voltage due to incomplete quenching. One incident ionizing particle may start a whole train of pulses.

Note:

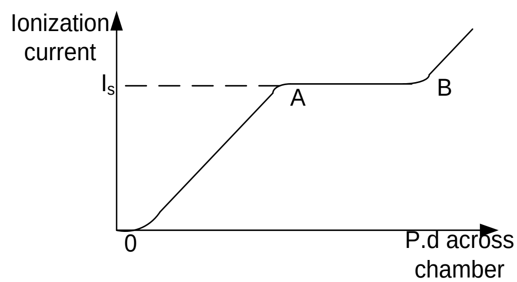
GM tubes should be operated in the plateau region (500 – 600V) preferably in the middle of the region. The sensitivity is then greatest and independent of supply voltage such that every particle that produced ionization is detected.

2. THE IONISATION CHAMBER



- The radiations enter through the thin wire gauze and ionises the gas molecules.
- The electrons move towards the anode and the positive ions towards the anode.
- Current is detected by an electrometer.
- The pulse per second (* count rate) gives a measure of the intensity of radiation.

Variation of the ionization current with $p.d$ (x-tic curve for ionization chamber)



- Between O and A, the $p.d$ is not large enough to draw all the electrons and positive ions to their respective electrodes. As the p.d increases more ions reach the electrode increasing the current.
- Between A and B, all the ions are attracted to their respective electrodes and there is no recombination. So the current reaches it's saturation value (Is) and remains constant as the p.d changes.
- Beyond B, the $p.d$ is large enough to cause secondary ionization. A point is reached when there is rapid multiplication of the ions in the chamber (gas amplification) thereby causing uncontrollable increase in the ionizing current.

Note:

(1) The $p.d$ at which an ionization is operated should be such that the ionization current has its saturation value. Under such condition;

(i) The ionization current is independent of fluctuations in supply voltage

(ii) The ionization current is proportional to the rate at which ionization is being produced in the chamber.

(2) Saturation current I_s is a measure of the rate of primary ionization.

$$\boxed{Is = ne}$$

Where e = 1.6 x10^{-19} C, n is the number of primary ion pair produced per second.

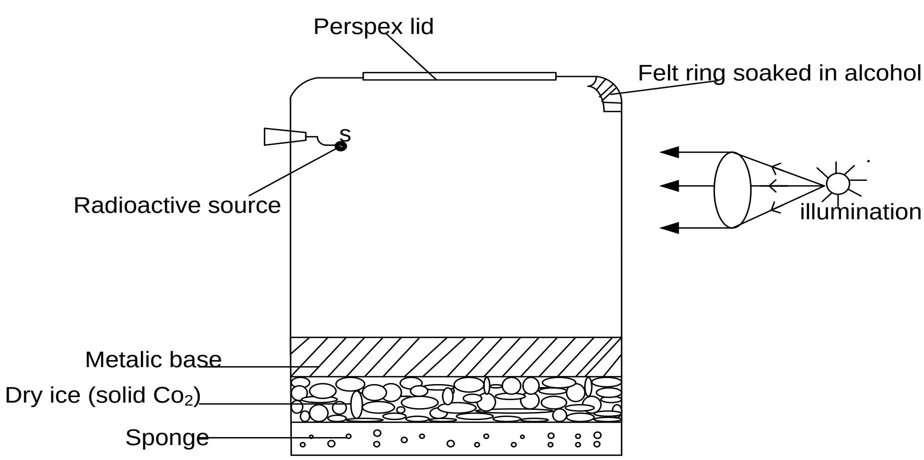
CLOUD CHAMBERS

In a cloud chamber a gas is cooled to a temperature slightly below its condensation point. It is said to be super cooled/super saturated. The gas molecules begin to condense on any ionized molecules present in the chamber. When a radiation(ionizing) particle catering the chamber passes through the saturated vapour /super cooled gas, the ions produced serve as centers on which tiny bubbles/ drop lets form (vapor condenses) a line of droplets/bubbles is formed along the track of the particles. Light scatters more from these bubbles/droplets than from the gas background when suitably illuminated. A photograph of the chambers at the right moment reveals the track of the particles. The length of the lines is proportional to particle energy.

The cloud chamber is used to show tracks of the radioactive particles rather than to measure the intensity of the cloud chambers are;

Expansion Wilson cloud chamber and Diffusion cloud chamber

3. DIFFUSION CLOUD CHAMBER



Structure and operation

- From the diagram above the base of the chamber is maintained at about -80^{0}C and the top is at room temperature so that there is a temperature gradient between the top and bottom.

- The air in the chamber is saturated with alcohol, where the vapour diffuses continuously from the top to the bottom and the air above the metal base becomes supersaturated.
- Then the radioactive particles cause ionisation of the air molecules
- The saturated vapour condenses on the ion formed producing tracks which can be seen by looking through the lid, hence radiation is detected.

4.3.0: THE RADIOACTIVE –DECAY LAW [N=$N_0e^{-\lambda t}$]

Radioactive decay is the spontaneous disintegration of unstable radioactive nuclei into more stable nuclei with emission of radiations.

The number of atoms delaying per second $\frac{dN}{dt} \propto N$

Where N is the number of un decayed atoms.

$\frac{dN}{dt} = -\lambda N$

Where $\lambda$is decay constant

$\frac{-dN}{dt} = \lambda N$

$$\boxed{A = -\lambda N}$$

Where A is activity or count rate per second the S.I unit for activity (A) is Becquel (Bq)

Definition

Activity is the number of decays per second. OR it is the number of radiations emitted per second.

Decay constant is the fraction of radioactive atoms which decay per second.

Given that No = number of radioactive atoms present at time t =0 (initially)

N = number of un decayed atoms after time t

From $\frac{dN}{dt} = -\lambda N$

$\int_{N0}^{N}\frac{dN}{N} = \int_0^t -\lambda d$

$[InN]_{N0}^{N} = [-\lambda t]_0^t$

$InN - InN0 = -\lambda t$

$In\left(\frac{N}{N0}\right) = -\lambda t$

$\frac{N}{N0} = e^{-\lambda t}$

$$\boxed{N = N_0e^{-\lambda t}}$$

4.3.1:HALF LIFE [$t_{\frac{1}{2}}$]

Half life of a radioactive element is the time taken for half of the atoms to decay

If N_0 is the number of original atoms

at t = $t_{\frac{1}{2}}$, N = $\frac{N_0}{2}$

From $N = N_0e^{-\lambda t}$

$$\frac{N_o}{2} = N_oe^{-\lambda t_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

Taking logs to base e on both sides

$$In(\frac{1}{2}) = Ine^{-\lambda t_{1/2}}$$

$$In\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$

$$t_{1/2} = \frac{-In(1/2)}{\lambda} = \frac{In2}{\lambda}$$

$$t_{1/2} = \frac{0.693}{\lambda}$$

$$\boxed{\lambda = \frac{0.693}{t_{\frac{1}{2}}}}$$

Note: Activity A at any one given time t is given **by** $\boldsymbol{A = A_0 e^{-\lambda t}}$

Examples

1. A sample of a radioactive material contains 10^{18} atoms. The half life of the material is 2.0days. Calculate
   (i) The fraction remaining after 5.0days
   (ii) The activity of the sample after 5.0days

Solution

$i)$ $t_{1/2} = 2\ days, t = 5\ days,$

$N_0 = 10^{18} atoms$

But $\lambda = \frac{0.693}{t_{1/2}}$

$$\lambda = \frac{0.693}{2} day^{-1}$$

But N = $N_0\ e^{-\lambda t}$

$$\frac{N}{N_0} = e^{\frac{-0.693}{2}x5}$$

$$\frac{N}{N_0} = 0.1768$$

Fraction remaining after 5 days = 0.1768

ii)Activity $\frac{dN}{dt} = \lambda N$

$$\lambda = \frac{0.693}{2x24x60x60} s^{-1}$$

$$\lambda = 4.0104x10^{-06} s^{-1}$$

$$\frac{dN}{dt} = \lambda N$$

But from $\frac{N}{N_0} = 0.1768$

N =0.1768N_0

$$\frac{dN}{dt} = 4.0104x10^{-06} x0.1768x10^{10}$$

$$= 7.09x10^{11} Bq$$

$$activity\ after\ 5\ days = 7.09x10^{11} Bq$$

2. Potassium ${}^{44}_{19}K$ has half life of 20 minutes and decays to form ${}^{14}_{20}Ca,$ a stable isotope of calcium
   i) How many atoms would there be in 10mg sample of potassium -44
   ii) What would be the activity of the sample?
   iii) What would be the activity be after one hour
   iv) What would the ratio of potassium atoms to calcium atoms be after one hour [N_A = 6.0x10^{23}mol^{-1}]

Solution

i) 44g of potassium =6 x 10^{23} atoms

10x10^{-3}g of potassium =$\frac{6x10^{23}}{44} x10x10^{-3} g$

= 1.364x10^{20}atoms

10mg of potassium - 44 has .364x10^{20}atoms

ii) A = $-\lambda N$

$A = \frac{0.693}{t_{1/2}}\ x\ 1.364x10^{20}$

$A = \frac{0.693}{20x60}\ x\ 1.364x10^{20}$

A = 7.88×10^{16}Bq

Activity of the sample = 7.88×10^{16}Bq

iii) When t = 1 hour

t = 3600s

$N = N_0e^{-\lambda t}$

N = $1.364 \times 10^{20} e^{\frac{-0693}{20x60}x3600}$

N = 1.706×10^{19} atoms

Number of atoms remaining after 1hour = 1.706×10^{19}atoms

But A = -λN

$A = \frac{0.693}{20x60}x1.706x10^{19}$

= 9.85×10^{15}Bq

Activity after one hour = 9.85×10^{15}Bq

iv) Let N_K = number of potassium atoms present after time t

N_C= number of Calcium atoms present after time t

Then N_K + N_C = Number of potassium atoms present initially

From $N = N_0e^{-\lambda t}$

N_K = (N_K + N_C)$e^{--\lambda t}$

$$\frac{N_k}{N_k + N_c} = e^{-\frac{In2}{20x60}x3600}$$

$$\frac{N_k}{N_k + N_c} = \frac{1}{8}$$

8N_K = N_K +N_C

7N_K = N_C

$$\frac{N_k}{N_c} = \frac{1}{7}$$

N_K : N_C = 1:7

Ratio would be = 1:7

3. An isotope of krypton ${}^{87}_{36}Kr$ has a half-life of 78 minutes. Calculate the activity of 10μg Of ${}^{87}_{36}kr$ [N_A =6×10^{23}mol^{-1}]

Solution

87 of ${}^{87}_{36}kr$ contains 6×10^{23}atoms

10×10^{-6}g of ${}^{87}_{36}kr$ contains $\frac{6x10^{23}}{87}x10x10^{-6}$

=6.9×10^{16}atoms

But $\frac{dN}{dt} = \lambda N$

= $\frac{In2}{78x60}x6.9x10^{16}$

= 1.022×10^{13}Bq

4. A sample of radioactive waste has a half-life of 80 years. How long will it take for its activity to fall to 20% of its current value

Solution

A = $\frac{20}{100}A_o$ but $\boldsymbol{A = A_0e^{-\lambda t}}$

$$\frac{20}{100}A_o = A_oe^{\left(\frac{-In2}{80}t\right)}$$

In(0.2) = $-t\left(\frac{In2}{80}\right)$

t = $-80\frac{In0.2}{In2}$

t= 185.75 years

it will take ≈ 186 years

5. A sample of radioactive material has an activity $9x10^{12}$Bq. The material has half life of 80s. how long will it take for the activity to fall to $2x10^{12}$Bq

Solution

$A_o = 9x10^{12}$Bq, $A = 2x10^{12}$Bq, t =?,

$t_{1/2}$ = 80

$$A = A_0e^{-\lambda t}$$

$2x10^{12} = 9x10^{12}e^{\left(\frac{In2}{80}t\right)}$

$\frac{2}{9} = e^{\left(\frac{In2}{80}t\right)}$

$In\left(\frac{2}{9}\right) = \frac{-t/n2}{80}$

t = $\frac{-80In(2/9)}{In2}$

t= 173.594

Time taken = 174s

6. A radioactive source contains $1.0\mu g$ of plutonium of mass number 239. If the source emits 2300 alpha particles per second. Calculate the half life of plutonium, assume [$N = N_0e^{-\lambda t}$]

Solution

239g of plutonium contains= $6.02x10^{23}$

$1x10^{-6}$g of plutonium contains= $\frac{6.02x10^{23}}{2.39}x10^{-6}$

= 2.519 x 10^{15}atoms

Since it emits 2300 alpha particles per second, then

$A = 2300s^{-1}$

A = -λN

2300 = λx$2.19x10^{15}$

2300 = $\left(\frac{In2}{t_{1/2}}\right)x2.519x10^{15}$

$t_{1/2} = \frac{2.519x10^{15}}{2300}N$

$$t_{1/2} = 7.591x10^{11}s$$

7. What mass of radium -227 would have an activity of $1x10^6$Bq. The half life of radium-227 is 41minutes ($N_A=6X10^{23}mol^{-1}$)

Solution

$t_{1/2} = 41minutes$ But A = -λN

$1x10^6 = \left(\frac{In2}{41x60}\right)N$

N = $3.55x10^9$atoms

But $6x10^{23}$atoms contains 227g

$3.55x10^9$atoms will contain

$\frac{227}{6x10^{23}}x3.55x10^9$

= $1.34x10^{-12}$g

8. A radioactive source has a half life of 20s and an initial activity of $7x10^{12}$Bq. Calculate its activity after 50s have elapsed

Solution

$t_{1/2}$ = 20s, t= 50s $A_0 = 7x10^{12}$

$$A = A_0e^{-\lambda t}$$

A = $7x10^{12}e^{\frac{-In2}{20}x50}$

A = 1.24x10^{12}Bq

9. The half-life of a particular radioactive material is 10minutes, determine what fraction of a sample of the material will decay in 30 minutes.

Solution

$t_{1/2}$ = 10minutes, t= 30minutes

using $N = N_0 e^{-\lambda t}$

$$\frac{N}{N_o} = e^{\frac{-In2}{20}x30}$$

$$\frac{N}{N_o} = \frac{1}{8}$$

The fraction remaining = $\frac{1}{8}$

The fraction that has decayed = 1-$\frac{1}{8}$

= $\frac{7}{8}$

10. Find the activity of 1g sample of radius $^{226}_{88}Ra$ whose half-life is 1620years

Solution

226g of $^{226}_{88}Ra$ contains 6.02x10^{23}atoms

1g of $^{226}_{88}Ra$ contains $\frac{6.02x10^{23}}{226}atoms$ = 2.664x10^{21}atoms

But A = -λN

A = $\frac{In2}{0.693}N$, A = $\frac{In2}{t_{1/2}}N$

A = $\left(\frac{In2}{1620x365x24x3600}\right)x2.664x10^{21}$

A = 3.61x$10^{10}$$s^{-1}$

Activity = 3.6x10^{10}Bq

11. A small volume of a solution which contains a radioactive isotope of sodium had an activity of 12000 disintegration per minute when it was injected into a blood stream of a patient. After 30 hours, the activity of 1.0cm^3 of the blood was found to be 0.50 disintegration per minute. If the half life of the sodium isotope is taken as 15 hours, estimate the volume of blood in a patient

Solution

At t = 0, activity A_0 = 12000min-^{1}

T= 15 (half life) A = 6000min^{-1}

T= 30 A = 3000min^{-1}

Total activity in the blood stream = 3000min^{-1}

$$Total\ volume\ of\ blood\ = \frac{blood\ in\ the\ blood\ stream}{activity\ in\ 1cm^3}$$

$$= \frac{3000}{0.5}$$

$$= 6000cm^3$$

$$Therefore\ volume\ of\ blood\ in\ a\ patient = 6litres$$

Calculation on ionization chamber

1. If 32eV is required to produce an ion-pair in air, calculate the current produced when an alpha particle per second from a radium source is stopped inside an ionization chamber, the energy of alpha particles from a radium source is 4.8MeV

Solution

32eV produces one ion pair

$4.8x10^6$eV will produce = $\frac{1}{32}x4.8x10^6$

= $1.5x10^5$ ion pairs

But I = ne

I = $1.5x10^5x1.6x10^{-19}$

I = $2.4x10^{-14}$A

2. A radioactive source emits $2x10^5$ alpha particles per second. The particles produce a saturated current of $1.1x10^{-8}$ in an ionization chamber. If the energy required to produce an ion pair is 32eV. Determine the energy in MeV of an alpha particle emitted by the source.

Solution

From I= ne

$$\frac{1.1x10^{-8}}{1.6x10^{-19}} = n$$

n= $6.875x10^{10}$ ion pairs

One ion pair produces 32eV

$6.875x10^{10}$ ion pairs will produce

$6.875x10^{10}$x32eV

= $2.2x10^{12}$eV

Energy of an alpha particle = $\frac{total\ energy}{no\ of\ \alpha-particle}$

$= \frac{1.1x10^{12}}{10^5}eV$

= $1.1x10^7$eV

= $\frac{1.1x10^7}{10^6}MeV$

Energy of an α-particle = 11MeV

3. A radioactive source produces alpha particles each of energy 60MeV. If 20% of the alpha particles enter the ionization chamber, a current of 0.2μA flows. Find the activity of the alpha source, if the energy needed to make an ion pair in the chamber is 32MeV.

Solution

I = ne

$$\frac{0.2x10^{-6}}{1.6x10^{-19}} = n$$

n = $1.25x10^{12}$ ion pairs

one ion pair requires 32MeV

$1.25x10^{12}$ ion pairs will require

$32x1.25x10^{12}$

= $4x10^{13}$MeV

Energy of an alpha particle = $\frac{total\ energy}{no\ of\ \alpha-particle}$

$$60 = \frac{32x1.25x10^{12}}{number\ of\ alpha\ particles}$$

Number of alpha particles = $\frac{32x1.25x10^{12}}{60}$

= $6.667x10^{11}$ alpha particles

If A is the activity then

Number of particles = $\frac{20}{100}A$

$6.667x10^{11}$ = $\frac{20}{100}A$

A = $3.33x10^{12}$s

Examples on carbon dating

1. Wood from a buried ship was forced to have a specific activity of 1.2x10^{2}Bqkg^{-1} due to ^{14}C whereas a comparable living wood has a specific activity of 2x10^{2}Bqkg^{-1} What is the age of the ship? [half life of ^{14}C = 5.7x10^{3}years]

Solution

A_o = 2x10^{2}BqKg^{-1},

A = 1.2x10^{2}BqKg^{-1}

$A = A_0e^{-\lambda t}$

1.2x10^2 = 2x10^2$e^{-\frac{In2}{t_{1/2}}t}$

$$\frac{1.2}{2} = e^{-\frac{In2}{t_{1/2}}t}$$

$$In(\frac{1.2}{2}) = t\frac{-In2}{(5.7x10^3)}$$

$$t = 4.2x10^3 years$$

2. Archeological wood was found to have an activity of 20 units due to ^{14}C. Recent wood gave an activity of 47.8units, estimate the age of the wood [half life of ^{14}C = 5600years]

Solution

A_o = 47.8, A = 20

Using

$A = A_0e^{-\lambda t}$

20 = 47.8$e^{-\frac{In2}{5600}t}$

$$In(\frac{20}{47.8}) = t\frac{-In2}{(5600)}$$

t = 7.4x10^{3}years

3. A rock containing$^{238}_{92}U$. Decays to produce a stable isotope of $^{206}_{82}Pb$. Estimate the age of the rock if the ratio of $^{206}_{82}Pb$. to $^{238}_{92}U$. Is 0.6 [half life of $^{238}_{92}U$. = 4.5x109years]

Solution

Let N_u = number of uranium atoms present at time t

N_{Pb} = number of lead atoms present at time t

(N_u + N_{Pb}) = number of uranium atoms present initially

From N = . $N_0e^{-\lambda t}$

N_u = (N_u + N_{Pb}) $e^{\frac{-In2}{4.5x10^9}t}$

$\frac{N_u}{N_u+N_{Pb}}$ =$e^{\frac{-In2}{4.5x10^9}t}$

$$In(\frac{N_u}{N_u + N_{Pb}}) = \frac{-tIn2}{4.5x10^9}$$

$$In(\frac{N_u + N_{Pb}}{N_u}) = t\frac{In2}{4.5x10^9}$$

$In\left(1 + \frac{N_u}{N_{Pb}}\right)$ = $t\frac{In2}{4.5x10^9}$

$In(1 + 0.6)$ = $t\frac{In2}{4.5x10^9}$

t= 3.1x10^{9}years

USES RADIOACTIVITY

- Treatment of deep-lying tumors
- Measurement of thickness of metal sheet during manufacture

- Used to determine the exact position of underground pipes and allows leaks to be detected
- Radioactive phosphorous is used to asses the different abilities of plants to take up -phosphorous from different types of phosphate fertilizer
- Used in radioactive dating

Health hazard

The cells of the body may under go dangerous physical and chemical changes as a result of exposure to the radiation causing;

- Mutation (genetic changes)
- Cancer cells

Precautions

- Lead aprons should be worn when dealing with radiations
- Avoid unnecessary exposure to the radiations
- Delicate parts should not be exposed to the radiations

EXERCISE12

1. A certain $\alpha - part$icle the track in a cloud chamber has length of 37mm. Given that the average energy required to produce an ion pair in air is 5.2×10^{-18}J and that $\alpha - part$icles in air produce on average 5×10^{3} such airs per mm of track. Find the initial energy of the $\alpha - part$icle . Express your answer in MeV [e=$1.6 \times 10^{-19}C$]

   An(6.0MeV)

2. A radioactive source has a half-life of 20days. Calculate the activity of the source after 70days have elapsed if it's initial activity is 10^{10}Bq **An(8.8×10^{8}Bq)**

3. The radioactive isotope ${}^{218}_{84}P_o$ has a half life of 3minutes, emitting $\alpha - part$icles according to the equation;

$${}^{218}_{84}Po \rightarrow \alpha + {}^{x}_{y}Pb$$

   (i) What are the values of x and y
   (ii) If N atoms of ${}^{218}_{84}P_o$ emit $\alpha - part$icles at a rate of $5.12 \times 10^{-4}s^{-1}$, what will be the rate of emission after $^1/_2$ hour. **An($50s^{-1}$)**

4. An isotope of the element radon has a half life of 4days . A sample of radon originally contains 10^{10} atoms.[Take 1day to be 86x103s]

Calculate;

(i) The number of radon atoms remaining after 16days
(ii) The radioactive decay constant for radon
(iii) The rate of decay of the radon sample after 16days

An(6.3×10^{8}atoms, $2\times10^{-6}s^{-1}$, 1.3×10^{3}Bq)

5. The half life of ${}^{30}_{15}P$ is 2.5 minutes. Calculate the mass of ${}^{30}_{15}P$ which has an activity of 10^{15}Bq. ($N_A = 6.0\times10^{23}$ mol^{-1}]
An[11μg]

6. The activity of a particular radioactive nuclide falls from 1×10^{11}Bq to 2×10^{10}Bq in 10 hours, calculate the half life of the nuclide [**An 4.3hours]**

7. Calculate the activity of 2μg of ${}^{64}_{29}Cu$. [half life of ${}^{64}_{29}Cu$ = 13hours, $N_A = 6.0\times10^{23}mol^{-1}$] **An [$2.8\times10^{11}$Bq**]

8. The radioactive isotope of iodine ${}^{131}I$ has a half life of 8 days and is used as a tracer in medicine, calculate;
   (a) The number of atoms of ${}^{131}I$ which must be present in the patient when she is tested to give a disintegration rate of $6\times10^{5}s^{-1}$
   (b) The number of atoms of ${}^{131}I$ which must have been present in a dose prepared 24 hours before. [**An 6.0×10^{11}, 6.5×10^{11}]**

9. The activity of a mass of ${}^{14}_{6}C$ is 5×10^{8}Bq and the half life is 5570 years. Estimate the number of ${}^{14}_{6}C$ nuclei present [In2 = 0.69] **[An 1.27×10^{20}]**

10. (a) What is meant by the decay constant ʎ and the half life $T_{1/2}$ for a radioactive isotope?.
   Show from first principles that $\lambda T_{1/2}$ = 0.69
   (b) At a certain time, two radioactive sources R and S contain the same number of radioactive nuclei. The half life is 2hours for R and 1 hour for S, calculate
   (i) The ratio of the rate of decay of R to that of S at this time
   (ii) The ratio of the rate of decay of R to that of S after 2 hours
   (iii) The proportion of the radioactive nuclei in S which have decayed in 2 hours

An [1:2, 1:1, 75%]

11. (a) The various isotopes of an element X are distinguished by using the notation $^{A}_{Z}X$. Explain the meaning of A, Z and of the term isotope

    (b) Radioactive sources which might be used in schools are ^{226}Ra which emits α, β, and γ-rays and ^{90}Sr which emits β-rays only

    (i) List three safety precautions which need to be taken into account when using such sources.

    (ii) The half-life of the ^{90}Sr is 28 years when its activity falls to 25% of its original value, it should be replaced. After how many years should it be replaced? **An [56 years]**

    (c) (i) When $^{226}_{88}Ra$ emits an α-particle, it decays to Radon (Rn). Write down a balanced equation for this change

    (ii) Radioactive isotopes have many applications merely by virtue of being isotopes, describe and explain one such application

12. (a) In 420 days, the activity of a sample of polonium P_0, fell to one – eighth of its initial value. Calculate the half life of polonium

    (c) Give the numerical values of a, b, c, d, d, e, f, in the nuclear equation
    $$^{a}_{b}Po \rightarrow \, ^{c}_{d}\alpha + \, ^{206}_{82}Pb + \, ^{e}_{f}\gamma$$

    An[140days, a = 210, b = 84, c = 4, d = 2, e = 0, f = 0]

13. A steel piston ring of mass 16g was irradiated with neutrons until its activity due to the formation of an isotope of iron was 10micro curie. Ten days later after the irradiation, the ring was installed in an engine and after 80 days of continuous use, the crankcase oil was found to have a total activity of 1.65×10^{3} disintegrations per second. Determine the average mass of iron worn off the ring per day, assuming that all the metal removed from the ring accumulated in the oil and that one curie is equivalent to 3.7×10^{10} disintegration per second.
    [half life of the isotope of iron = 45days] **An[4.0mg per day]**

14. A tube containing an isotope of radon, $^{222}_{86}Rn$ is to be implanted in a patient. The radon has an initial activity of 1.6×10^{4}Bq, a half life of 4 days and it delays by a

alpha emission. To provide the correct dose, the tube, containing a freshly for 8 days

(a) What are the protons and nucleon number of the daughter nucleus produced by the daughter of the radon?

(b) Determine;

(i) The decay constant for radon in S^{-1}

(ii) The initial number of radioactive radon atoms in the tube

An[$2.0 \times 10^{-6} s^{-1}$, 8.0×10^{9}]

15. At the start of an experiment a mixture of radioactive materials contain $20\mu g$ of a radio isotope A, which has a half-life of 70S and $40\mu g$ of radio isotopes β has a half life of 35s

(i) After what period of time will the mixture contain equal masses of each isotope. What is the mass of each isotope at this time?

(ii) Calculate the rate at which the atoms of isotope A are delaying when the masses are the same [molar mass of isotope A = 234g, $N_A = 6 \times 10^{23} mol^{-1}$

An[70s, $10\mu g$, $2.5 \times 10^{14} s^{-1}$]

16. The isotope of bismuth of mass number 200 has a half life of 5.4×10^{3}s. It emits alpha particles with an energy of 8.2×10^{-13}J.

(a) State the meaning of the term half life

(b) Calculate for this isotope;

(i) Decay constant

(ii) The initial activity of 1×10^{-6} mole of the isotope

(iii) the initial power output of this quantity of the isotope

[$N_A = 6 \times 10^{23} mol^{-1}$]

[Hint, power = activity x Energy] [An $1.3 \times 10^{-4} s^{-1}$, $7.7 \times 10^{13} S^{-1}$, 63W]

17. The radioactive isotope ^{60}Co decays to ^{60}Ni which spontaneously decays to give two gamma-ray photons, the half life of ^{60}Co is 5.27years.

(i) find the activity of 20g of ^{60}Co

(ii) estimate the power obtainable from 20g of ^{60}Co

[Mass of ^{60}Co= 59.93381U, mass of ^{60}Ni = 59.93079U] **[An $8.35 \times 10^{14} s^{-1}$, $3.76 \times 10^{2} Js^{-1}$]**

UNEB 2013 Q.10

(d) **(i)** What is a **decay constant** (01mark)

(ii) A sample from fresh wood of a certain species of tree has an activity of 16.0 counts per minute per gram. However, the activity of 5g of dead wood of the same species of tree is 10.0 counts per minute. Calculate the age of the dead wood (Assume half-life of 5730years) **An($1.72x10^4$years)** (04 marks)

UNEB 2012 Q9

a)(i) What is meant by the terms radioactive decay, half life and decay constant.

(ii)Show that the half life $t_{1/2}$ of a radio isotope is given by $t_{1/2} = \frac{0.693}{\lambda}$

Where λ is the decay constant [assume the decay law $N = N_o e^{-\lambda t}$] [03 marks]

(b) With the aid of a labeled diagram, describe the structure and action of a cloud chamber (05 marks)

(c). A radioactive isotope $^{99}_{43}X$ decays by emission of a gamma ray. The half life of the isotope is 360 minutes. What is the activity of 1mg of the isotope (06 marks)

[An $1.95x10^{14}$Bq]

d. Explain the term avalanche as applied to an ionization chamber (03 marks)

UNEB 2011 Q10

a)What is meant by unified atomic mass unit (1 mark)

b)(i) Distinguish between nuclear fission and nuclear fusion (2 marks)

ii) State the condition necessary for each of the nuclear reactions in b(i) to occur

c)(i) With the aid of a labeled diagram, describe the operation of an ionization chamber (6 marks)

ii)Sketch the curve of ionization current against applied p.d and explain its main features (4 marks)

d)A typical nuclear reaction is given by $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{95}_{42}Mo + ^{139}_{57}La + 2^{1}_{0}n + 7^{0}_{-1}e$

Calculate the total energy released by 1g of uranium

mass of $^{1}_{0}n$ = 1.009U of $^{0}_{-1}e$ = 0.00055U

$^{95}_{92}Mo = 94.906U\ of\ ^{139}_{57}La = 138.906U$

$^{235}_{92}U = 235.044U \quad 1U = 1.66x10^{-27}kg$

$\boldsymbol{Ans}$ **[$8.387x10^{10}J$]**

UNEB 2010 Q 10

a)(i) What is meant by mass defect? (1 mark)

(ii) Sketch a graph showing how binding energy per nucleon varies with mass number and explain its main features (3 marks)

iii)Find the binding energy per nucleon of $^{56}_{26}Fe$ given that mass of 1proton = 1.007825U. Mass of 1neutron=1.008665U, [1U = 931MeV] **[Ans 7.7Mev]**

b)With the aid of a diagram, explain how an ionization chamber works (6 marks)

UNEB 2008 Q9

a)(i) Define the term binding energy (1 mark)

(ii) Sketch a graph showing the variation of binding energy per nucleon with mass number (2 marks)

(iii) Use the sketch graph you have drawn in a(ii) to explain how energy is released during fission and fusion (3 marks)

b)Explain why high temperature is required during fusion of nuclides (1 mark)

c)The isotope $^{238}_{92}U$ emits an alpha particle and forms an isotope of thorium (Th) while the isotope $^{235}_{92}U$ when bombarded by a neutron, forms $^{144}_{56}Ba$, $^{90}_{36}Kr$ and neutrons

i) Write the nuclear equations for the reactions of $^{238}_{92}U$ and $^{235}_{92}U$ (2 marks)

ii) How does the reaction of $^{235}_{92}U$ differ from that of $^{238}_{92}U$ (3 marks)

d)A steel piston ring contains 15g of radioactive iron, $^{54}_{26}Fe$. The activity of $^{54}_{26}Fe$. Is 3.7×10^5 disintegrations per second. After 100 days of continuous use, the crank case oil was found to have a total activity of 1.23×10^3 disintegration per second. Find the;

i)Half life of $^{54}_{26}Fe$ ` (5 marks)

ii)Average mass of iron worn off the ring per day, assuming that all the metal removed from the ring accumulates in the oil. **An[3.13×10^{17}s, 4.9×10^{-4}g]**

UNEB 2007 Q9

c)Explain the purpose of each of the following in a Geiger muller tube

i) A thin mica window

ii) Argon gas at low pressure

iii)Halogen gas mixed with argon gas

iv)An anode in the form of a wire (4 marks)

d)(i) What is meant by binding energy per nucleon of a nucleus (1 mark)

ii)Sketch a graph of binding energy per nucleon against mass number for naturally occurring nuclides (1 mark)

iii)State one similarity between nuclear fusion and nuclear fission (1 mark)

e)(i) At a certain time, an α-particle detector registers account rate of $32s^{-1}$. Exactly 10 days later the count rate dropped to $8s^{-1}$. Find the decay constant. (4 marks)

[Ans 0.139 per day]

ii)State two industrial uses and two health hazards of radioactivity (2 marks)

UNEB 2006Q10

a)i) What is meant by half life of a radioactive material (1 mark)

ii) Given the radioactive law $N_t = N_o e^{-\lambda t}$, obtain the relation between λ and half life $T_{1/2}$

iii) What are radio isotopes (1 mark)

iv) The radio isotope $^{90}_{38}Sr$ decays by emission of β-particles. The half life of the radio isotope is 28.8years, determine the activity of 1g of the isotope (5 marks)

An[$5.1 \times 10^{12} s^{-1}$]

c) i) With aid of a diagram, describe the structure and action of a Geiger Muller tube (06 marks)

ii) Sketch the count rate –voltage characteristic of the Geiger muller tube and explain it's main features

(3mk)

(iii) I identify, giving reasons, the suitable range in (b)(ii) of operation of the tube(2mk)

UNEB 2005 Q10

a)Define Binding energy of nuclide (1mk)

b)i) Sketch a graph showing how binding energy per nucleon varies with mass number (1mk)

(ii) Describe the main features of the graph in (b)(i) (3 marks)

c)Distinguish between nuclear fission and nuclear fusion; and account for the energy released. (3 marks)

(i) With the aid of a labeled diagram, the working of the Geiger-Muller tube(5 marks)

(ii) How would you use a Geiger-Muller tube to determine the half life of a radioactive sample (4 marks)

UNEB 2004 Q10

b)Describe with the aid of a labeled diagram the structure and action of diffusion cloud chamber (6 marks)

c)i) Define the terms radio activity and half life of radioactive substance (2 marks)

(ii) A radioactive isotope of strontium of mass $5\mu g$ has half-life of 28 years, find the mass of the isotope left after 14years. **An[$3.54\mu g$]**

UNEB 2003 Q10

a)What is meant by the following terms

i)Nuclear number

ii)Binding energy (2mk)

b)Calculate the energy released during the decay of ${}^{220}_{86}Rn$ nucleus into ${}^{216}_{84}Po$ and an alpha-particle

Mass of ${}^{220}_{86}Ra$ = 219.964176U

Mass of ${}^{216}_{84}Po$ = 215.955794U

Mass of ${}^{216}_{84}Po$ = 4.001566U

(1U = 931MeV) **Ans**

[6.35MeV]

UNEB 2002 Q10

a)What is meant by

i)Half life of a radioactive element (1mk)

ii)Nuclear fission (1mk)

iii)Nuclear fusion (1mk)

b)An atom of ${}^{222}Ra$ emits an alpha-particle of energy 5.3MeV. Given that the half life of ${}^{222}Ra$ is 3.8 days, use the decay law to calculate the

i)Decay constant (3mk)

ii)Amount of energy released by 3.0x10^{-9}kg of ${}^{222}Ra$ after 3.8 days(5mk)

An[2.11x$10^{-6}$$S^{-1}$,2.16x$10^{16}$MeV]

UNEB 2001 Q9

a)What is meant by the following

i)An alpha particle (1mk)

ii)Radioactivity (1mk)

a)Describe the structure and actions of a cloud chamber (6 marks)

d)State four uses of radioactive isotopes (2 marks)

UNEB 2000 Q9

a)i) Define the term half life and decay constant as applied to radio activity (2 marks)

(ii) State the relationship between half life and delay constant (1 mark)

b)The radio isotope ${}^{60}Co$ decays by emission of a β-particles and γ-rays. Its half-life is 5.3years

i)Find the activity of a source containing 0.1g of ${}^{60}Co$

ii)In what ways do γ-rays differ from β-particles? **[Ans 4.15x$10^{12}$$s^{-1}$]**

c)i) What is meant by mass defect in nuclear physics (1 mark)

ii)Calculate the mass defect of ${}^{59}_{26}Fe$. Given the following information.

Mass of ${}^{59}_{26}Fe$ nucleus = 58.93488u

Mass of proton = 1.00728u

Mass of neutron = 1.00867u **Ans [0.54051U]** (4 marks)

d)Describe the structure and action an ionization chamber.